Computer Vision – Lecture 17

Epipolar Geometry & Stereo Basics

19.01.2016

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Announcements

- Exam Dates
  - 1st try: 29.02. 13:30 - 17:30h in AH I/II + AH VI, UMIC 025
  - 2nd try: 31.03. 09:40 - 12:40h in UMIC 025 + AH IV

  - *We will send around an email announcing the precise start/end times and your assigned exam rooms.*

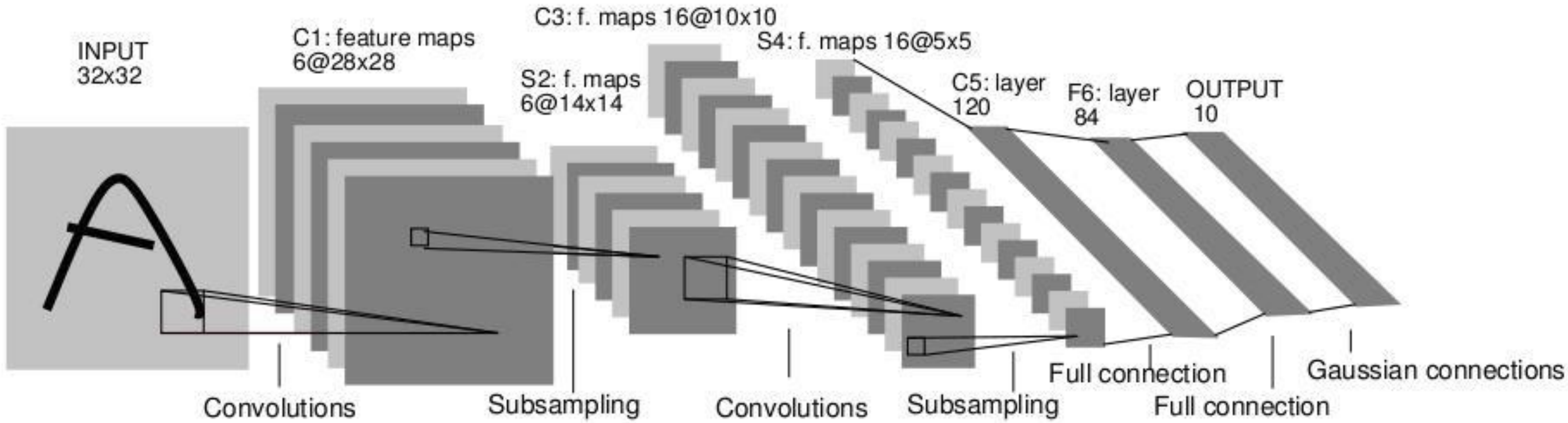
Announcements (2)

- **Seminar in the summer semester**
  - *"Current Topics in Computer Vision and Machine Learning"*
  - Block seminar, presentations at beginning of semester break
  - Registration period: 14.01.2016 – 27.01.2016
  - https://www.graphics.rwth-aachen.de/apse/check.php

Course Outline

- Image Processing Basics
- Segmentation & Grouping
- Object Recognition
- Local Features & Matching
- Object Categorization
- 3D Reconstruction
  - Epipolar Geometry and Stereo Basics
  - Camera calibration & Uncalibrated Reconstruction
  - Multi-view Stereo
- Optical Flow

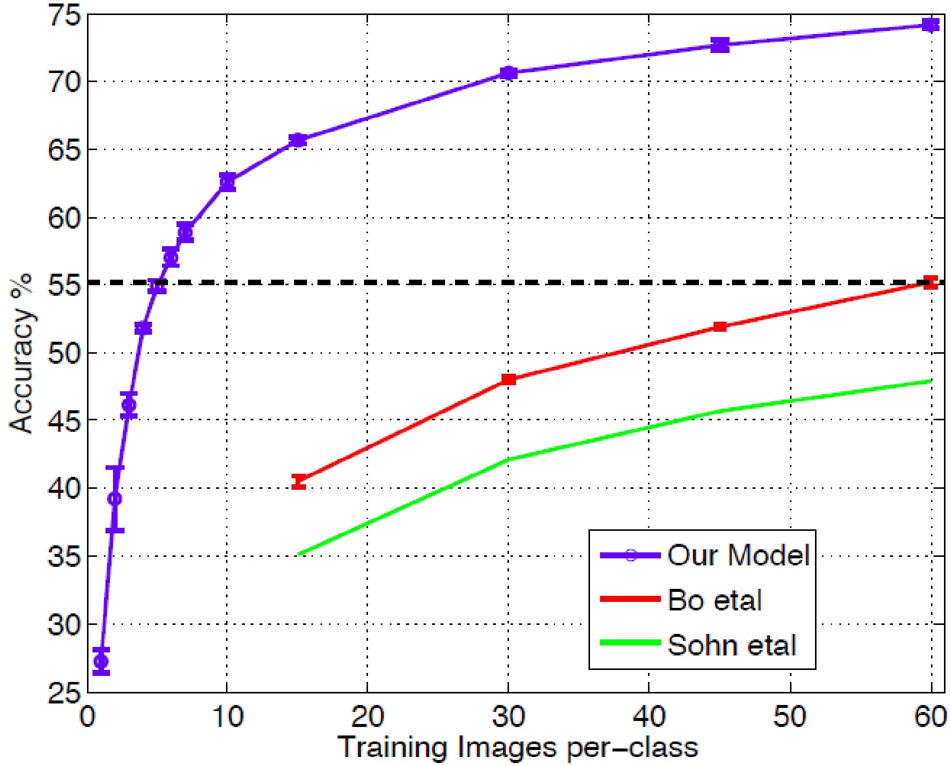
Recap: Convolutional Neural Networks



- Neural network with specialized connectivity structure
  - Stack multiple stages of feature extractors
  - Higher stages compute more global, more invariant features
  - Classification layer at the end

Y. LeCun, L. Bottou, Y. Bengio, and P. Haffner, Gradient-based learning applied to document recognition, Proceedings of the IEEE 86(11): 2278–2324, 1998.

