Computer Vision 2
WS 2018/19

Part 3 – Template-based Tracking
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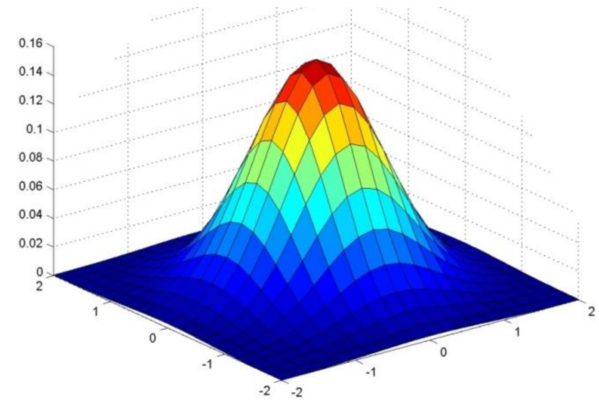
Course Outline

- Single-Object Tracking
  - Background modeling
  - Template based tracking
  - Tracking by online classification
  - Tracking-by-detection
- Bayesian Filtering
- Multi-Object Tracking
- Visual Odometry
- Visual SLAM & 3D Reconstruction
- Deep Learning for Video Analysis

Image source: Robert Collins

Recap: Gaussian Background Model

- Statistical model
  - Value of a pixel represents a measurement of the radiance of the first object intersected by the pixel's optical ray.
  - With a static background and static lighting, this value will be a constant affected by i.i.d. Gaussian noise.



- Idea
  - Model the background distribution of each pixel by a single Gaussian centered at the mean pixel value:

$$\mathcal{N}(\mathbf{x}|\boldsymbol{\mu},\boldsymbol{\Sigma}) = \frac{1}{(2\pi)^{D/2}|\boldsymbol{\Sigma}|^{1/2}} \exp\left\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right\}$$

  - Test if a newly observed pixel value has a high likelihood under this Gaussian model.

$\Rightarrow$ *Automatic estimation of a sensitivity threshold for each pixel.*

Recap: Stauffer-Grimson Background Model

- Idea
  - Model the distribution of each pixel by a mixture of K Gaussians

$$p(\mathbf{x}) = \sum_{k=1}^{K} \pi_k \mathcal{N}(\mathbf{x}_n | \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k) \quad \text{where} \quad \boldsymbol{\Sigma}_k = \sigma_k^2 \mathbf{I}$$

  - Check every new pixel value against the existing K components until a match is found (pixel value within $2.5\ \sigma_k$ of μ_k).
  - If a match is found, adapt the corresponding component.
  - Else, replace the least probable component by a distribution with the new value as its mean and an initially high variance and low prior weight.
  - Order the components by the value of w_k/σ_k and select the best B components as the background model, where

$$B = \arg\min_b \left(\sum_{k=1}^{b} \frac{w_k}{\sigma_k} > T \right)$$

[C. Stauffer, W.E.L. Grimson, CVPR’99]

Recap: Stauffer-Grimson Background Model

- Online adaptation
  - Instead of estimating the MoG using EM, use a simpler online adaptation, assigning each new value only to the matching component.
  - Let $M_{k,t} = 1$ iff component k is the model that matched, else 0.

$$\pi_k^{(t+1)} = (1-\alpha)\pi_k^{(t)} + \alpha M_{k,t}$$

  - Adapt only the parameters for the matching component

$$\boldsymbol{\mu}_k^{(t+1)} = (1-\rho)\boldsymbol{\mu}_k^{(t)} + \rho x^{(t+1)}$$

$$\boldsymbol{\Sigma}_k^{(t+1)} = (1-\rho)\boldsymbol{\Sigma}_k^{(t)} + \rho(x^{(t+1)} - \boldsymbol{\mu}_k^{(t+1)})(x^{(t+1)} - \boldsymbol{\mu}_k^{(t+1)})^T$$

where

$$\rho = \alpha\mathcal{N}(\mathbf{x}_n|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$$

(i.e., the update is weighted by the component likelihood)

[C. Stauffer, W.E.L. Grimson, CVPR'99]

Recap: Kernel Background Modeling

- Nonparametric density estimation
  - Estimate a pixel's background distribution using the kernel density estimator $K(\cdot)$ as

$$p(\mathbf{x}^{(t)}) = \frac{1}{N}\sum_{i=1}^{N} K(\mathbf{x}^{(t)} - \mathbf{x}^{(i)})$$

  - Choose K to be a Gaussian $\mathcal{N}(0, \mathbf{\Sigma})$ with $\mathbf{\Sigma} = \mathrm{diag}\{\sigma_j\}$. Then

$$p(\mathbf{x}^{(t)}) = \frac{1}{N}\sum_{i=1}^{N}\prod_{j=1}^{d}\frac{1}{\sqrt{2\pi\sigma_j^2}} e^{-\frac{1}{2}\frac{(x_j^{(t)} - x_j^{(i)})^2}{\sigma_j^2}}$$

  - A pixel is considered foreground if $p(\mathbf{x}^{(t)}) < \theta$ for a threshold θ.
    - This can be computed very fast using lookup tables for the kernel function values, since all inputs are discrete values.
    - Additional speedup: partial evaluation of the sum usually sufficient

[A. Elgammal, D. Harwood, L. Davis, ECCV'00]

Practical Issues: Background Model Update

- Kernel background model
  - Sample N intensity values taken over a window of W frames.
- FIFO update mechanism
  - Discard oldest sample.
  - Choose new sample randomly from each interval of length W/N frames.
- When should we update the distribution?
  - Selective update: add new sample only if it is classified as a background sample
  - Blind update: always add the new sample to the model.

Updating Strategies

- Selective update
  - Add new sample only if it is classified as a background sample.
  - Enhances detection of new objects, since the background model remains uncontaminated.
  - But: Any incorrect detection decision will result in persistent incorrect detections later.

  $\Rightarrow$ Deadlock situation.
- Blind update
  - Always add the new sample to the model.
  - Does not suffer from deadlock situations, since it does not involve any update decisions.
  - But: Allows intensity values that do not belong to the background to be added to the model.

  $\Rightarrow$ Leads to bad detection of the targets (more false negatives).

Solution: Combining the Two Models

- Short-term model
  - Recent model, adapts to changes quickly to allow very sensitive detection
  - Consists of the most recent N background sample values.
  - Updated using a selective update mechanism based on the detection mask from the final combination result.
- Long-term model
  - Captures a more stable representation of the scene background and adapts to changes slowly.
  - Consists of N samples taken from a much larger time window.
  - Updated using a blind update mechanism.
- Combination
  - Intersection of the two model outputs.

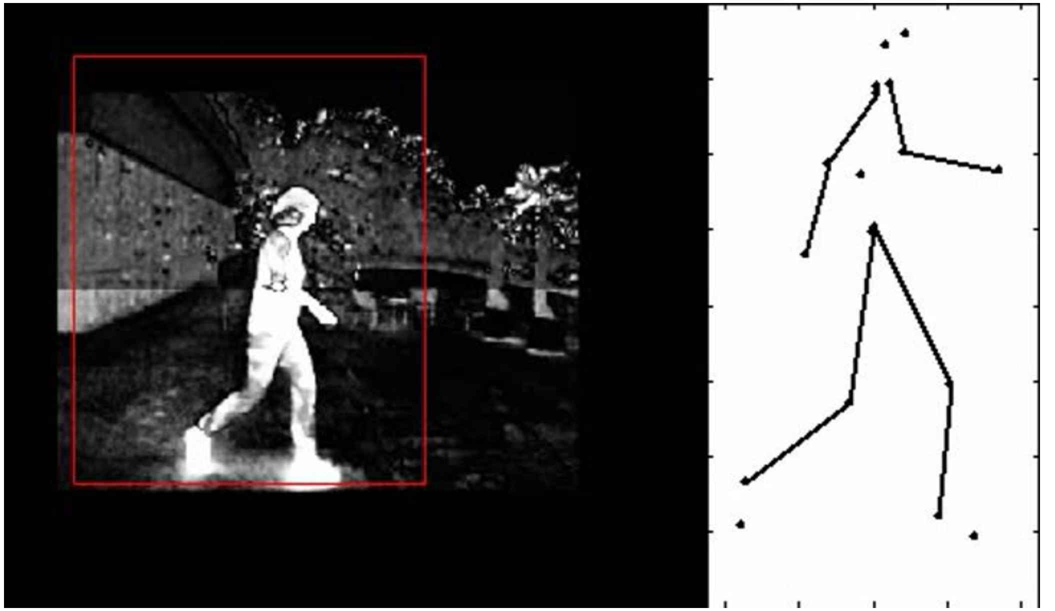
[A. Elgammal, D. Harwood, L. Davis, ECCV'00]

Applications: Visual Surveillance

- Background modeling to detect objects for tracking
  - Extension: Learning a foreground model for each object.

Video source: Ian Reid, Univ. of Oxford

Applications: Articulated Tracking



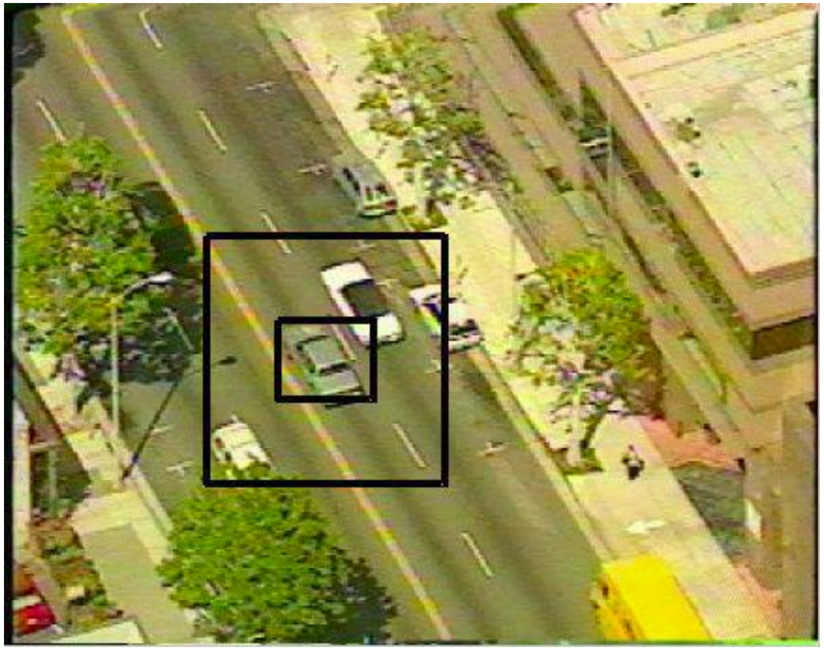
- Background modeling as preprocessing step
  - Track a person's location through the scene
  - Extract silhouette information from the foreground mask.
  - Perform body pose estimation based on this mask.

