This Lecture: Advanced Machine Learning

- Regression Approaches
  - Linear Regression
  - Regularization (Ridge, Lasso)
  - Kernels (Kernel Ridge Regression)
  - Gaussian Processes

- Approximate Inference
  - Sampling Approaches
  - MCMC

- Deep Learning
  - Linear Discriminants
  - Neural Networks
  - Backpropagation & Optimization
  - CNNs, ResNets, RNNs, etc.
Topics of This Lecture

• Recap: Recurrent Neural Networks (RNNs)
  - Backpropagation through Time (BPTT)
  - Problems with RNN Training
  - Handling Vanishing Gradients

• Improved hidden units for RNNs
  - Long Short-Term Memory (LSTM)
  - Gated Recurrent Units (GRU)

• Applications of RNNs
Recurrent Neural Networks

• Up to now
  - Simple neural network structure: 1-to-1 mapping of inputs to outputs

• This lecture: Recurrent Neural Networks
  - Generalize this to arbitrary mappings
Recap: Recurrent Neural Networks (RNNs)

- RNNs are regular NNs whose hidden units have additional connections over time.
  - You can unroll them to create a network that extends over time.
  - When you do this, keep in mind that the weights for the hidden layers are shared between temporal layers.

- RNNs are very powerful
  - With enough neurons and time, they can compute anything that can be computed by your computer.

Image source: Andrej Karpathy
Recap: Backpropagation Through Time (BPTT)

- **Configuration**

  \[ h_t = \sigma \left( W_{xh} x_t + W_{hh} h_{t-1} + b \right) \]

  \[ \hat{y}_t = \text{softmax} \left( W_{hy} h_t \right) \]

- **Backpropagated gradient**

  For weight \( w_{ij} \):

  \[
  \frac{\partial E_t}{\partial w_{ij}} = \sum_{1 \leq k \leq t} \left( \frac{\partial E_t}{\partial h_t} \frac{\partial h_t}{\partial h_k} \frac{\partial h_k}{\partial w_{ij}} \right)
  \]
Recap: Backpropagation Through Time (BPTT)

- Analyzing the terms
  - For weight $w_{ij}$: \[ \frac{\partial E_t}{\partial w_{ij}} = \sum_{1 \leq k \leq t} \left( \frac{\partial E_t}{\partial h_t} \frac{\partial h_t}{\partial h_k} \frac{\partial^+ h_k}{\partial w_{ij}} \right) \]
  - This is the “immediate” partial derivative (with $h_{k-1}$ as constant)
Recap: Backpropagation Through Time (BPTT)

- Analyzing the terms

  - For weight $w_{ij}$:
    \[
    \frac{\partial E_t}{\partial w_{ij}} = \sum_{1 \leq k \leq t} \left( \frac{\partial E_t}{\partial h_t} \frac{\partial h_t}{\partial h_k} \frac{\partial^+ h_k}{\partial w_{ij}} \right)
    \]

  - Propagation term:
    \[
    \frac{\partial h_t}{\partial h_k} = \prod_{t \geq i > k} \frac{\partial h_i}{\partial h_{i-1}}
    \]
Recap: Exploding / Vanishing Gradient Problem

- **BPTT equations:**

  \[
  \frac{\partial E_t}{\partial w_{ij}} = \sum_{1 \leq k \leq t} \left( \frac{\partial E_t}{\partial h_t} \frac{\partial h_t}{\partial h_k} \frac{\partial^+ h_k}{\partial w_{ij}} \right)
  \]

  \[
  \frac{\partial h_t}{\partial h_k} = \prod_{t \geq i > k} \frac{\partial h_i}{\partial h_{i-1}} = \prod_{t \geq i > k} W_{hh}^\top \text{diag} (\sigma'(h_{i-1}))
  \]

  \[
  = (W_{hh}^\top)^l
  \]

  (if \( t \) goes to infinity and \( l = t - k \).)

⇒ We are effectively taking the weight matrix to a high power.