The Learned Features are Generic



state of the art level (pre-CNN)

- **Experiment: feature transfer**
  - Train AlexNet-like network on ImageNet
  - Chop off last layer and train classification layer on CalTech256

⇒ State of the art accuracy already with only 6 training images!

Image source: M. Zeiler, R. Fergus

Transfer Learning with CNNs

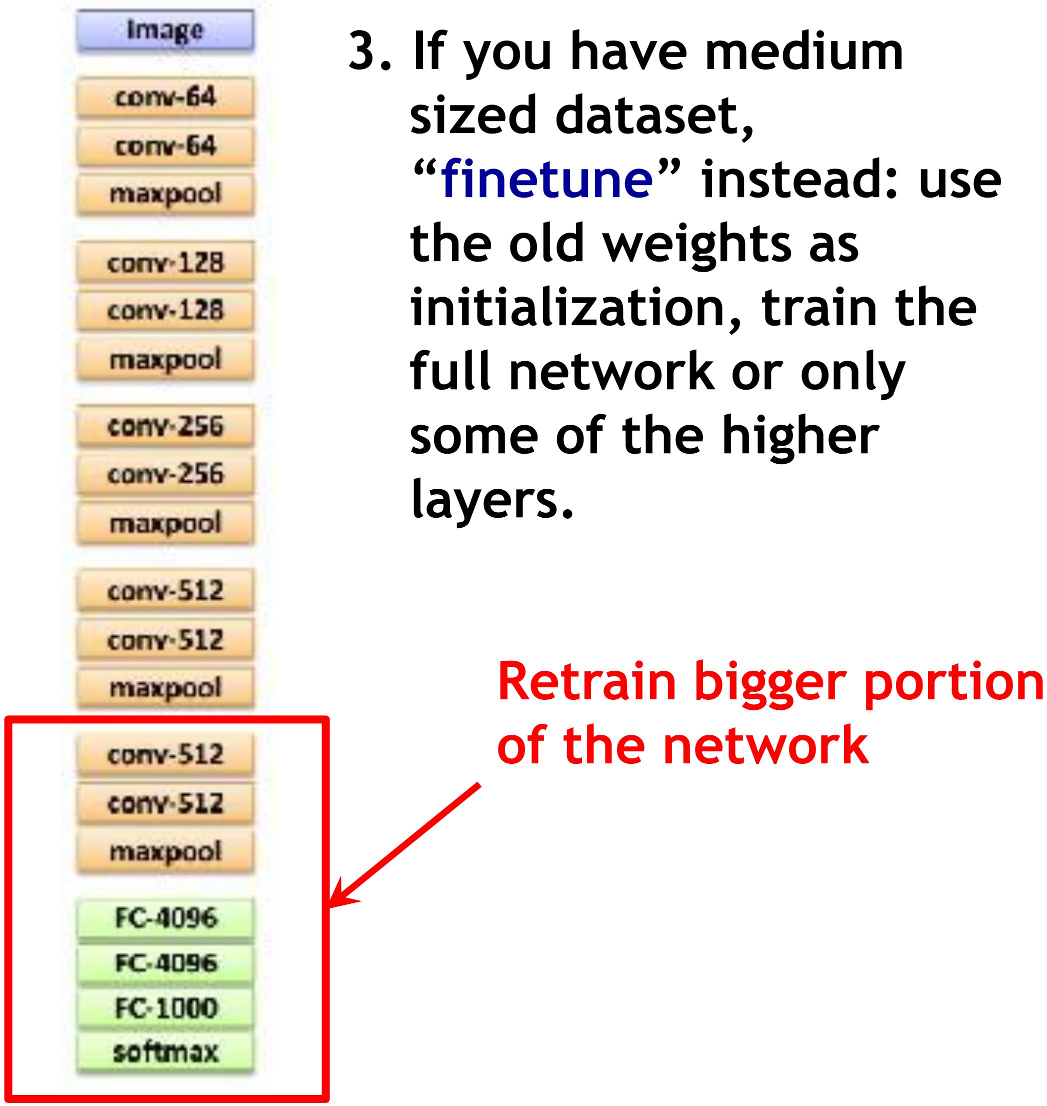
1. Train on ImageNet

2. If small dataset: fix all weights (treat CNN as fixed feature extrac-tor), retrain only the classifier

I.e., swap the Softmax layer at the end

Transfer Learning with CNNs

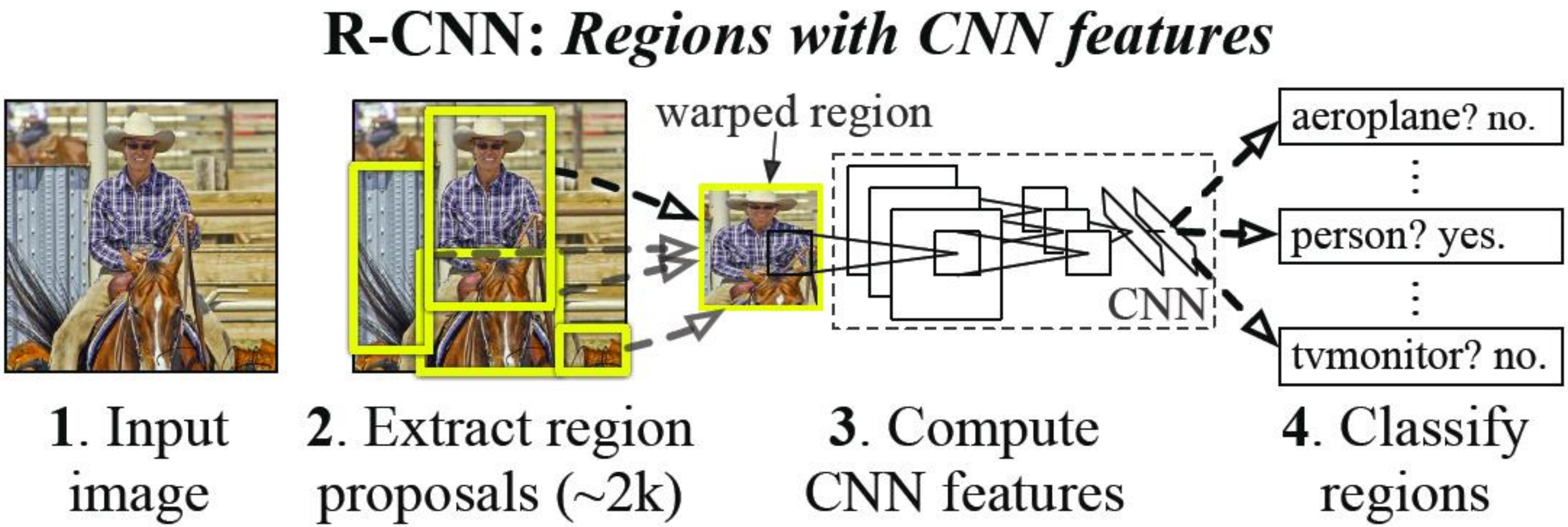
1. Train on ImageNet



3. If you have medium sized dataset, "finetune" instead: use the old weights as initialization, train the full network or only some of the higher layers.