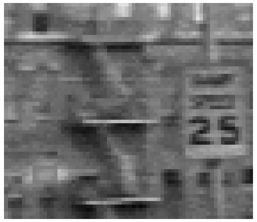
Video source: Hedvik Kjellstroem, Tobias Jaeggli

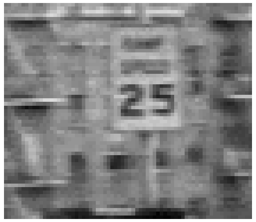
Summary

- Background Modeling
  - Fast and simple procedure to detect moving object in static camera footage.
  - Makes subsequent tracking *much* easier!
  - $\Rightarrow$ *If applicable, always make use of this information source!*
- We've looked at two models in detail
  - Adaptive MoG model (Stauffer-Grimson model)
  - Kernel background model (Elgammal et al.)
  - Both perform well in practice, have been used extensively.
- Many extensions available
  - Learning object-specific foreground color models
  - Background modeling for moving cameras

Today: Template based Tracking







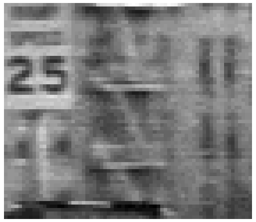
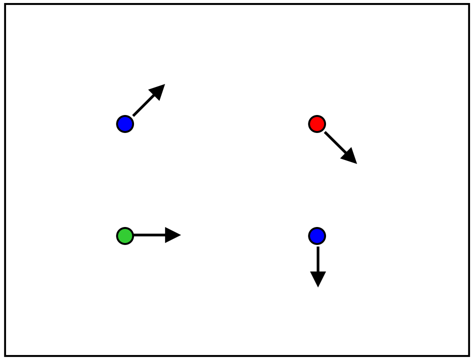


Image source: Robert Collins, Shi & Tomasi

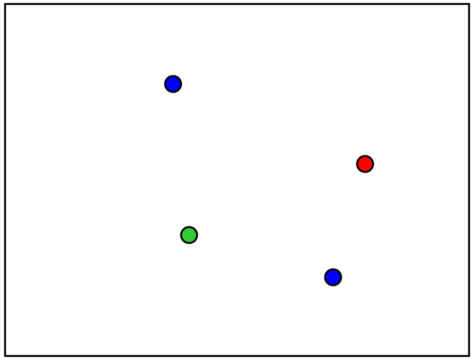
Topics of This Lecture

- Lucas-Kanade Optical Flow
  - Brightness Constancy constraint
  - LK flow estimation
  - Coarse-to-fine estimation
- Feature Tracking
  - KLT feature tracking
- Template Tracking
  - LK derivation for templates
  - Warping functions
  - General LK image registration
- Applications

Estimating Optical Flow



$I(x,y,t-1)$

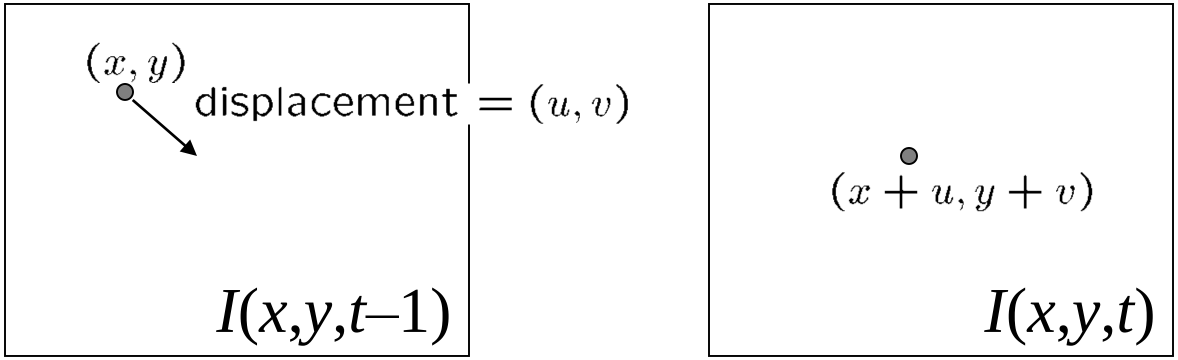


$I(x,y,t)$

- Optical Flow
  - Given two subsequent frames, estimate the apparent motion field $u(x,y)$ and $v(x,y)$ between them.
- Key assumptions
  - Brightness constancy: projection of the same point looks the same in every frame.
  - Small motion: points do not move very far.
  - Spatial coherence: points move like their neighbors.

Slide credit: Svetlana Lazebnik

The Brightness Constancy Constraint



- Brightness Constancy Equation:

$$I(x, y, t-1) = I(x+u(x, y), y+v(x, y), t)$$

- Linearizing the right hand side using Taylor expansion:

$$I(x, y, t-1) \approx I(x, y, t) + I_x \cdot u(x, y) + I_y \cdot v(x, y)$$

- Hence, $$I_x \cdot u + I_y \cdot v + I_t \approx 0$$

Spatial derivatives

Temporal derivative

Slide credit: Svetlana Lazebnik

The Brightness Constancy Constraint

$$I_x \cdot u + I_y \cdot v + I_t = 0$$

- How many equations and unknowns per pixel?
  - One equation, two unknowns

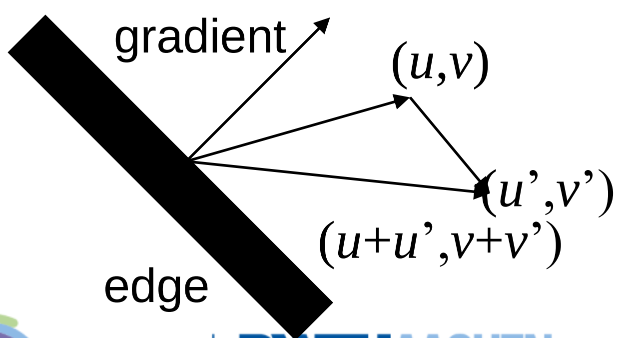
- Intuitively, what does this constraint mean?

$$\nabla I \cdot (u, v) + I_t = 0$$

– It gives us a constraint on the component of the flow in the direction of the gradient.

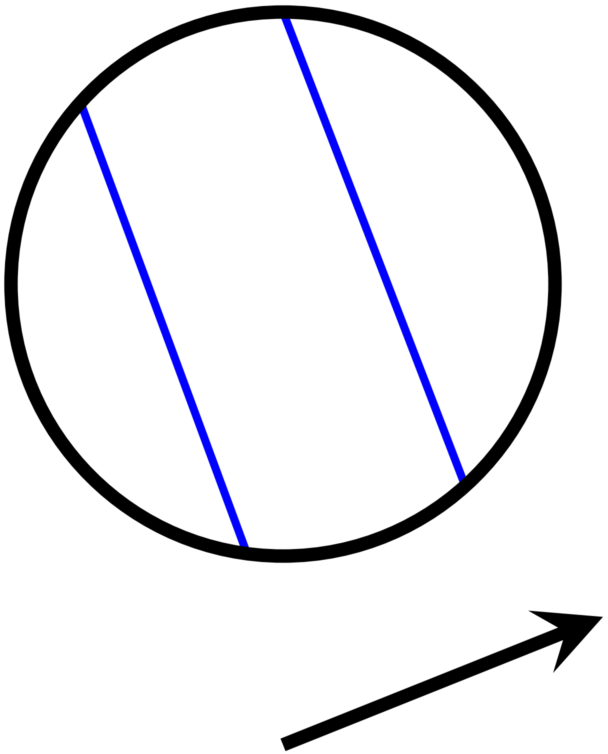
$\Rightarrow$ The component of the flow perpendicular to the gradient (i.e., parallel to the edge) is unknown!

If (u,v) satisfies the equation,
so does $(u+u', v+v')$ if $\nabla I \cdot (u', v') = 0$