- The result will depend on the eigenvalues of \( W_{hh} \).
  - Largest eigenvalue > 1 ⇒ Gradients *may* explode.
  - Largest eigenvalue < 1 ⇒ Gradients *will* vanish.
  - This is very bad...
Recap: Gradient Clipping

- Trick to handle exploding gradients
  - If the gradient is larger than a threshold, clip it to that threshold.

Algorithm 1 Pseudo-code

\[
\hat{g} \leftarrow \frac{\partial \mathcal{E}}{\partial \theta} \\
\text{if } \|\hat{g}\| \geq \text{threshold} \text{ then} \\
\hat{g} \leftarrow \frac{\text{threshold}}{\|\hat{g}\|} \hat{g} \\
\text{end if}
\]

- This makes a big difference in RNNs
Handling Vanishing Gradients

• Vanishing Gradients are a harder problem
  ➢ They severely restrict the dependencies the RNN can learn.
  ➢ The problem gets more severe the deeper the network is.
  ➢ It can be very hard to diagnose that Vanishing Gradients occur (you just see that learning gets stuck).

• Ways around the problem
  ➢ Glorot/He initialization (more on that in Lecture 21)
  ➢ ReLU
  ➢ More complex hidden units (LSTM, GRU)
ReLU to the Rescue

• Idea
  - Initialize $W_{hh}$ to identity matrix
  - Use Rectified Linear Units (ReLU)
    \[ g(a) = \max \{0, a\} \]

• Effect
  - The gradient is propagated with a constant factor
    \[ \frac{\partial g(a)}{\partial a} = \begin{cases} 
    1, & a > 0 \\
    0, & \text{else} 
  \end{cases} \]
  - $\Rightarrow$ Huge difference in practice!
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• Applications of RNNs
More Complex Hidden Units

• Target properties
  - Want to achieve constant error flow through a single unit
  - At the same time, want the unit to be able to pick up long-term connections or focus on short-term ones, as the problem demands.

• Ideas behind LSTMs
  - Take inspiration from the design of memory cells
  - Keep around memories to capture long distance dependencies
  - Allow error messages to flow at different strengths depending on the inputs
Long Short-Term Memory

- RNNs can be seen as chains of repeating modules
  - In a standard RNN, the repeating module has a very simple structure (e.g., a tanh)

Image source: Christopher Olah, [http://colah.github.io/posts/2015-08-Understanding-LSTMs/](http://colah.github.io/posts/2015-08-Understanding-LSTMs/)
Long Short-Term Memory

- **LSTMs**
  - Repeating modules have 4 layers, interacting in a special way.

Image source: Christopher Olah, [http://colah.github.io/posts/2015-08-Understanding-LSTMs/](http://colah.github.io/posts/2015-08-Understanding-LSTMs/)
LSTMs: Core Ideas

- **Cell state**
  - This is the key to LSTMs.
  - It acts like a conveyor belt, information can flow along it unchanged.

- **Gates**
  - The cell state can be modified through gates.
  - Structure: sigmoid net layer + pointwise multiplication
  - The sigmoid outputs values between 0 and 1
    - 0: Let nothing through
    - 1: Let everything through
  - The gate layers are learned together with all other parameters.

Source: Christopher Olah, [http://colah.github.io/posts/2015-08-Understanding-LSTMs/](http://colah.github.io/posts/2015-08-Understanding-LSTMs/)
Elements of LSTMs

• **Forget gate layer**
  - Look at $h_{t-1}$ and $x_t$ and output a number between 0 and 1 for each dimension in the cell state $C_{t-1}$.
    - 0: completely delete this,
    - 1: completely keep this.

  $$f_t = \sigma (W_f \cdot [h_{t-1}, x_t] + b_f)$$

• **Example**
  - Task: try to predict the next word
  - Cell state could include the gender of the present subject
  - When we see a new subject, want to forget the gender of the old subject.
Elements of LSTMs

• **Update gate layer**
  - Decide what information to store in the cell state.
  - Sigmoid network (input gate layer) decides which values are updated.
  - tanh layer creates a vector of new candidate values $\tilde{C}_t$ that could be added to the state.