Slide credit: Andrej Karpathy

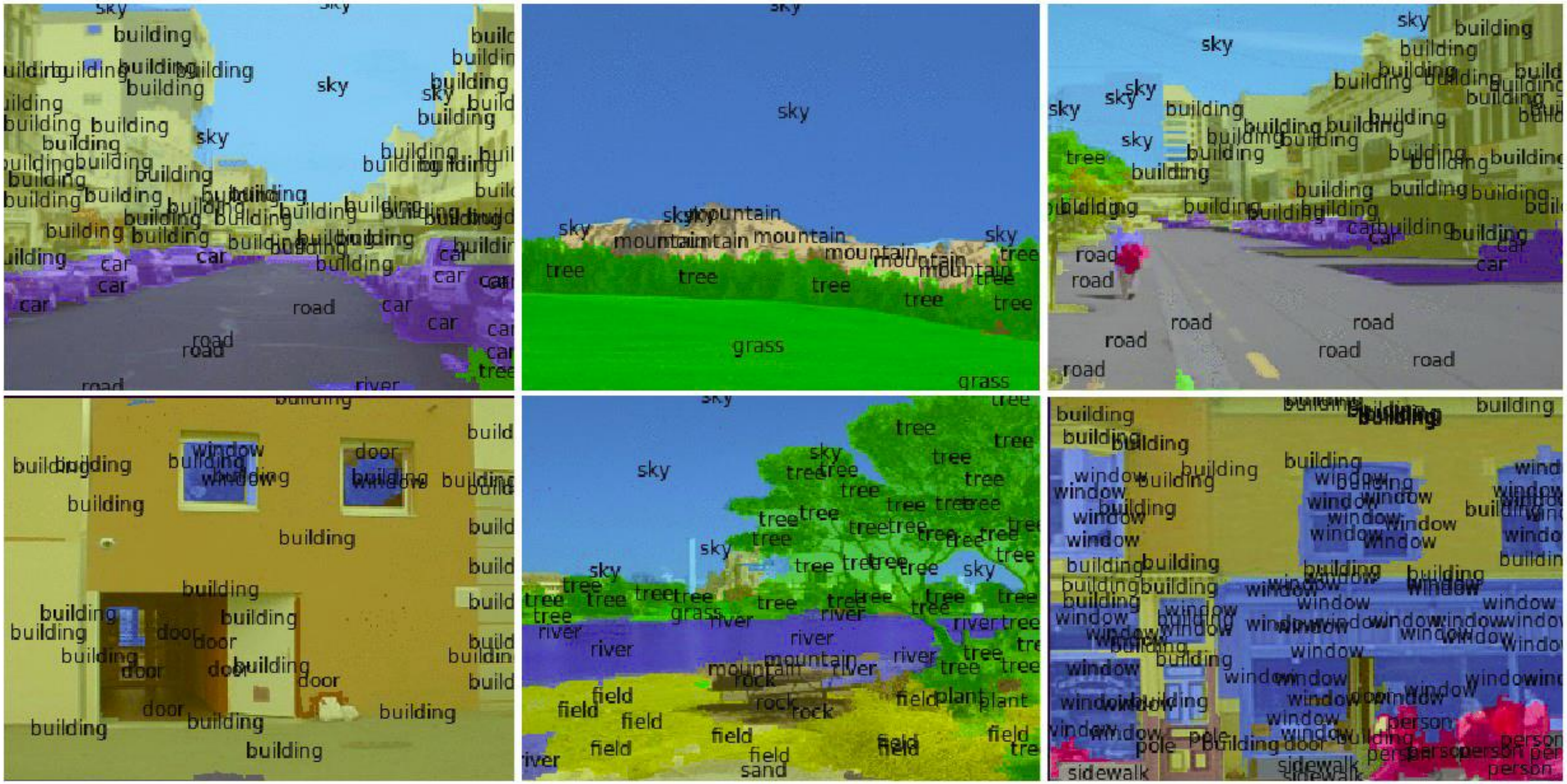
Other Tasks: Detection



- **Results on PASCAL VOC Detection benchmark**
  - **Pre-CNN state of the art: 35.1% mAP [Uijlings et al., 2013]**
    33.4% mAP DPM
  - **R-CNN: 53.7% mAP**

R. Girshick, J. Donahue, T. Darrell, and J. Malik, Rich Feature Hierarchies for Accurate Object Detection and Semantic Segmentation, CVPR 2014