Slide credit: Svetlana Lazebnik

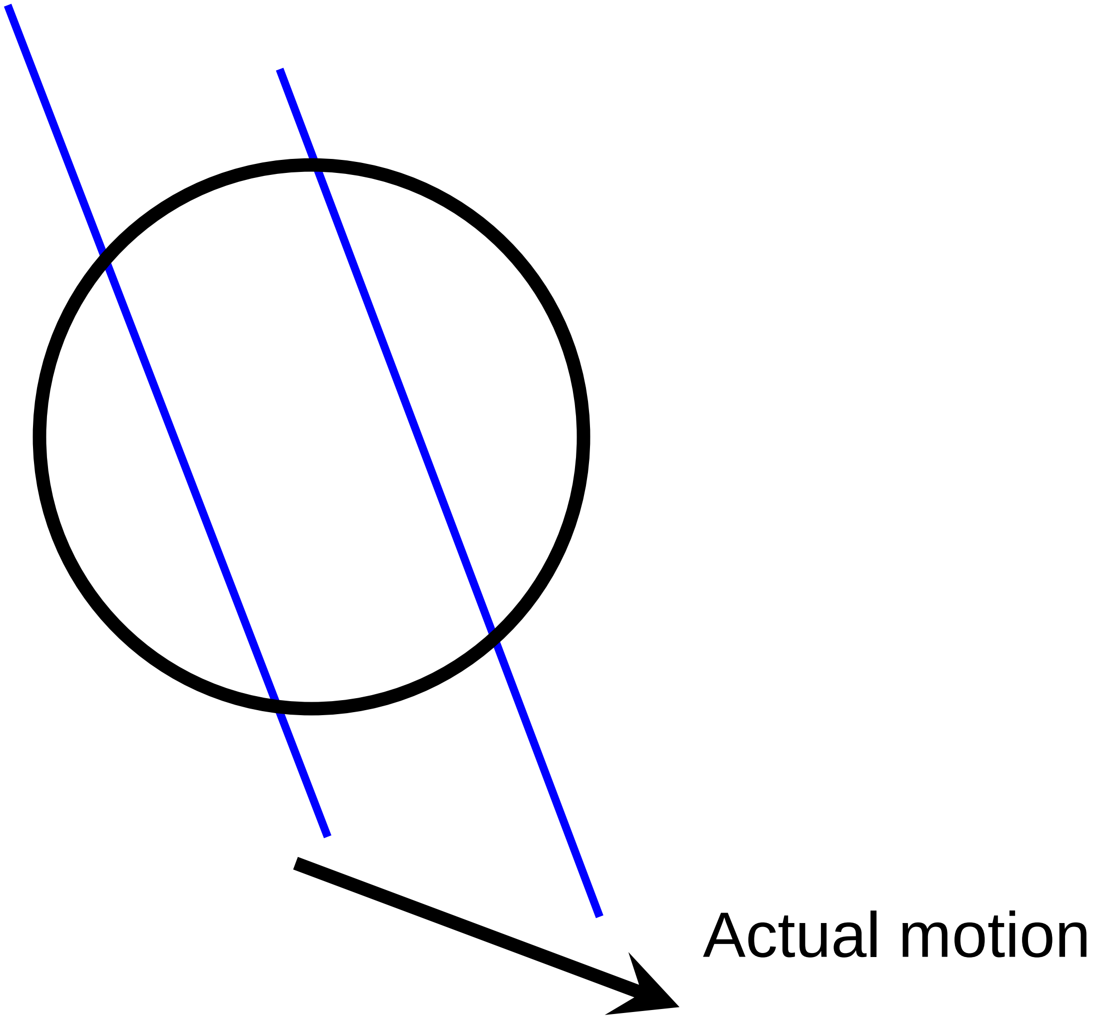
The Aperture Problem



Perceived motion

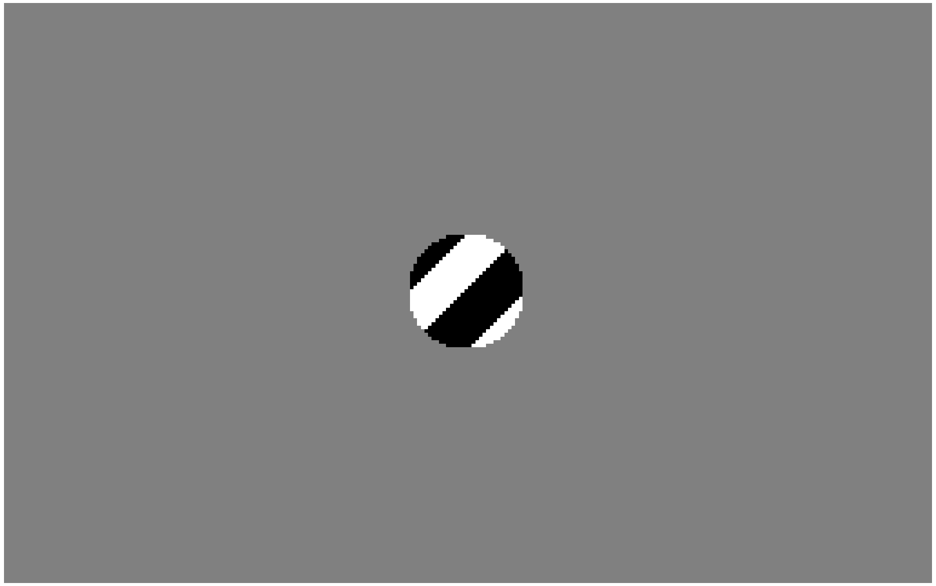
Slide credit: Svetlana Lazebnik

The Aperture Problem



Slide credit: Svetlana Lazebnik

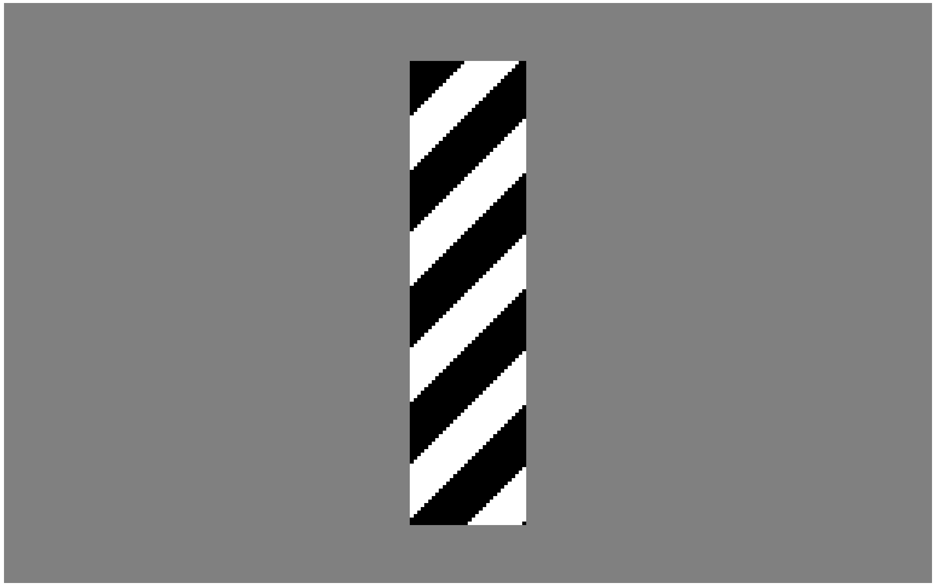
The Barber Pole Illusion



http://en.wikipedia.org/wiki/Barberpole_illusion

Slide credit: Svetlana Lazebnik

The Barber Pole Illusion



http://en.wikipedia.org/wiki/Barberpole_illusion

Slide credit: Svetlana Lazebnik

The Barber Pole Illusion

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Slide credit: Svetlana Lazebnik

Solving the Aperture Problem

- How to get more equations for a pixel?
- Spatial coherence constraint
  - Pretend the pixel's neighbors have the same (u,v).
  - If we use a 5×5 window, that gives us 25 equations per pixel

$$0 = I_t(\mathbf{p_i}) + \nabla I(\mathbf{p_i}) \cdot [u \;\; v]$$

$$\begin{bmatrix} I_x(\mathbf{p_1}) & I_y(\mathbf{p_1}) \\ I_x(\mathbf{p_2}) & I_y(\mathbf{p_2}) \\ \vdots & \vdots \\ I_x(\mathbf{p_{25}}) & I_y(\mathbf{p_{25}}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{p_1}) \\ I_t(\mathbf{p_2}) \\ \vdots \\ I_t(\mathbf{p_{25}}) \end{bmatrix}$$

B. Lucas and T. Kanade. An iterative image registration technique with an application to stereo vision. In *Proc. IJCAI'81*, pp. 674–679, 1981.

Slide credit: Svetlana Lazebnik

Solving the Aperture Problem

- Least squares problem:

$$\begin{bmatrix} I_x(\mathbf{p}_1) & I_y(\mathbf{p}_1) \\ I_x(\mathbf{p}_2) & I_y(\mathbf{p}_2) \\ \vdots & \vdots \\ I_x(\mathbf{p}_{25}) & I_y(\mathbf{p}_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{p}_1) \\ I_t(\mathbf{p}_2) \\ \vdots \\ I_t(\mathbf{p}_{25}) \end{bmatrix} \qquad \underset{25\text{x}2}{A} \; \underset{2\text{x}1}{d} = \underset{25\text{x}1}{b}$$

- Minimum least squares solution given by solution of

$$\underset{2\text{x}2}{(A^T A)} \; \underset{2\text{x}1}{d} = \underset{2\text{x}1}{A^T b}$$

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

(The summations are over all pixels in the $K \times K$ window)

Slide credit: Svetlana Lazebnik

Conditions for Solvability

- Optimal (u, v) satisfies Lucas-Kanade equation

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

- When is this solvable?
  - $A^T A$ should be invertible.
  - $A^T A$ entries should not be too small (noise).
  - $A^T A$ should be well-conditioned.

  $\Rightarrow$ Looking for cases where A has two large eigenvalues (i.e., corners and highly textured areas).

Slide credit: Svetlana Lazebnik

Iterative LK Refinement

1. Estimate velocity at each pixel using one iteration of LK estimation.

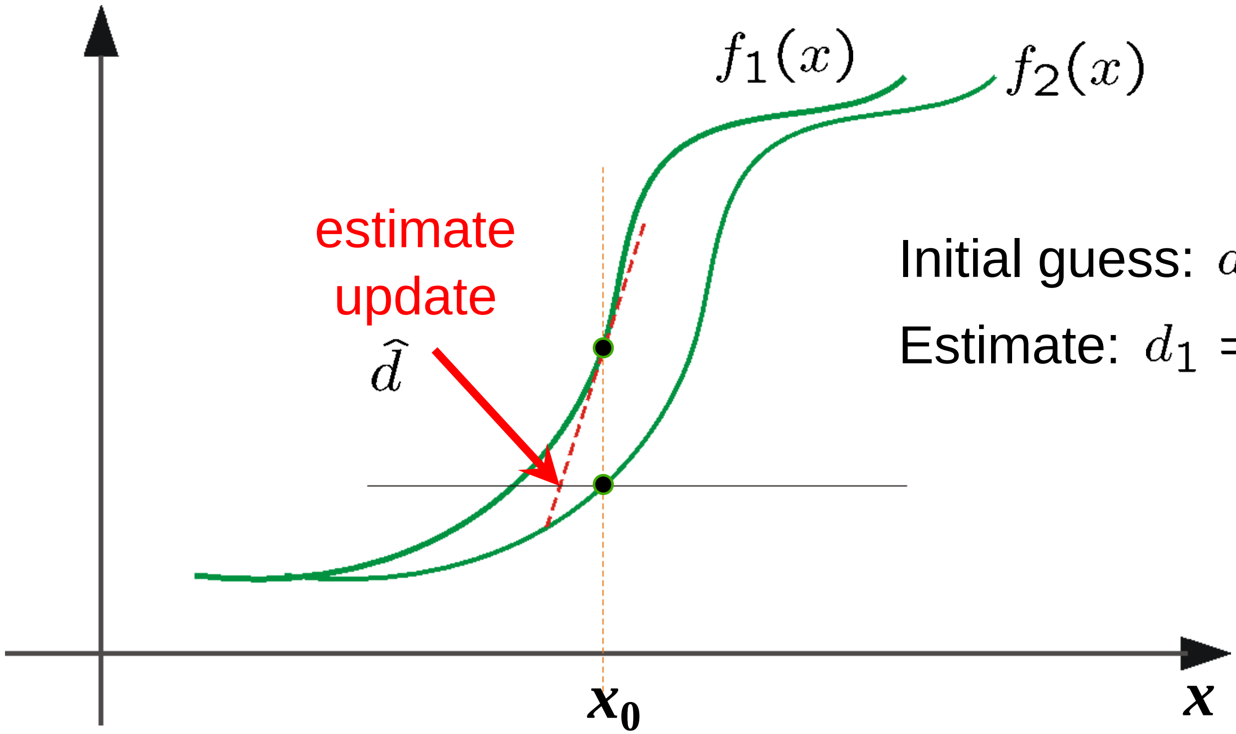
$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

2. Warp one image toward the other using the estimated flow field.
   – *(Easier said than done)*

3. Refine estimate by repeating the process.

Slide adapted from Steve Seitz

Iterative LK Refinement



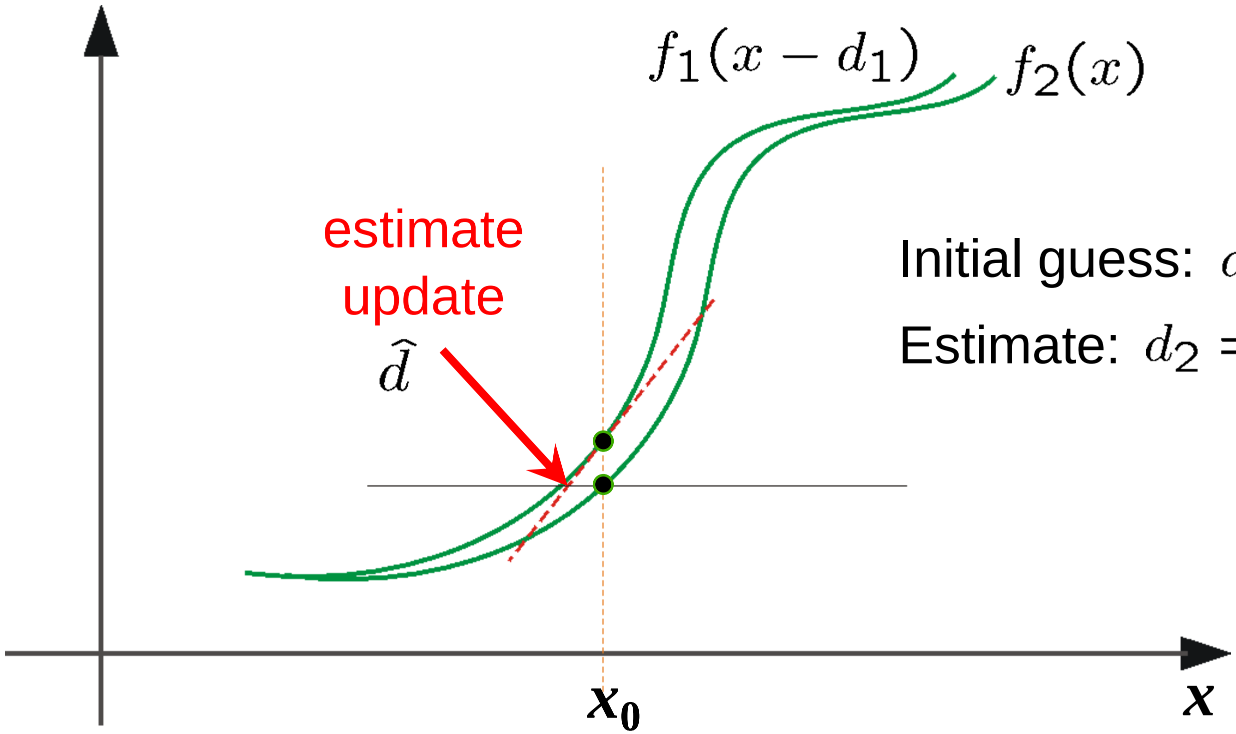
Initial guess: $d_0 = 0$

Estimate: $d_1 = d_0 + \hat{d}$

(using d for *displacement* here instead of u)