$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i)$$
$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$

• **In the example**
  - Add the gender of the new subject to the cell state.

Source: Christopher Olah, [http://colah.github.io/posts/2015-08-Understanding-LSTMs/](http://colah.github.io/posts/2015-08-Understanding-LSTMs/)
Elements of LSTMs

- **Updating the state**
  - Multiply the old state by $f_t$, forgetting the things we decided to forget.
  - Then add $i_t \cdot \tilde{C}_t$, the new candidate values, scaled by how much we decided to update each value.

- **In the example**
  - Combined effect: replace the old gender by the new one.

$$C_t = f_t \times C_{t-1} + i_t \times \tilde{C}_t$$

Source: Christopher Olah, [http://colah.github.io/posts/2015-08-Understanding-LSTMs/](http://colah.github.io/posts/2015-08-Understanding-LSTMs/)
Elements of LSTMs

- **Output gate layer**
  - Output is a filtered version of our gate state.
  - First, apply sigmoid layer to decide what parts of the cell state to output.
  - Then, pass the cell state through a tanh (to push the values to be between -1 and 1) and multiply it with the output of the sigmoid gate.

\[
o_t = \sigma (W_o \ [h_{t-1}, x_t] + b_o)
\]
\[
h_t = o_t \times \tanh (C_t)
\]

- In the example
  - Since we just saw a subject, might want to output information relevant to a verb (e.g., whether the subject is singular or plural).

Source: Christopher Olah, [http://colah.github.io/posts/2015-08-Understanding-LSTMs/](http://colah.github.io/posts/2015-08-Understanding-LSTMs/)
RNN vs. LSTM

- LSTM just changes the form of the equation for $h$ such that:
  1. More expressive multiplicative interactions become possible
  2. Gradients flow nicer
  3. The network can explicitly decide to reset the hidden state

- Those changes have a huge effect in practice
  - LSTMs perform much better than regular RNNs
  - Many applications have become possible with LSTMs that weren’t feasible before.
LSTMs in Practice

- LSTMs are currently highly en vogue
  - Popular default model for most sequence labeling tasks.
  - Very powerful, especially when stacked and made even deeper.
  - Most useful if you have lots and lots of data.
  ⇒ Very active research field

- Here are also some other ways of illustrating them
Extension: Gated Recurrent Units (GRU)

- Simpler model than LSTM
  - Combines the forget and input gates into a single update gate $z_t$.
  - Similar definition for a reset gate $r_t$, but with different weights.
  - In both cases, merge the cell state and hidden state.

- Empirical results
  - Performance similar to LSTM (no clear winner yet)
  - But GRU has fewer parameters.

\[
\begin{align*}
    z_t &= \sigma (W_z \cdot [h_{t-1}, x_t]) \\
    r_t &= \sigma (W_r \cdot [h_{t-1}, x_t]) \\
    \tilde{h}_t &= \tanh (W \cdot [r_t \ast h_{t-1}, x_t]) \\
    h_t &= (1 - z_t) \ast h_{t-1} + z_t \ast \tilde{h}_t
\end{align*}
\]
GRUs: Intuition

• Effects
  ➢ If reset is close to 0, ignore previous hidden state.
    ⇒ Allows model to drop information that is irrelevant in the future.
  ➢ Update gate $z$ controls how much of past state should matter now.
    ⇒ If $z$ is close to 0, then we can copy information in that unit through many time steps!
    ⇒ Less vanishing gradients!

\[
\begin{align*}
  z_t &= \sigma (W_z \cdot [h_{t-1}, x_t]) \\
  r_t &= \sigma (W_r \cdot [h_{t-1}, x_t]) \\
  \tilde{h}_t &= \tanh (W \cdot [r_t * h_{t-1}, x_t]) \\
  h_t &= (1 - z_t) * h_{t-1} + z_t * \tilde{h}_t
\end{align*}
\]
GRUs: Intuition

- Typical learned behaviors
  - Units with short-term dependencies often have active reset gate
  - Units with long-term dependencies have inactive update gates.