Other Tasks: Semantic Segmentation

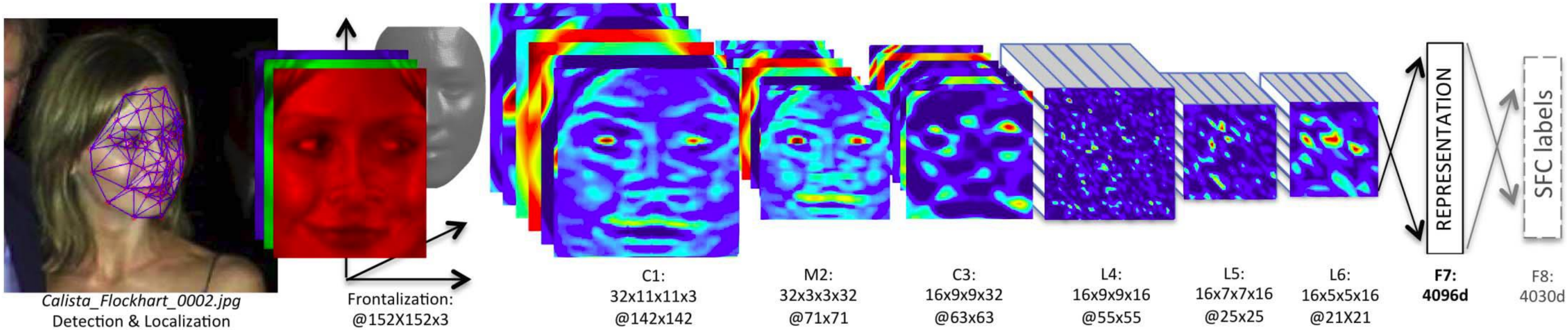


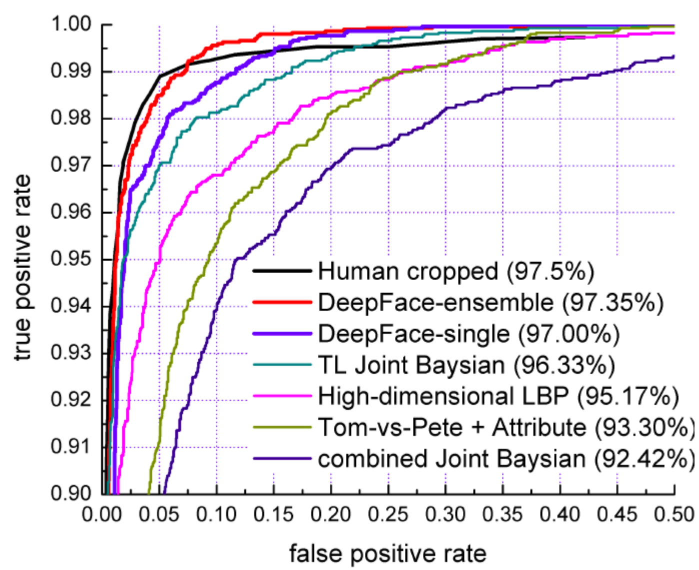
[Farabet et al. ICML 2012, PAMI 2013]

Other Tasks: Semantic Segmentation

[Farabet et al. ICML 2012, PAMI 2013]

Other Tasks: Face Verification





Y. Taigman, M. Yang, M. Ranzato, L. Wolf, DeepFace: Closing the Gap to Human-Level Performance in Face Verification, CVPR 2014