Slide credit: Steve Seitz

Iterative LK Refinement



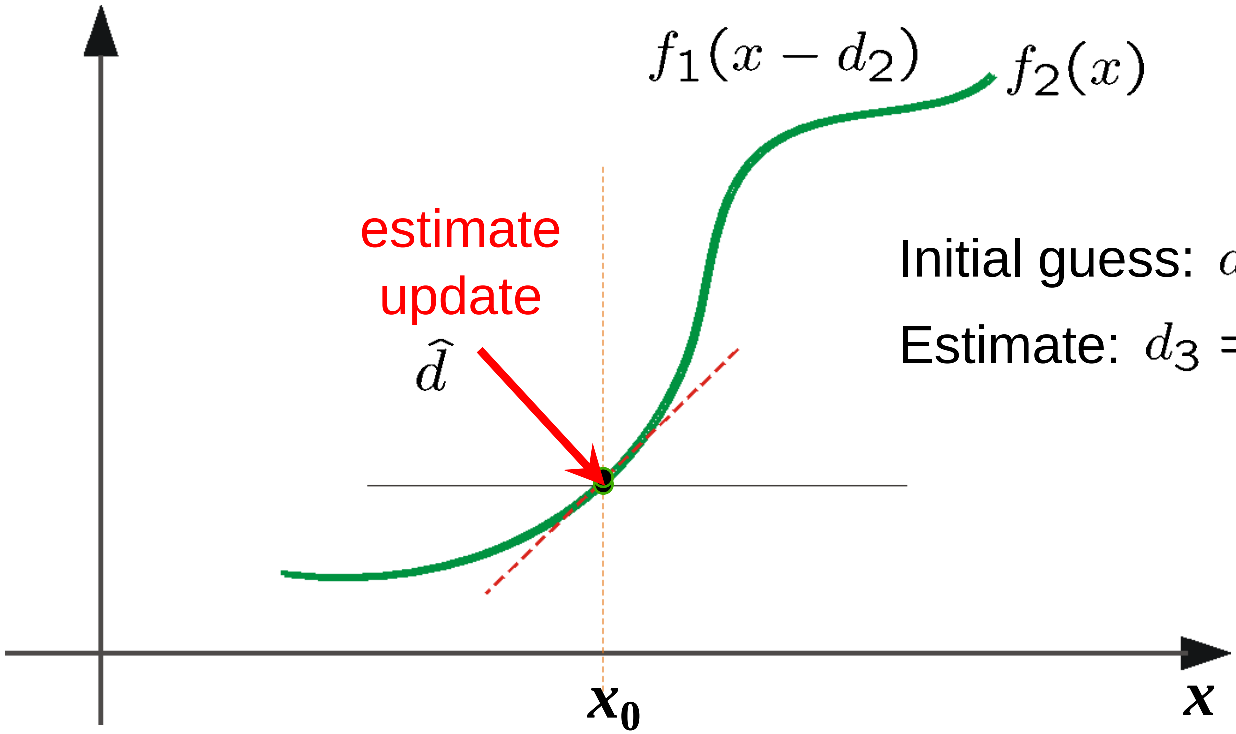
Initial guess: d_1

Estimate: $d_2 = d_1 + \hat{d}$

(using d for *displacement* here instead of u)

Slide credit: Steve Seitz

Iterative LK Refinement



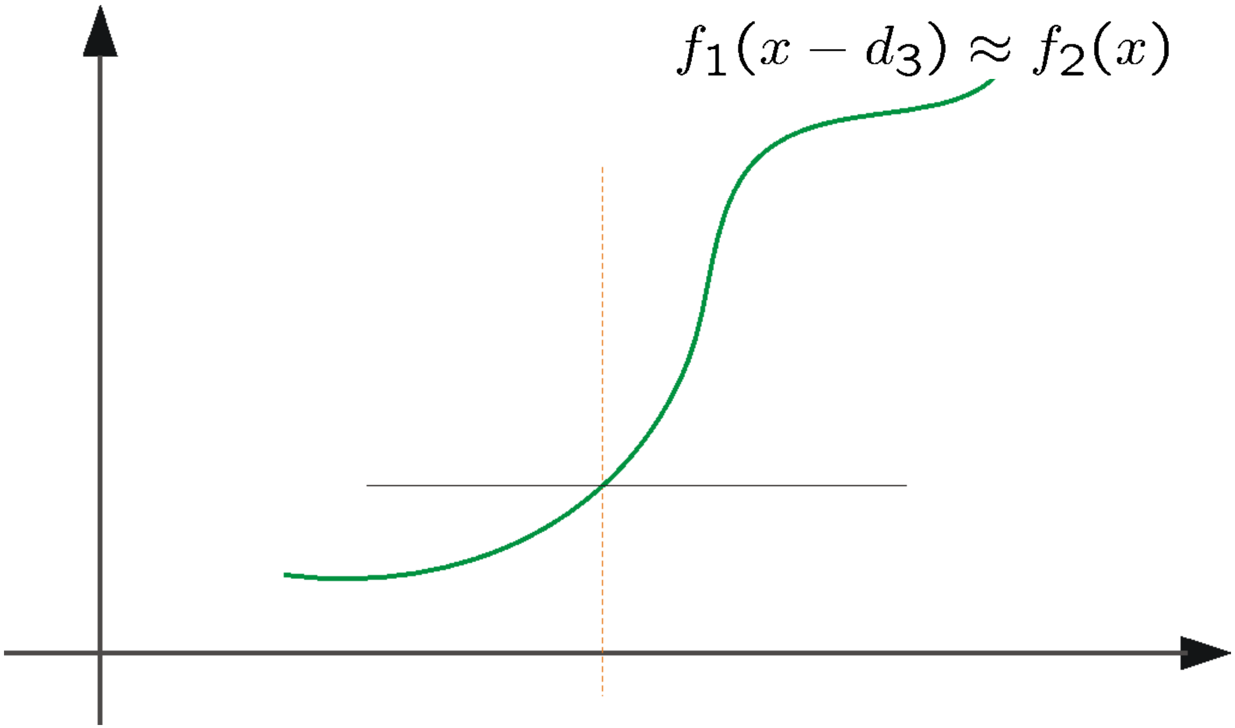
Initial guess: d_2

Estimate: $d_3 = d_2 + \hat{d}$

(using d for *displacement* here instead of u)

Slide credit: Steve Seitz

Iterative LK Refinement



(using d for *displacement* here instead of u)

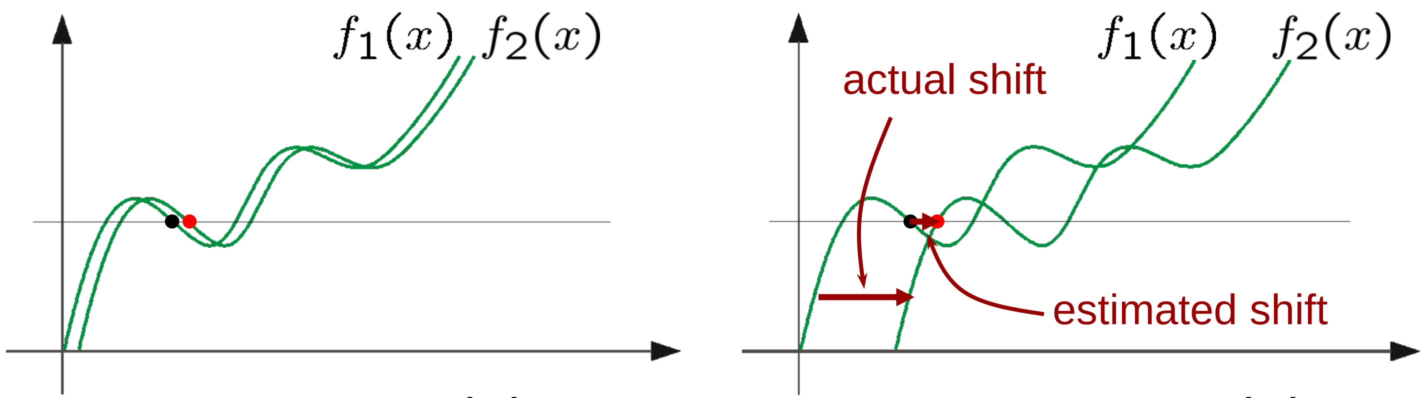
Slide credit: Steve Seitz

Problem Case: Large Motions

Slide credit: Svetlana Lazebnik

Temporal Aliasing

- Temporal aliasing causes ambiguities in optical flow because images can have many pixels with the same intensity.
- I.e., how do we know which ‘correspondence’ is correct?



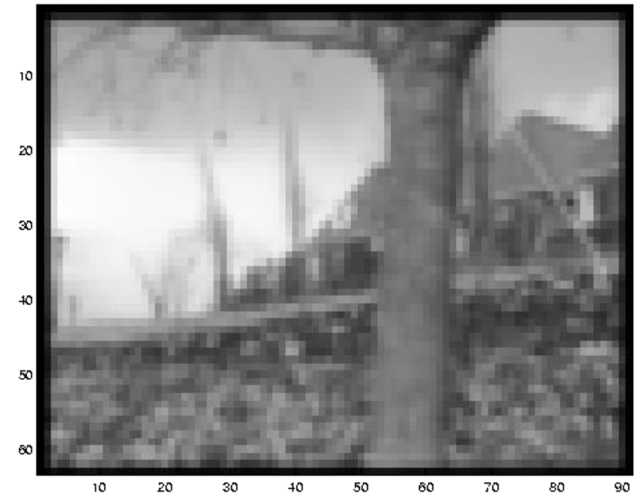
Nearest match is correct (no aliasing)

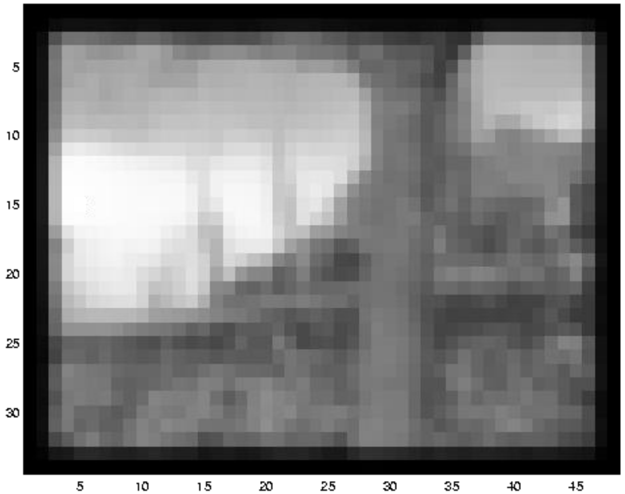
Nearest match is incorrect (aliasing)

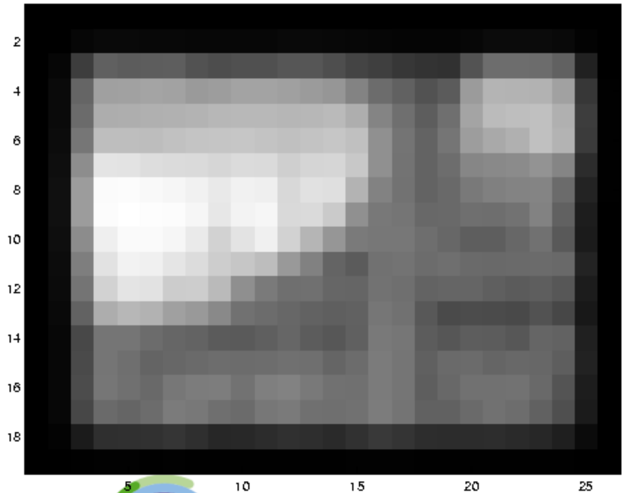
- To overcome aliasing: coarse-to-fine estimation.