\[
\begin{align*}
    z_t &= \sigma (W_z \cdot [h_{t-1}, x_t]) \\
    r_t &= \sigma (W_r \cdot [h_{t-1}, x_t]) \\
    \tilde{h}_t &= \tanh (W \cdot [r_t \ast h_{t-1}, x_t]) \\
    h_t &= (1 - z_t) \ast h_{t-1} + z_t \ast \tilde{h}_t
\end{align*}
\]
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• Applications of RNNs
Applications

- Machine Translation [Sutskever et al., 2014]
Application: Character-Level Language Model

- **Setup**
  - RNN trained on huge amounts of text
  - Task: model the probability distribution of the next character in the sequence.

- **Main advantage of RNN here**
  - RNN can learn varying amount of context
Language Model Results

PANDARUS:
Alas, I think he shall be come approached and the day
When little srain would be attain’d into being never fed,
And who is but a chain and subjects of his death,
I should not sleep.

Second Senator:
They are away this miseries, produced upon my soul,
Breaking and strongly should be buried, when I perish
The earth and thoughts of many states.

DUKE VINCENTIO:
Well, your wit is in the care of side and that.

Second Lord:
They would be ruled after this chamber, and
my fair nues begun out of the fact, to be conveyed,
Whose noble souls I’ll have the heart of the wars.

- Example: Generating Shakespeare
  - Trained on all works of Shakespeare (4.4 MB of data)
  - Using a 3-Layer RNN with 512 hidden units per layer
Language Model Results

Naturalism and decision for the majority of Arab countries' capitalide was grounded by the Irish language by [[John Clair]], [[An Imperial Japanese Revolt]], associated with Guangzham's sovereignty. His generals were the powerful ruler of the Portugal in the [[Protestant Immininners]], which could be said to be directly in Cantonese Communication, which followed a ceremony and set inspired prison, training. The emperor travelled back to [[Antioch, Perth, October 25|21]] to note, the Kingdom of Costa Rica, unsuccessful fashioned the [[Thrales]], [[Cynth's Dajoard]], known in western [[Scotland]], near Italy to the conquest of India with the conflict. Copyright was the succession of independence in the slop of Syrian influence that was a famous German movement based on a more popular servocious, non-doctrinal and sexual power post. Many governments recognize the military housing of the [[Civil Liberalization and Infantry Resolution 265 National Party in Hungary]], that is sympathetic to be to the [[Punjab Resolution]]

(PJS)[http://www.humah.yahoo.com/guardian.cfm/7754800786d17551963s89.htm Official economics Adjoint for the Nazism, Montgomery was swear to advance to the resources for those Socialism's rule, was starting to signing a major tripad of aid exile.]]

- Example: Generating Wikipedia pages
  - Trained on 100MB of Wikipedia data
  - Using an LSTM
Language Model Results

For $\bigoplus_{n=1,...,m}$ where $\mathcal{L}_m = 0$, hence we can find a closed subset $\mathcal{H}$ in $\mathcal{H}$ and any sets $\mathcal{F}$ on $X$, $U$ is a closed immersion of $S$, then $U \to T$ is a separated algebraic space.

Proof. Proof of (1). It also start we get

$$S = \text{Spec}(R) = U \times_X U \times_X U$$

and the coproduct in the fibre product covering we have to prove the lemma generated by $\bigsqcup Z \times_U U \to V$. Consider the maps $M$ along the set of points $\text{Sch}_{fppf}$ and $U \to U$ is the fibre category of $S$ in $U$ in Section, ?? and the fact that any $U$ affine, see Morphisms, Lemma ???. Hence we obtain a scheme $S$ and any open subset $W \subset U$ in $\text{Sh}(G)$ such that $\text{Spec}(R') \to S$ is smooth or an

$$U = \bigcup U_i \times_{S_i} U_i$$

which has a nonzero morphism we may assume that $f_i$ is of finite presentation over $S$. We claim that $\mathcal{O}_{X,x}$ is a scheme where $x, x', s'' \in S'$ such that $\mathcal{O}_{X,x} \to \mathcal{O}_{X',x'}$ is