Slide credit: Svetlana Lazebnik

Commercial Recognition Services

- E.g., clarifai

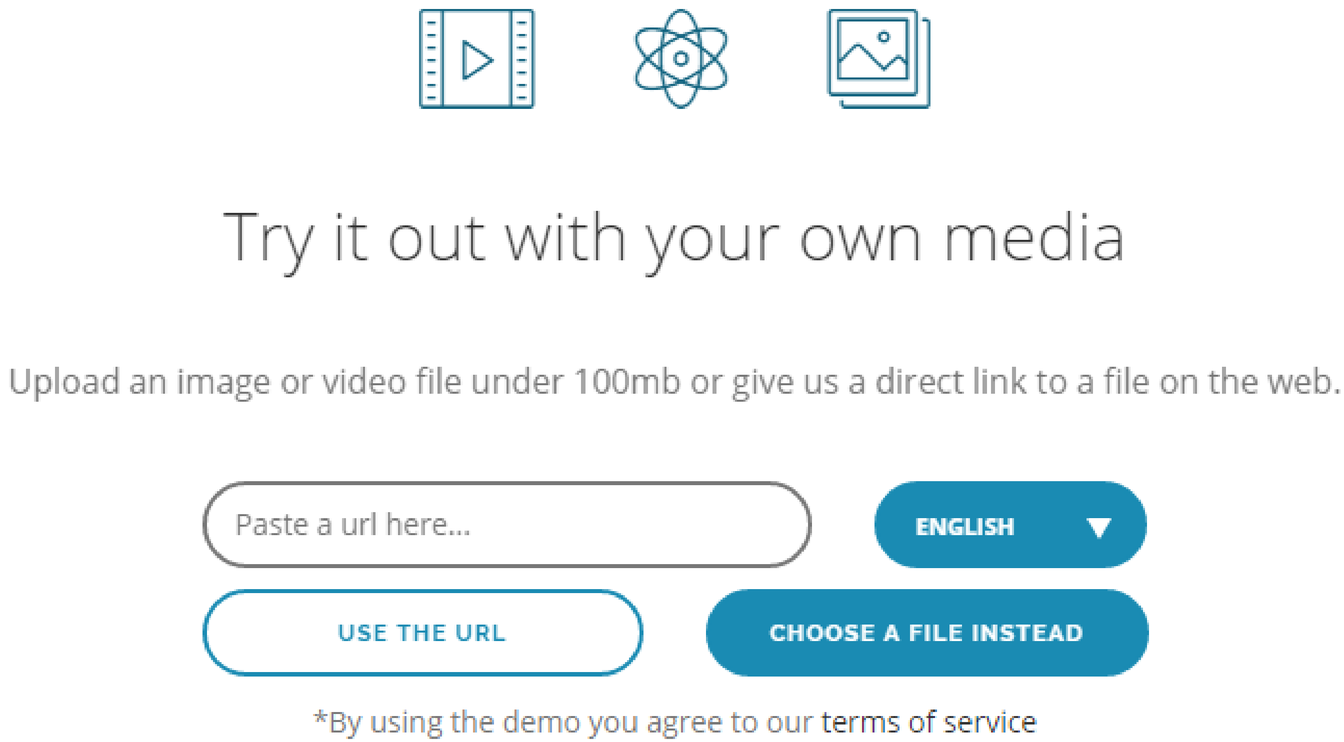
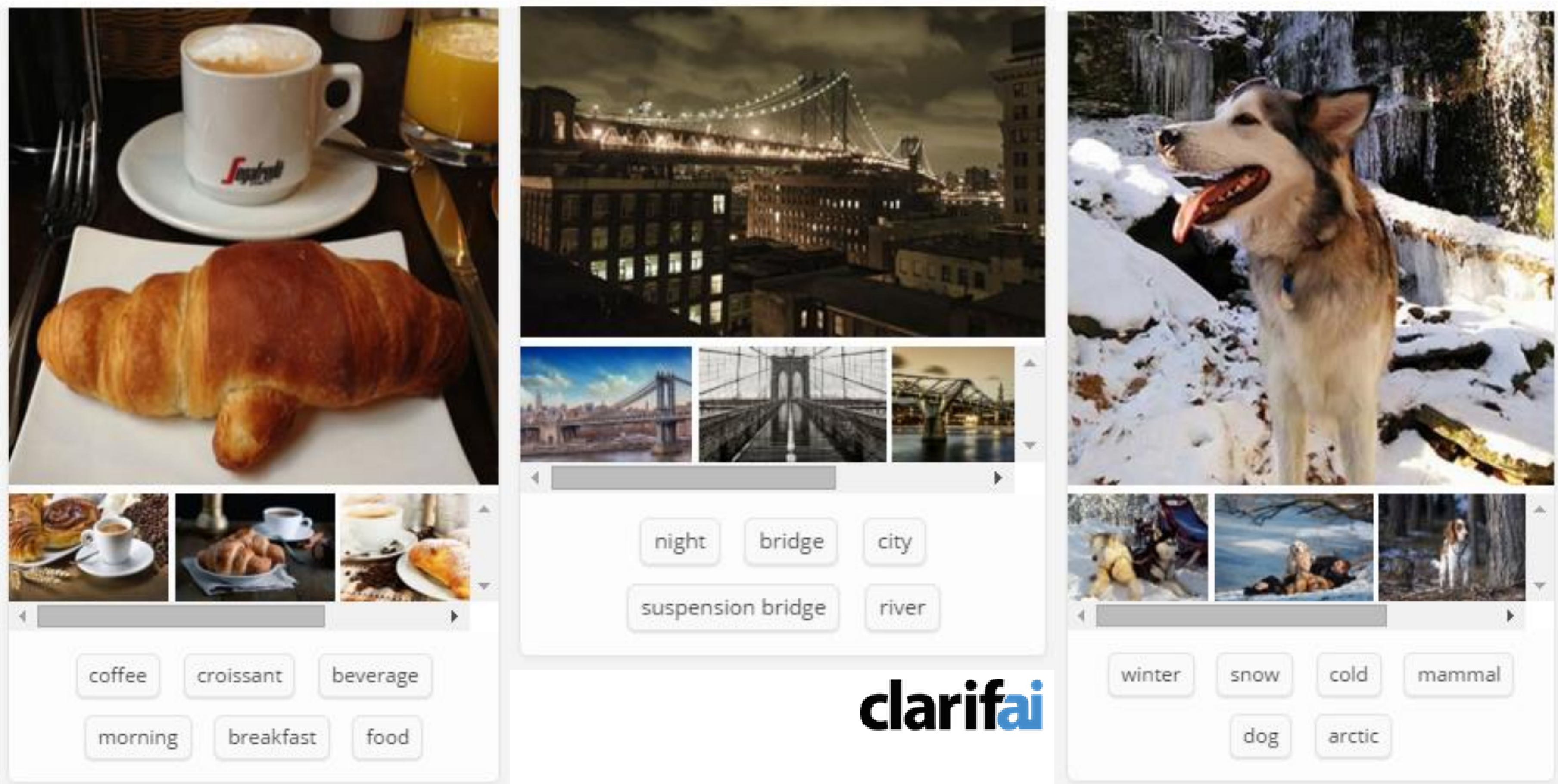