Slide credit: Steve Seitz

Idea: Reduce the Resolution!

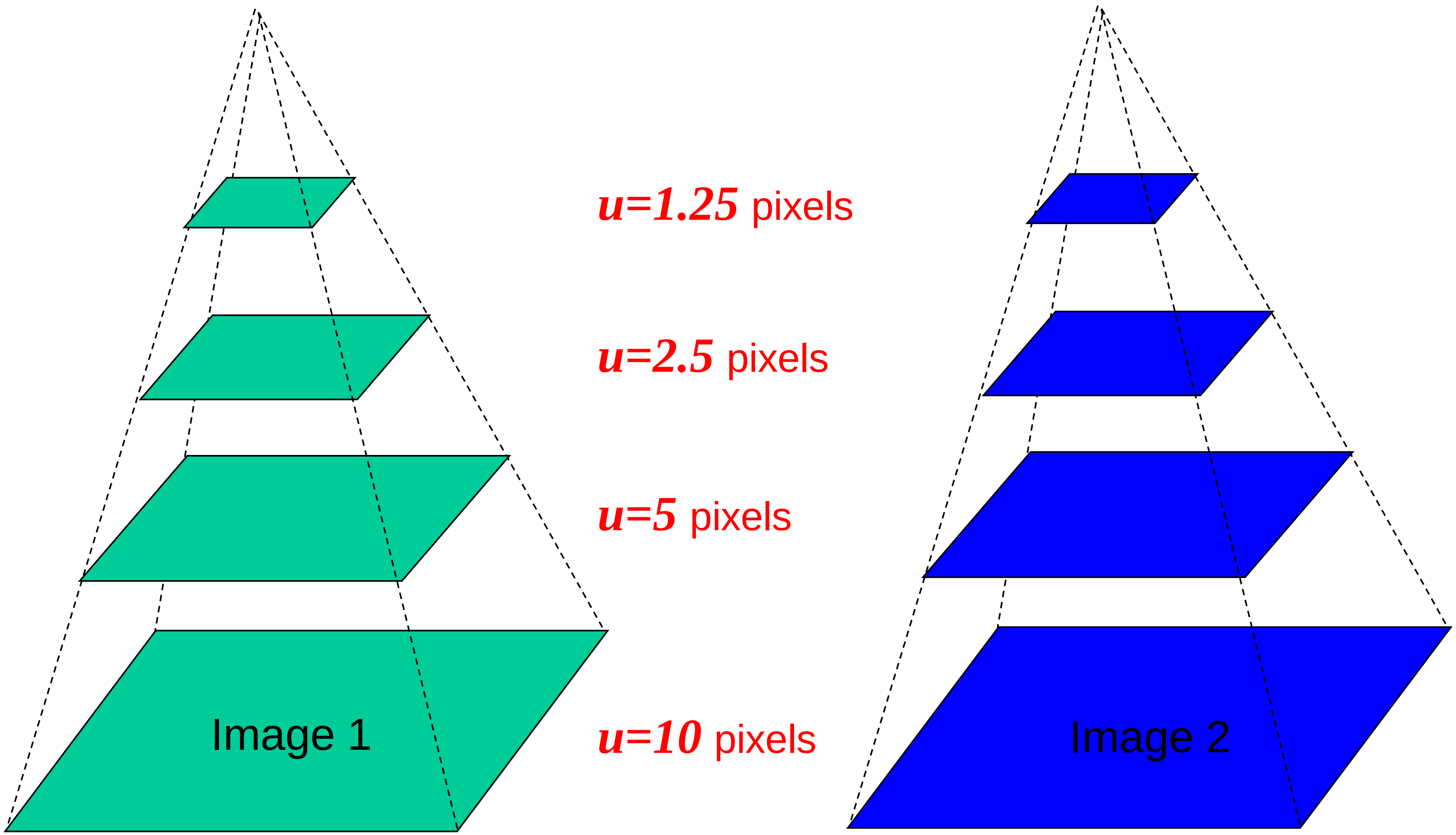






Slide credit: Svetlana Lazebnik

Coarse-to-fine Optical Flow Estimation

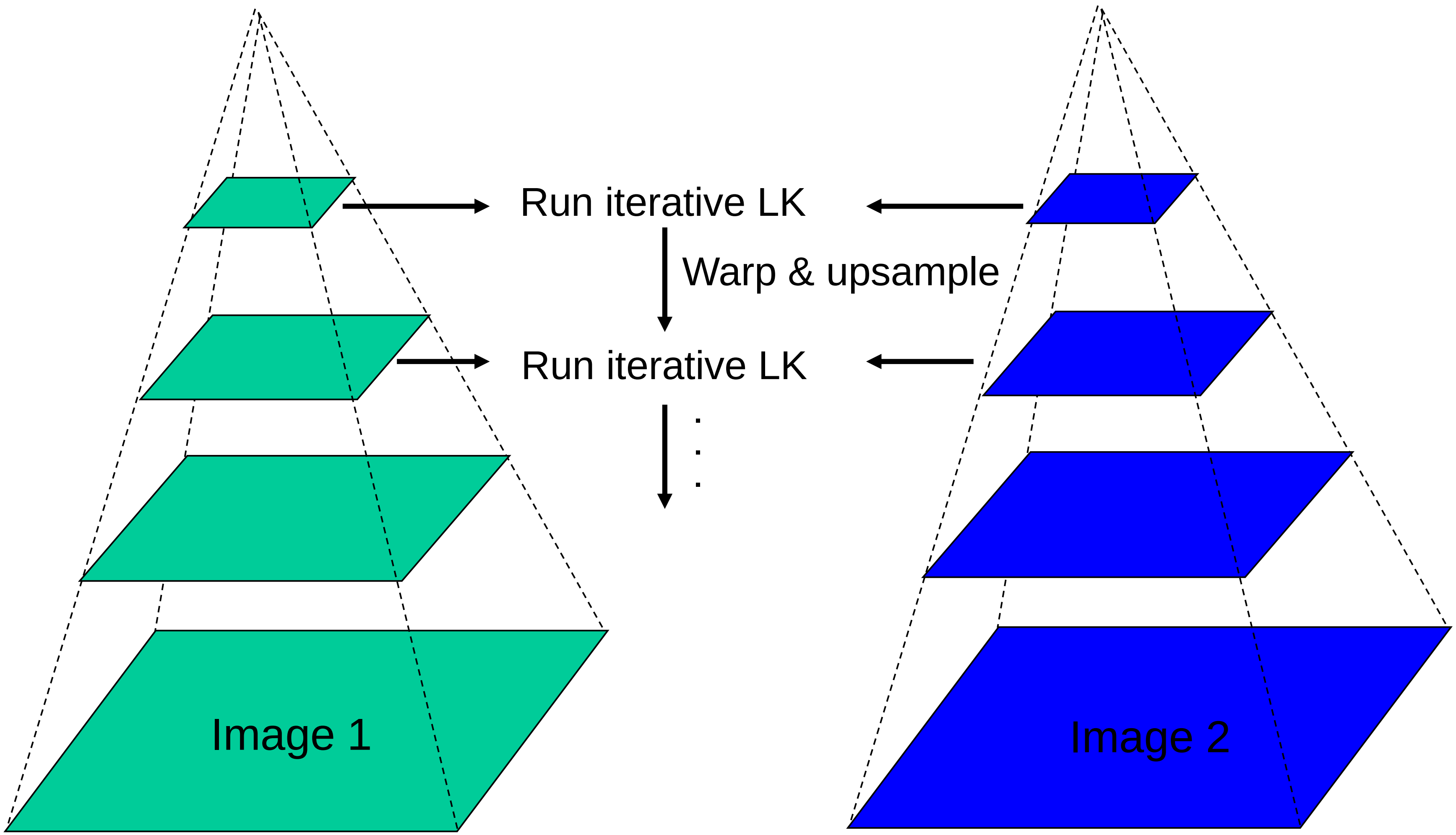


Gaussian pyramid of image 1

Gaussian pyramid of image 2

Slide credit: Steve Seitz

Coarse-to-fine Optical Flow Estimation



Gaussian pyramid of image 1

Gaussian pyramid of image 2

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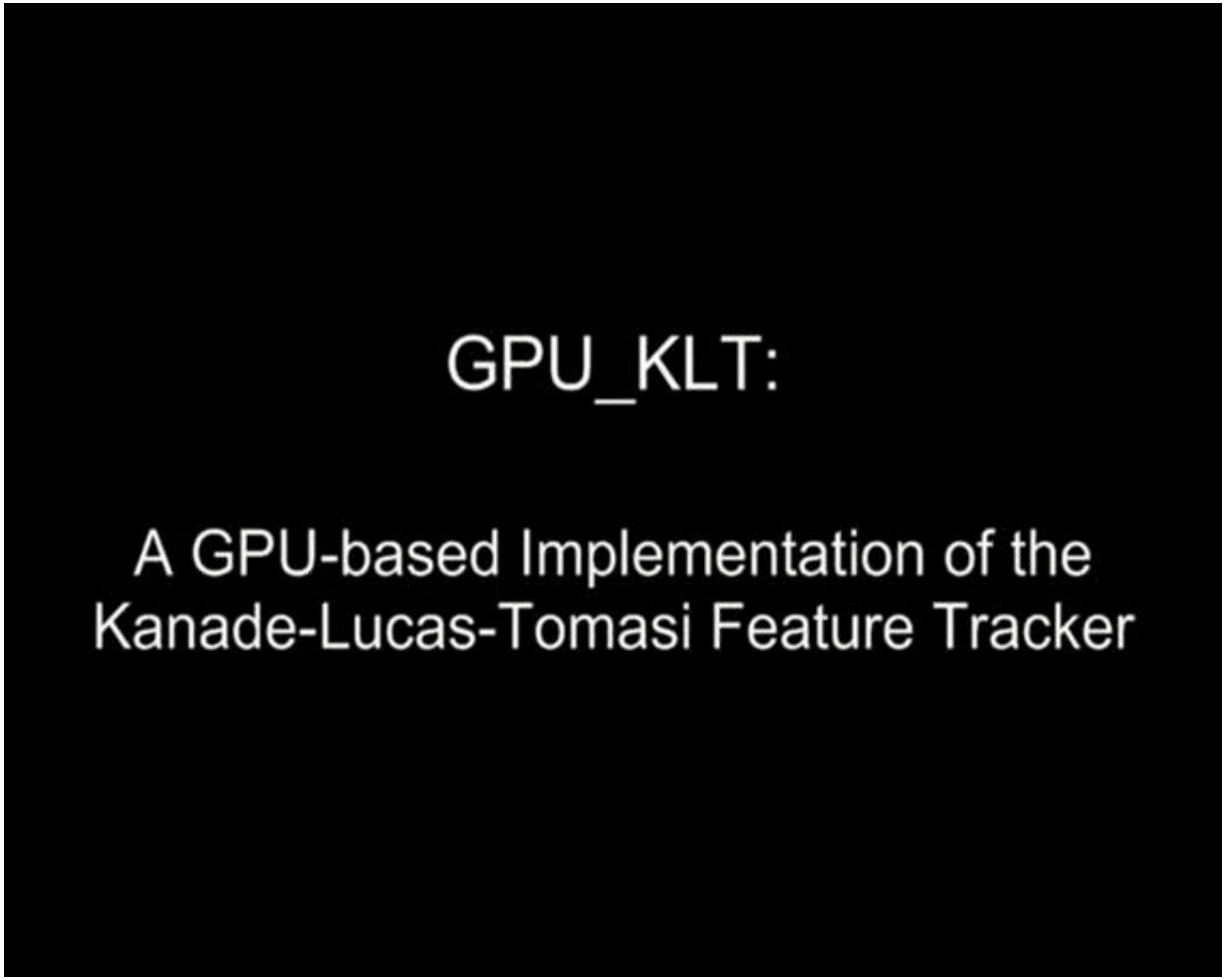
KLT Feature Tracking



http://www.cs.unc.edu/~ssinha/Research/GPU_KLT/

Shi-Tomasi Feature Tracker

- Idea
  - Find good features using eigenvalues of second-moment matrix
  - Key idea: “good” features to track are the ones that can be tracked reliably.
- Frame-to-frame tracking
  - Track with LK and a pure *translation* motion model.
  - More robust for small displacements, can be estimated from smaller neighborhoods (e.g., 5×5 pixels).
- Checking consistency of tracks
  - *Affine* registration to the first observed feature instance.
  - Affine model is more accurate for larger displacements.
  - Comparing to the first frame helps to minimize drift.