- Example: Hallucinating Algebraic Geometry
  - Trained on an Algebraic Geometry book
  - Using a multilayer LSTM
Language Model Results

```c
/*
 * Increment the size file of the new incorrect UI_FILTER group information
 * of the size generatively.
 */

static int indicate_policy(void)
{
    int error;
    if (fd == MARN_EPT) {
        /*
         * The kernel blank will coedl it to userspace.
         */
        if (ss->segment < mem_total)  
            unblock_graph_and_set_blocked();
    }
    else
        ret = 1;
    goto bail;
}

segaddr = in_SB(in.addr);
selector = seg / 16;
setup_works = true;
for (i = 0; i < blocks; i++) {
    seq = buf[i++];
    bpf = bd->bd.next + i * search;
    if (fd) {
```

- **Example:**
  - Hallucinating C Code
  - Trained on the Linux source code (474MB from github)
  - Using a large 3-layer LSTM
Applications: Image Tagging

- Simple combination of CNN and RNN
  - Use CNN to define initial state $h_0$ of an RNN.
  - Use RNN to produce text description of the image.
Applications: Image Tagging

- Setup
  - Train on corpus of images with textual descriptions
  - E.g. Microsoft CoCo
    - 120k images
    - 5 sentences each
Results: Image Tagging

Spectacular results!

Slide adapted from Andrej Karpathy

B. Leibe
Results: Image Tagging

- Wrong, but one can still see why those results were selected...
Results: Image Tagging

- Not sure what happened here...
Fun Application: Image to Story

Later on the eighth day, Billy was a friend of a man who lived on his own. He didn't know how far away they were, and if he was to survive the fall. His mind raced, trying not to show any signs of weakness. The wind ruffled the snow and ice in the snow. He had no idea how many times he was going to climb into the mountains. He told me to stay on the ground for a while, but if I find out what's going on, we should go on foot. Sam and Si Lei joined us in the army.

- Example: Generating a story from an image
  - Trained on corpus of adventure novels

Having lain on the bed, I didn't know what to say. He turned his attention to the room and saw a large room. The room was furnished with a single bed, a dresser and a large bed with a table in the center of the room. It was a long time ago. The room was designed with the most powerful and efficient ones. As far as I'm concerned, it was a long time ago. On the other side of the room was a beautiful picture of a woman who had been abducted by the fireplace and their own personal belongings in order to keep it safe, but it didn't take too long. Feeling helpless, he turned his attention back to me.```
Only Prince Darin knew how to run from the mountains, and once more, he could see the outline of a rider on horseback. The wind ruffled his hair in an attempt to locate the forest. He hadn't been in such a state of mind before, but it was a good thing. All of them seemed to be doing the same thing. They didn't know where they came from. The wind blew up the mountain peaks and disappeared into the sky, leaving trails behind the peaks of the mountains on Mount Fuji.
Application: Video to Text Description

Our LSTM network is connected to a CNN for RGB frames or a CNN for optical flow images.

- Raw Frames
- CNN - Object pretrained
- CNN Outputs
- LSTMs
  - man is cutting a bottle
  - <eos>

Source: Subhashini Venugopalan, ICCV'15
Video-to-Text Results

**Correct descriptions.**

S2VT: A man is doing stunts on his bike.
S2VT: A herd of zebras are walking in a field.
S2VT: A young woman is doing her hair.
S2VT: A man is shooting a gun at a target.

**Relevant but incorrect descriptions.**

S2VT: A small bus is running into a building.
S2VT: A man is cutting a piece of a pair of a paper.
S2VT: A cat is trying to get a small board.
S2VT: A man is spreading butter on a tortilla.

**Irrelevant descriptions.**

S2VT: A man is pouring liquid in a pan.
S2VT: A polar bear is walking on a hill.
S2VT: A man is doing a pencil.
S2VT: A black clip to walking through a path.

Source: Subhashini Venugopalan, ICCV'15
References and Further Reading

- **RNNs**

- **LSTM**
  - C. Olah, *Understanding LSTM Networks*, blog post, August 2015.