Image source: clarifai.com

Commercial Recognition Services



- Be careful when testing with images from Google Search
  - Chances are they may have been seen in the training set...

Image source: clarifai.com

Topics of This Lecture

- Geometric vision
  - Visual cues
  - Stereo vision
- Epipolar geometry
  - Depth with stereo
  - Geometry for a simple stereo system
  - Case example with parallel optical axes
  - General case with calibrated cameras
- Stereopsis & 3D Reconstruction
  - Correspondence search
  - Additional correspondence constraints
  - Possible sources of error
  - Applications

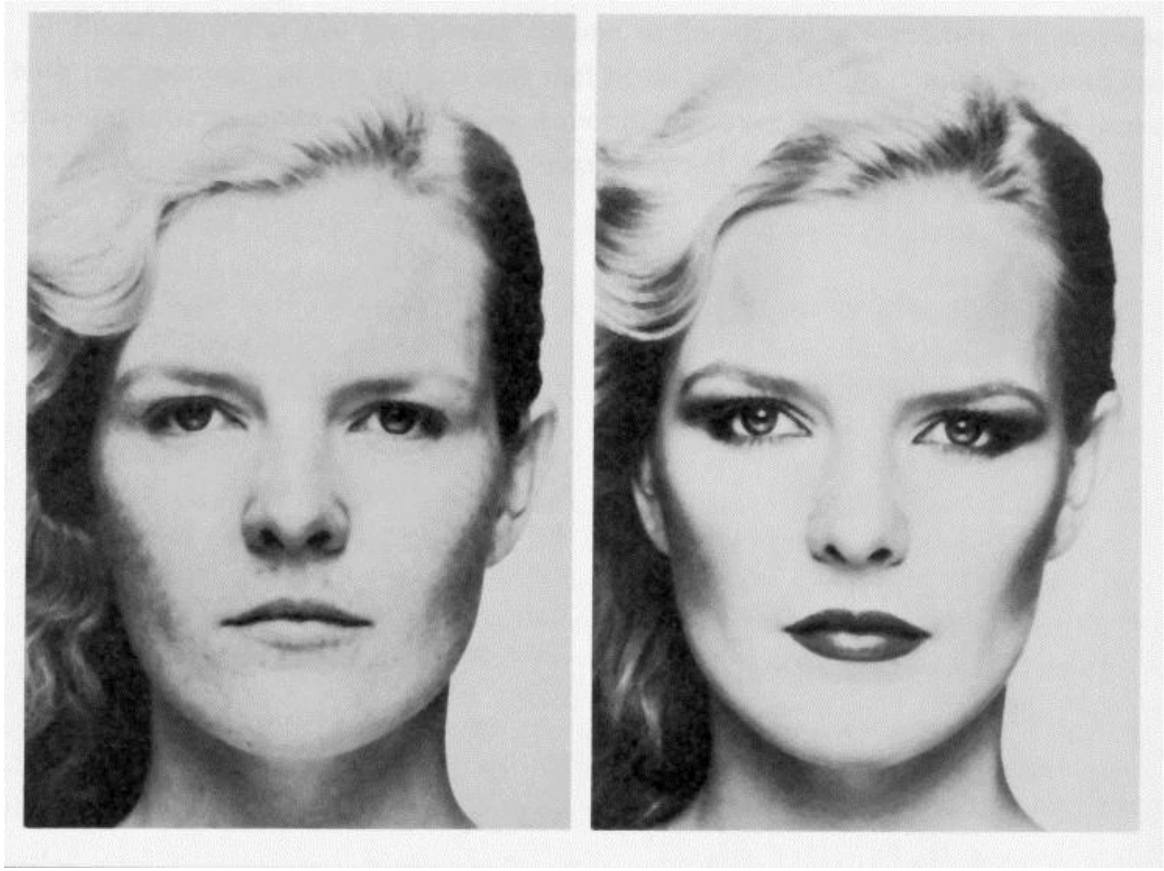
Geometric vision

- Goal: Recovery of 3D structure
  - What cues in the image allow us to do this?