J. Shi and C. Tomasi. Good Features to Track. CVPR 1994.

Slide credit: Svetlana Lazebnik

Tracking Example

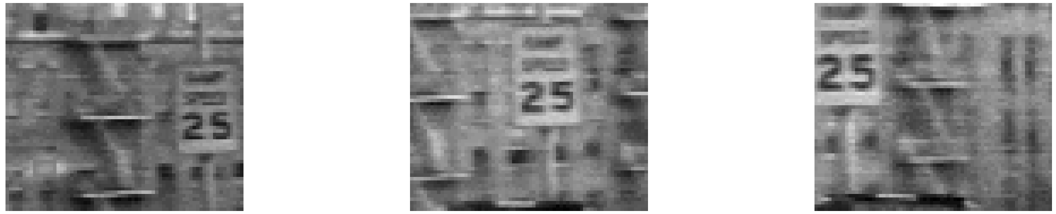


Figure 1: Three frame details from Woody Allen's *Manhattan*. The details are from the 1st, 11th, and 21st frames of a subsequence from the movie.

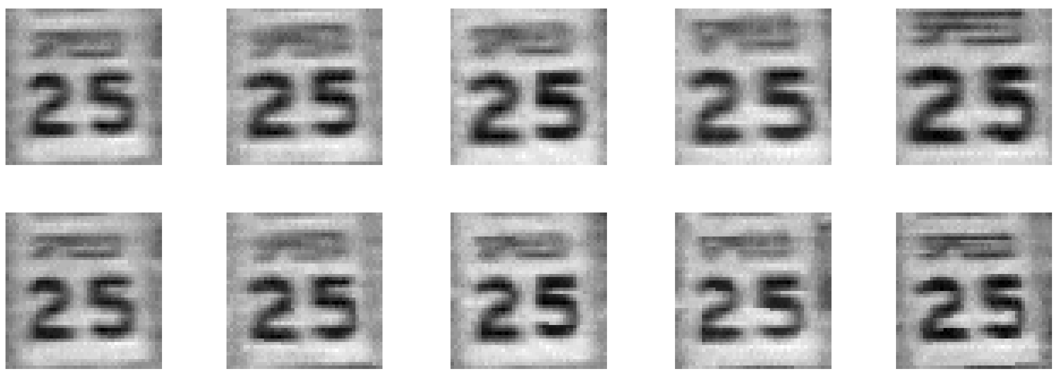


Figure 2: The traffic sign windows from frames 1,6,11,16,21 as tracked (top), and warped by the computed deformation matrices (bottom).

J. Shi and C. Tomasi. [Good Features to Track](). CVPR 1994.

Slide credit: Svetlana Lazebnik

Real-Time GPU Implementations

- This basic feature tracking framework (Lucas-Kanade + Shi-Tomasi) is commonly referred to as “KLT tracking”.
  - Often used as first step in SfM/SLAM pipelines
  - Lends itself to easy parallelization
- Very fast GPU implementations available, e.g.,
  - C. Zach, D. Gallup, J.-M. Frahm,
    Fast Gain-Adaptive KLT tracking on the GPU.
    In CVGPU’08 Workshop, Anchorage, USA, 2008
  - 216 fps with automatic gain adaptation
  - 260 fps without gain adaptation

http://www.cs.unc.edu/~ssinha/Research/GPU_KLT/

http://www.inf.ethz.ch/personal/chzach/opensource.html

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Lucas-Kanade Template Tracking

- Traditional LK
  - Typically run on small, corner-like features (e.g., 5×5 patches) to compute optical flow ($\rightarrow$ KLT).
  - However, there is no reason why we can't use the same approach on a larger window around the tracked object.

Slide credit: Robert Collins

Basic LK Derivation for Templates

$$E(u,v) = \sum_{\mathbf{x}} \left[I(x+u, y+v) - T(x,y) \right]^2$$

Current frame

Template model

(u,v) = hypothesized location of template in current frame

Slide credit: Robert Collins

Basic LK Derivation for Templates

- Taylor expansion

$$E(u,v) = \sum_{\mathbf{x}} \left[I(x+u, y+v) - T(x,y) \right]^2$$

$$\approx \sum_{\mathbf{x}} \left[I(x,y) + uI_x(x,y) + vI_y(x,y) - T(x,y) \right]^2$$

$$= \sum_{\mathbf{x}} \left[uI_x(x,y) + vI_y(x,y) + D(x,y) \right]^2 \quad \text{with} \quad D = I - T$$

- Taking partial derivatives

$$\frac{\partial E}{\partial u} = 2 \sum_{\mathbf{x}} \left[uI_x(x,y) + vI_y(x,y) + D(x,y) \right] I_x(x,y) \stackrel{!}{=} 0$$

$$\frac{\partial E}{\partial v} = 2 \sum_{\mathbf{x}} \left[uI_x(x,y) + vI_y(x,y) + D(x,y) \right] I_y(x,y) \stackrel{!}{=} 0$$

- Equation in matrix form

$$\sum_{\mathbf{x}} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \sum_{\mathbf{x}} \begin{bmatrix} I_x D \\ I_y D \end{bmatrix} \quad \Rightarrow$$

Solve via least-squares

Slide credit: Robert Collins

One Problem With This...

- Problematic Assumption
  - Assumption of constant flow (pure translation) for all pixels in a larger window is unreasonable for long periods of time.

- However...
  - We can easily generalize the LK approach to other 2D parametric motion models (like affine or projective) by introducing a “warp” function $\mathbf{W}$ with parameters $\mathbf{p}$.

$$E(u,v) = \sum_{\mathbf{x}} \left[I(x+u, y+v) - T(x,y) \right]^2$$

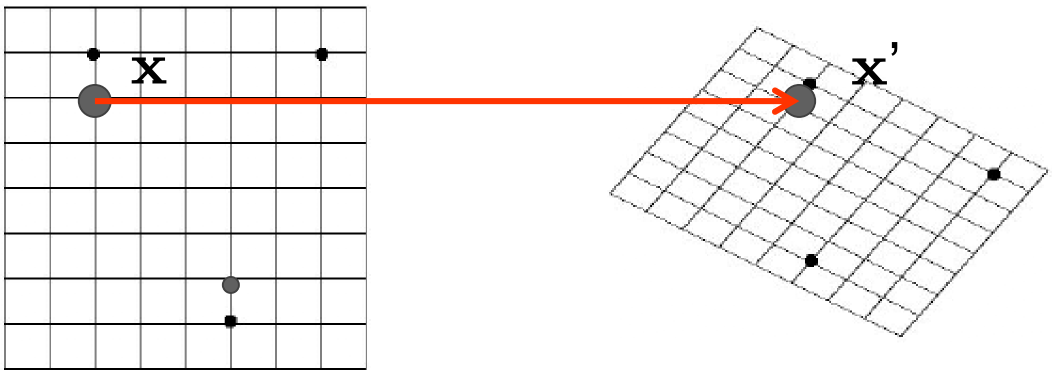
$$\downarrow$$

$$E(\mathbf{p}) = \sum_{\mathbf{x}} \left[I(\mathbf{W}([x,y];\mathbf{p})) - T([x,y]) \right]^2$$

Slide credit: Robert Collins

Geometric Image Warping

- The warp $\mathbf{W}(\mathbf{x};\, \mathbf{p})$ describes the geometric relationship between two images



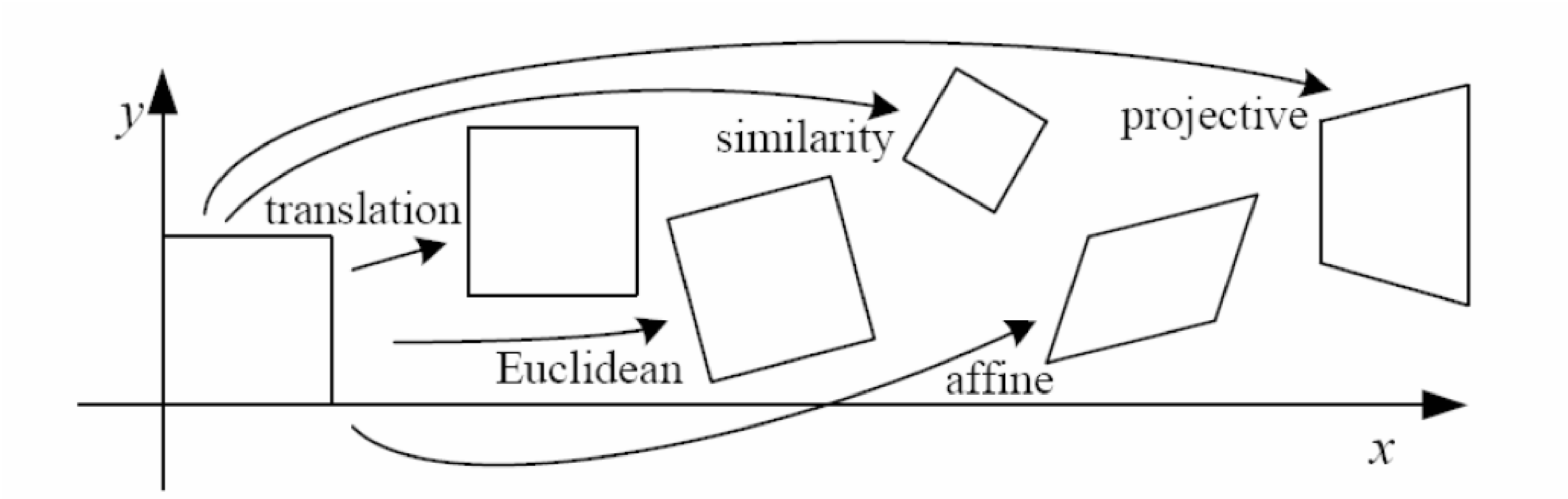
Input Image Transformed Image

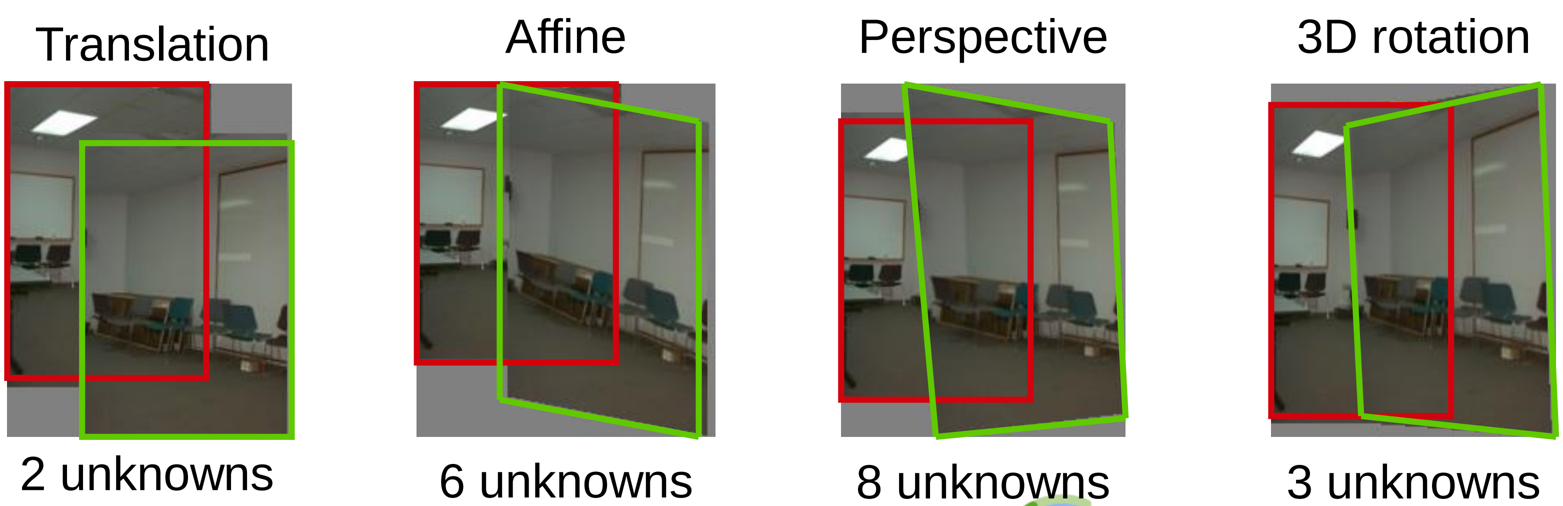
$$\mathbf{x}' = \mathbf{W}(\mathbf{x}; \boxed{\mathbf{p}}) = \begin{bmatrix} W_x(\mathbf{x};\mathbf{p}) \\ W_y(\mathbf{x};\mathbf{p}) \end{bmatrix}$$

Parameters of the warp

Slide credit: Jinxiang Chai

Example Warping Functions





Slide credit: Steve Seitz

Example Warping Functions

- Translation

$$\mathbf{W}([x,y];\mathbf{p}) = \begin{bmatrix} x+p_1 \\ y+p_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & p_1 \\ 0 & 1 & p_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Affine

$$\mathbf{W}([x,y];\mathbf{p}) = \begin{bmatrix} x+p_1x+p_3y+p_5 \\ y+p_2x+p_4y+p_6 \end{bmatrix} = \begin{bmatrix} 1+p_1 & p_3 & p_5 \\ p_2 & 1+p_4 & p_6 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Perspective

$$\mathbf{W}([x,y];\mathbf{p}) = \frac{1}{p_7x+p_8y+1} \begin{bmatrix} x+p_1x+p_3y+p_5 \\ y+p_2x+p_4y+p_6 \end{bmatrix}$$

  – Note: Other parametrizations are possible; the above ones are just particularly convenient here.

Slide credit: Jinxiang Chai

General LK Image Registration

- Goal
  - Find the warping parameters $\mathbf{p}$ that minimize the sum-of-squares intensity difference between the template image and the warped input image.

- LK formulation
  - Formulate this as an optimization problem

$$\arg\min_{\mathbf{p}} \sum_{\mathbf{x}} \left[I(\mathbf{W}(\mathbf{x};\mathbf{p})) - T(\mathbf{x}) \right]^2$$

  - We assume that an initial estimate of $\mathbf{p}$ is known and iteratively solve for increments to the parameters $\Delta\mathbf{p}$:

$$\arg\min_{\Delta\mathbf{p}} \sum_{\mathbf{x}} \left[I(\mathbf{W}(\mathbf{x};\mathbf{p}+\Delta\mathbf{p})) - T(\mathbf{x}) \right]^2$$

Step-by-Step Derivation

- Key to the derivation
  - Taylor expansion around $\Delta\mathbf{p}$

$$I(\mathbf{W}(\mathbf{x};\mathbf{p}+\Delta\mathbf{p})) \approx I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta\mathbf{p} + \mathcal{O}(\Delta\mathbf{p}^2)$$

  - Using pixel coordinates $\mathbf{x} = [x,y]$

$$\begin{aligned} I(\mathbf{W}([x,y];\mathbf{p}+\Delta\mathbf{p})) \approx \;& I(\mathbf{W}([x,y];p_1,\ldots,p_n)) \\ &+ \left[\frac{\partial I}{\partial x}\frac{\partial W_x}{\partial p_1} + \frac{\partial I}{\partial y}\frac{\partial W_y}{\partial p_1}\right]_{p_1} \Delta p_1 \\ &+ \left[\frac{\partial I}{\partial x}\frac{\partial W_x}{\partial p_2} + \frac{\partial I}{\partial y}\frac{\partial W_y}{\partial p_2}\right]_{p_1} \Delta p_2 \\ &+ \;\ldots \\ &+ \left[\frac{\partial I}{\partial x}\frac{\partial W_x}{\partial p_n} + \frac{\partial I}{\partial y}\frac{\partial W_y}{\partial p_n}\right]_{p_n} \Delta p_n \end{aligned}$$

Slide credit: Robert Collins

Step-by-Step Derivation

- Rewriting this in matrix notation

$$
\begin{aligned}
I(\mathbf{W}([x,y];\mathbf{p}+\Delta\mathbf{p})) \approx \;& I(\mathbf{W}([x,y];p_1,\ldots,p_n)) \\
&+ \begin{bmatrix} \frac{\partial I}{\partial x} & \frac{\partial I}{\partial y} \end{bmatrix} \begin{bmatrix} \frac{\partial W_x}{\partial p_1} \\ \frac{\partial W_y}{\partial p_1} \end{bmatrix}_{p_1} \Delta p_1 \\
&+ \begin{bmatrix} \frac{\partial I}{\partial x} & \frac{\partial I}{\partial y} \end{bmatrix} \begin{bmatrix} \frac{\partial W_x}{\partial p_2} \\ \frac{\partial W_y}{\partial p_2} \end{bmatrix}_{p_2} \Delta p_2 \\
&+ \;\ldots \\
&+ \begin{bmatrix} \frac{\partial I}{\partial x} & \frac{\partial I}{\partial y} \end{bmatrix} \begin{bmatrix} \frac{\partial W_x}{\partial p_n} \\ \frac{\partial W_y}{\partial p_n} \end{bmatrix}_{p_n} \Delta p_n
\end{aligned}
$$

Slide credit: Robert Collins

Step-by-Step Derivation

- And further collecting the derivative terms

$$I(\mathbf{W}([x,y];\mathbf{p}+\Delta\mathbf{p})) \approx I(\mathbf{W}([x,y];p_1,\ldots,p_n))$$

$$+\begin{bmatrix}\frac{\partial I}{\partial x} & \frac{\partial I}{\partial y}\end{bmatrix}\begin{bmatrix}\frac{\partial W_x}{\partial p_1} & \frac{\partial W_x}{\partial p_2} & \cdots & \frac{\partial W_x}{\partial p_n}\\ \frac{\partial W_y}{\partial p_1} & \frac{\partial W_y}{\partial p_2} & \cdots & \frac{\partial W_y}{\partial p_n}\end{bmatrix}\begin{bmatrix}\Delta p_1\\ \Delta p_2\\ \vdots\\ \Delta p_n\end{bmatrix}$$

Gradient	Jacobian	Increment parameters to solve for
∇I	$\frac{\partial \mathbf{W}}{\partial \mathbf{p}}$	$\Delta\mathbf{p}$

– Written in matrix form

$$I(\mathbf{W}(\mathbf{x};\mathbf{p}+\Delta\mathbf{p})) \approx I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta\mathbf{p}$$

Slide credit: Robert Collins

Example: Jacobian of Affine Warp

- General equation of Jacobian

$$\frac{\partial \mathbf{W}}{\partial \mathbf{p}} = \begin{bmatrix} \frac{\partial W_x}{\partial p_1} & \frac{\partial W_x}{\partial p_2} & \cdots & \frac{\partial W_x}{\partial p_n} \\ \frac{\partial W_y}{\partial p_1} & \frac{\partial W_y}{\partial p_2} & \cdots & \frac{\partial W_y}{\partial p_n} \end{bmatrix}$$

- Affine warp function (6 parameters)

$$\mathbf{W}([x,y];\mathbf{p}) = \begin{bmatrix} 1+p_1 & p_3 & p_5 \\ p_2 & 1+p_4 & p_6 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Result

$$\frac{\partial \mathbf{W}}{\partial \mathbf{p}} = \frac{\partial \begin{bmatrix} x + p_1 x + p_3 y + p_5 \\ p_2 x + y + p_4 y + p_6 \end{bmatrix}}{\partial \mathbf{p}}$$

$$= \begin{bmatrix} x & 0 & y & 0 & 1 & 0 \\ 0 & x & 0 & y & 0 & 1 \end{bmatrix}$$

Minimizing the Registration Error

- Optimization function after Taylor expansion

$$\arg\min_{\Delta\mathbf{p}} \sum_{\mathbf{x}} \left[I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta\mathbf{p} - T(\mathbf{x}) \right]^2$$

- Minimizing this function
  - *How?*

Minimizing the Registration Error

- Optimization function after Taylor expansion

$$\arg\min_{\Delta \mathbf{p}} \sum_{\mathbf{x}} \left[I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta \mathbf{p} - T(\mathbf{x}) \right]^2$$

- Minimizing this function
  – Taking the partial derivative and setting it to zero

$$\frac{\partial}{\partial \Delta \mathbf{p}} \stackrel{!}{=} 0 \rightarrow 2 \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta \mathbf{p} - T(\mathbf{x}) \right] \stackrel{!}{=} 0$$

  – Closed-form solution for $\Delta \mathbf{p}$ (Gauss-Newton):