Slide credit: Svetlana Lazebnik

Visual Cues

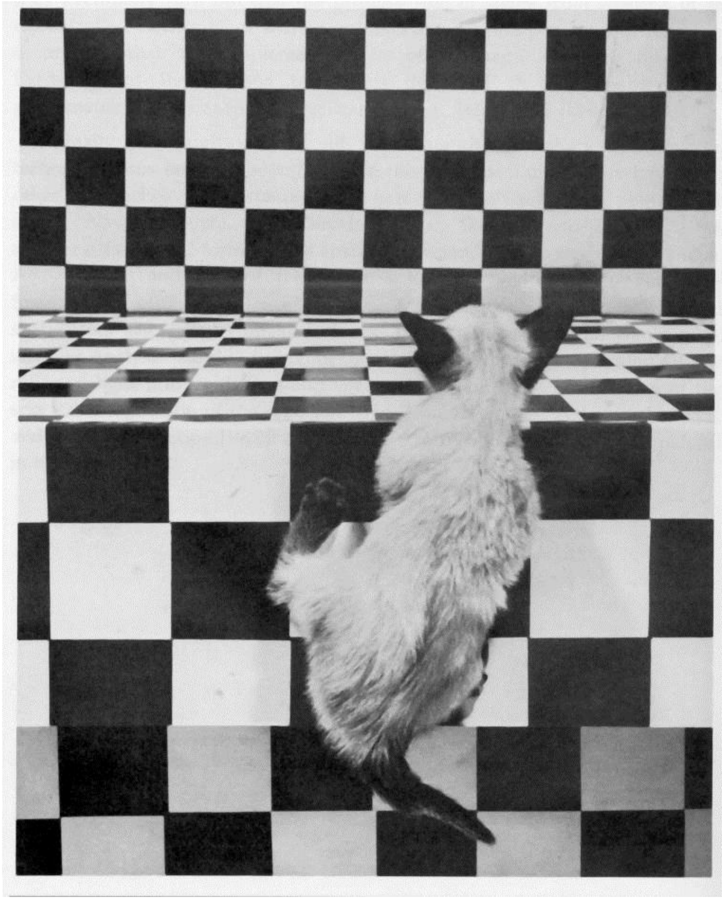
- Shading



Merle Norman Cosmetics, Los Angeles

Visual Cues

- Shading
- **Texture**



The Visual Cliff, by William Vandivert, 1960

Visual Cues

- Shading
- Texture
- **Focus**

From *The Art of Photography*, Canon

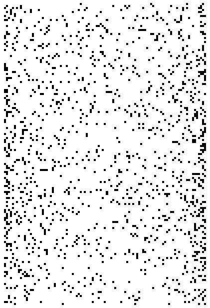
Visual Cues

- Shading
- Texture
- Focus
- **Perspective**

Visual Cues

- Shading
- Texture
- Focus
- Perspective
- **Motion**

Figures from L. Zhang

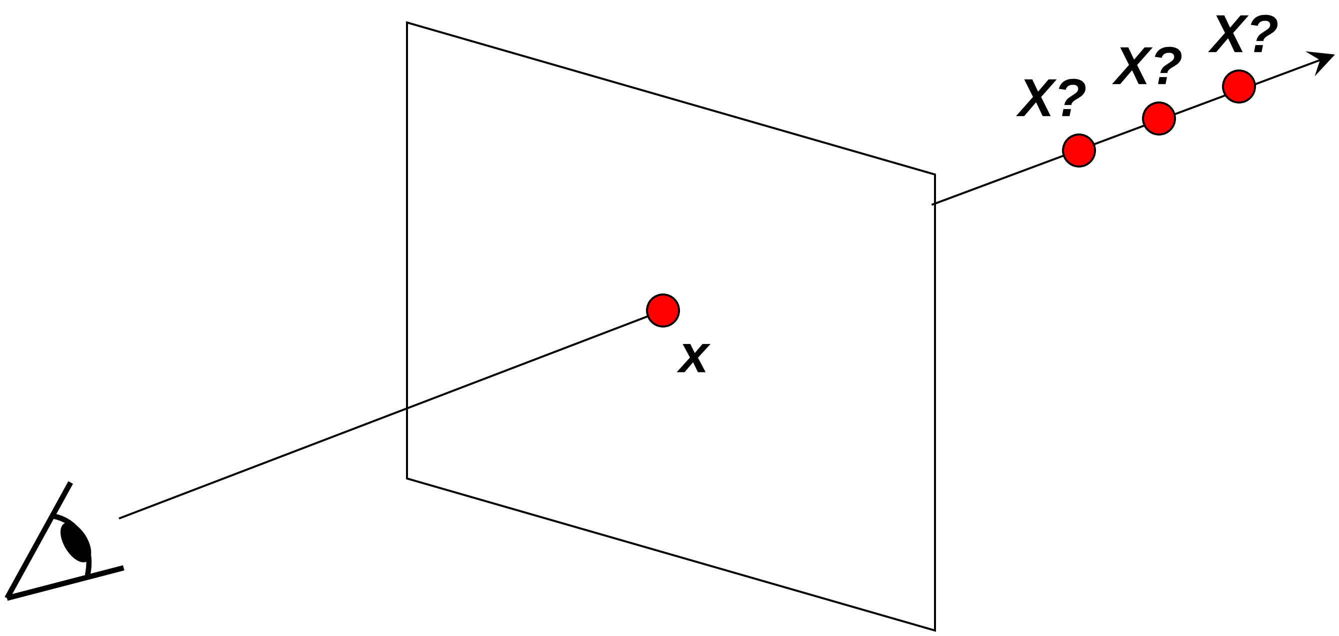


Slide credit: Steve Seitz, Kristen Grauman

http://www.brainconnection.com/teasers/?main=illusion/motion-shape

Our Goal: Recovery of 3D Structure

- We will focus on perspective and motion
- We need *multi-view geometry* because recovery of structure from one image is inherently ambiguous



To Illustrate This Point…

- **Structure and depth are inherently ambiguous from single views.**

Stereo Vision

http://www.well.com/~jimg/stereo/stereo_list.html

Slide credit: Kristen Grauman

What Is Stereo Vision?