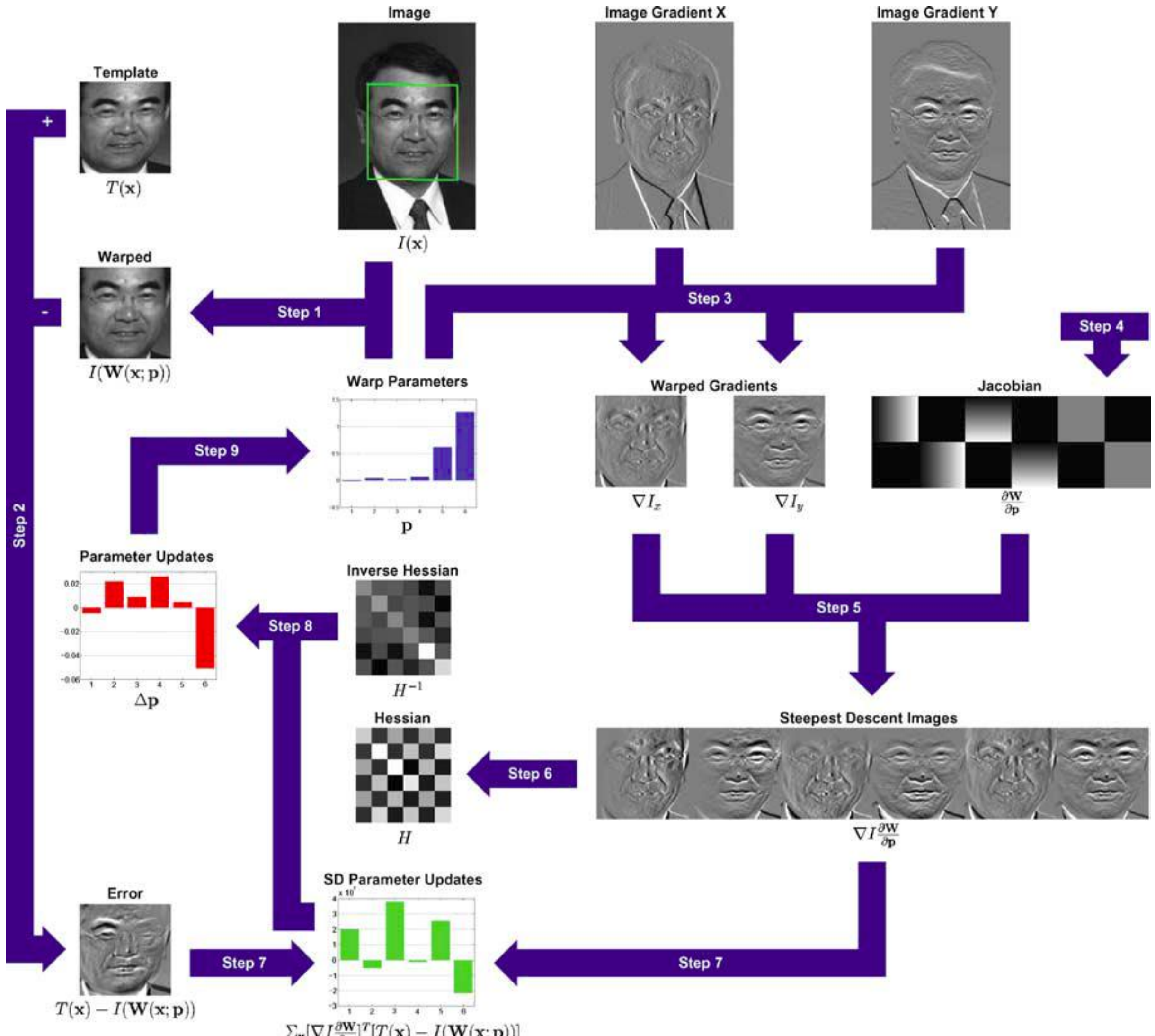
$$\Delta \mathbf{p} = \mathbf{H}^{-1} \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[T(\mathbf{x}) - I(\mathbf{W}(\mathbf{x};\mathbf{p})) \right]$$

  – where $\mathbf{H}$ is the Hessian $\quad \mathbf{H} = \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]$

Summary: Inverse Compositional LK Algorithm

- Iterate
  - Warp I to obtain $I(\mathbf{W}([x,\ y];\ \mathbf{p}))$
  - Compute the error image $T([x,\ y])$ – $I(\mathbf{W}([x,\ y];\ \mathbf{p}))$
  - Warp the gradient ∇I with $\mathbf{W}([x,\ y];\ \mathbf{p})$
  - Evaluate $\frac{\partial \mathbf{W}}{\partial \mathbf{p}}$ at $([x,\ y];\ \mathbf{p})$ **(Jacobian)**
  - Compute steepest descent images $\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}}$
  - Compute Hessian matrix $\mathbf{H} = \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}}\right]^T \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}}\right]$
  - Compute $\sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}}\right]^T \left[T([x,y]) - I(\mathbf{W}([x,y];\mathbf{p}))\right]$
  - Compute $\Delta \mathbf{p} = \mathbf{H}^{-1} \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}}\right]^T \left[T([x,y]) - I(\mathbf{W}([x,y];\mathbf{p}))\right]$
  - Update the parameters $\mathbf{p} \leftarrow \mathbf{p} + \Delta \mathbf{p}$
- Until $\Delta \mathbf{p}$ magnitude is negligible

Inverse Compositional LK Algorithm Visualization



[S. Baker, I. Matthews, IJCV'04]

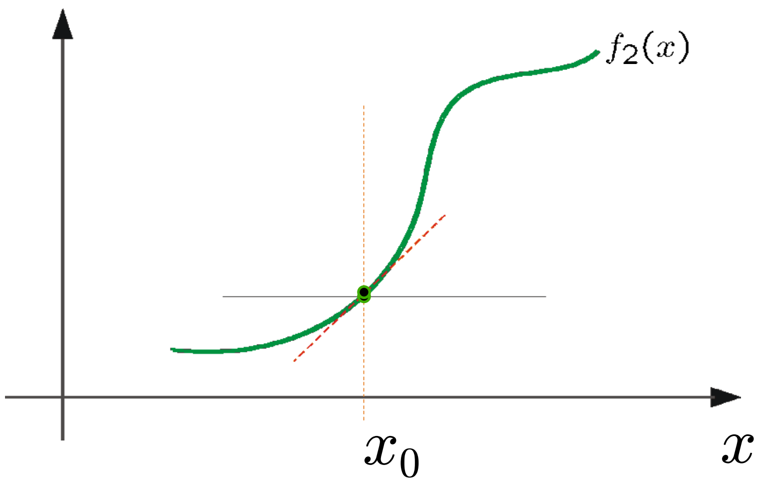
Discussion LK Alignment

- Pros
  - All pixels get used in matching
  - Can get sub-pixel accuracy (important for good mosaicking)
  - Fast and simple algorithm
  - Applicable to Optical Flow estimation, stereo disparity estimation, parametric motion tracking, etc.
- Cons
  - Prone to local minima.
  - Relatively small movement.

  $\Rightarrow$ Good initialization necessary

Side Note

- LK Registration needs a good initialization
  - Taylor expansion corresponds to a linearization around the initial position $\mathbf{p}$.
  - This linearization is only valid in a small neighborhood around $\mathbf{p}$.



- When tracking templates...
  - We typically use the previous frame's result as initialization.

$\Rightarrow$ The higher the frame rate, the smaller the warp will be.

$\Rightarrow$ This means we get better results and need fewer LK iterations.

$\Rightarrow$ *Tracking becomes easier (and faster!) with higher frame rates.*

Discussion

- Beyond 2D Tracking/Registration
  - So far, we focused on registration between 2D images.
  - The same ideas can be used when performing registration between a 3D model and the 2D image (model-based tracking).
  - The approach can also be extended for dealing with articulated objects and for tracking in subspaces.

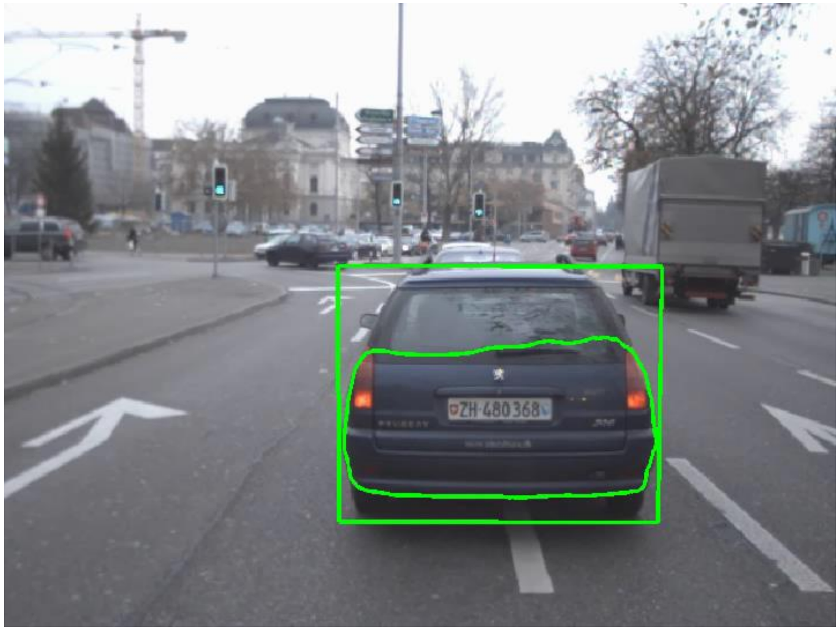
$\Rightarrow$ We will come back to this in later lectures when we talk about model-based 3D tracking...

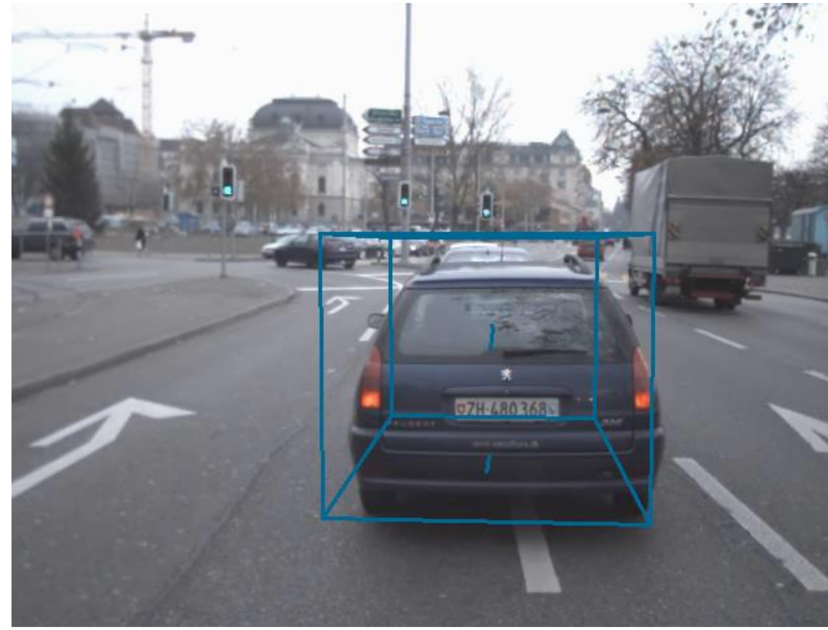
Slide credit: Jinxiang Chai

Topics of This Lecture

- Lucas-Kanade Optical Flow
  - Brightness Constancy constraint
  - LK flow estimation
  - Coarse-to-fine estimation
- Feature Tracking
  - KLT feature tracking
- Template Tracking
  - LK derivation for templates
  - Warping functions
  - General LK image registration
- Applications

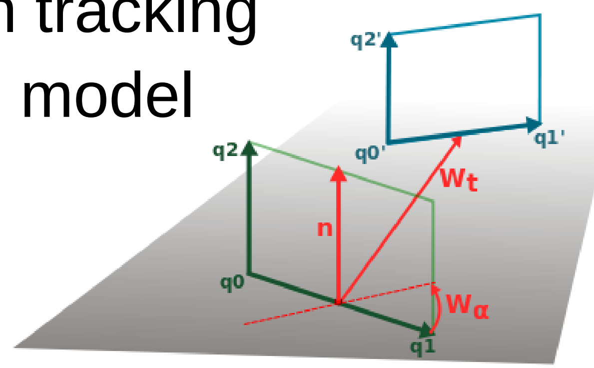
Example of a More Complex Warping Function





- Encode geometric constraints into region tracking
- Constrained homography transformation model
  - Translation parallel to the ground plane
  - Rotation around the ground plane normal
  - $\mathbf{W}(\mathbf{x}) = \mathbf{W}_{obj}\ \mathbf{P}\ \mathbf{W}_t\ \mathbf{W}_\alpha\ \mathbf{Q}\ \mathbf{x}$

$\Rightarrow$ Input for high-level tracker with car steering model.



[E. Horbert, D. Mitzel, B. Leibe, DAGM'10]

References and Further Reading

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  - B. Lucas and T. Kanade. An iterative image registration technique with an application to stereo vision. In *Proc. IJCAI*, pp. 674–679, 1981.
- A more recent paper giving a better explanation
  - S. Baker, I. Matthews. Lucas-Kanade 20 Years On: A Unifying Framework. In IJCV, Vol. 56(3), pp. 221-255, 2004.
- The original KLT paper by Shi & Tomasi
  - J. Shi and C. Tomasi. Good Features to Track. CVPR 1994.