- Generic problem formulation: given several images of the same object or scene, compute a representation of its 3D shape

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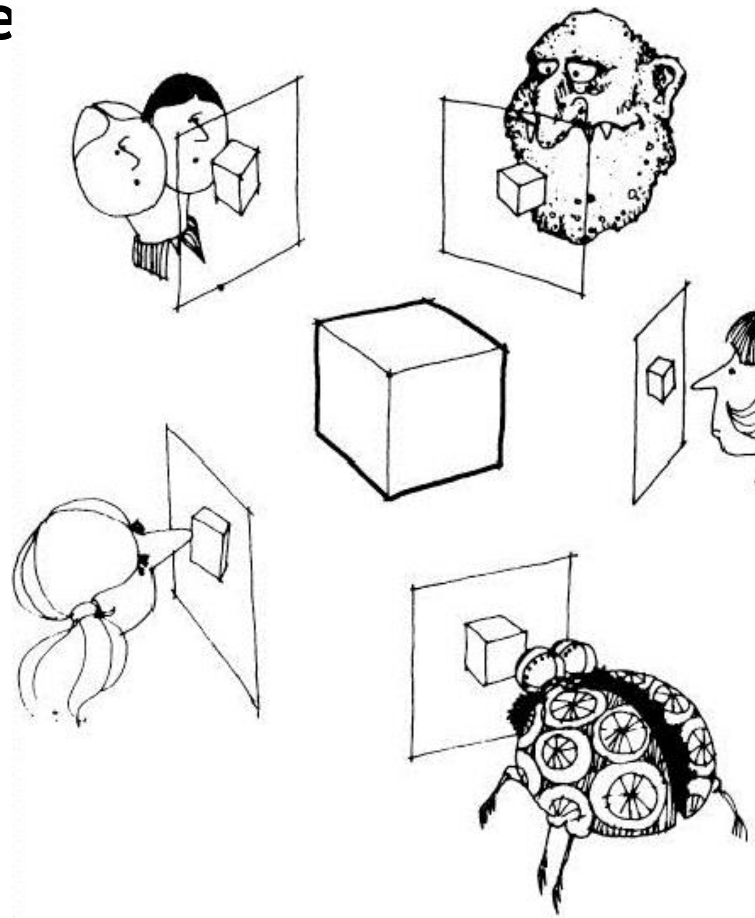


What Is Stereo Vision?

- **Narrower formulation: given a calibrated binocular stereo pair, fuse it to produce a depth image**

Image 1

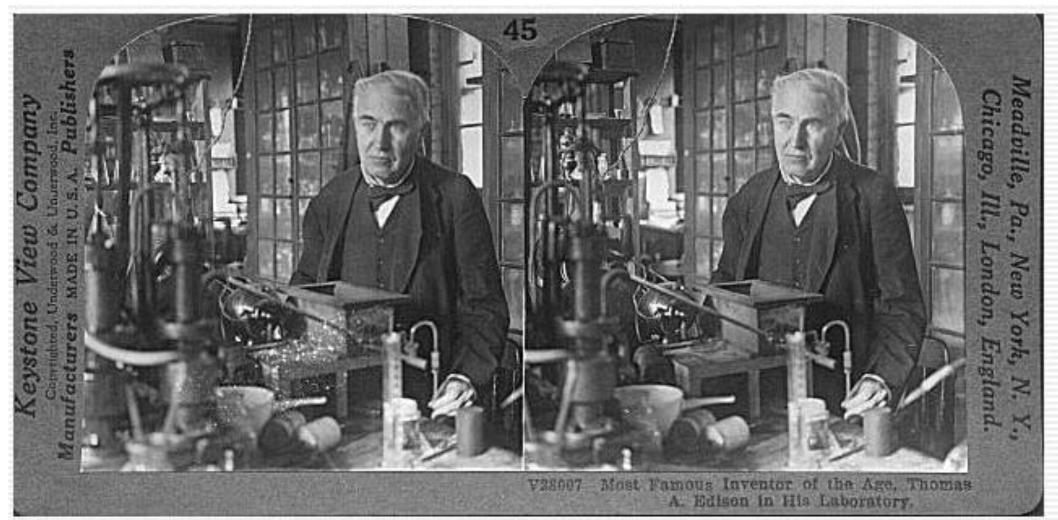
Image 2

Dense depth map

Slide credit: Svetlana Lazebnik, Steve Seitz

What Is Stereo Vision?

- **Narrower formulation: given a calibrated binocular stereo pair, fuse it to produce a depth image.**
  - Humans can do it



Stereograms: Invented by Sir Charles Wheatstone, 1838

What Is Stereo Vision?

- **Narrower formulation: given a calibrated binocular stereo pair, fuse it to produce a depth image.**
  - Humans can do it

Autostereograms: http://www.magiceye.com

Slide credit: Svetlana Lazebnik, Steve Seitz

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- **Narrower formulation: given a calibrated binocular stereo pair, fuse it to produce a depth image.**
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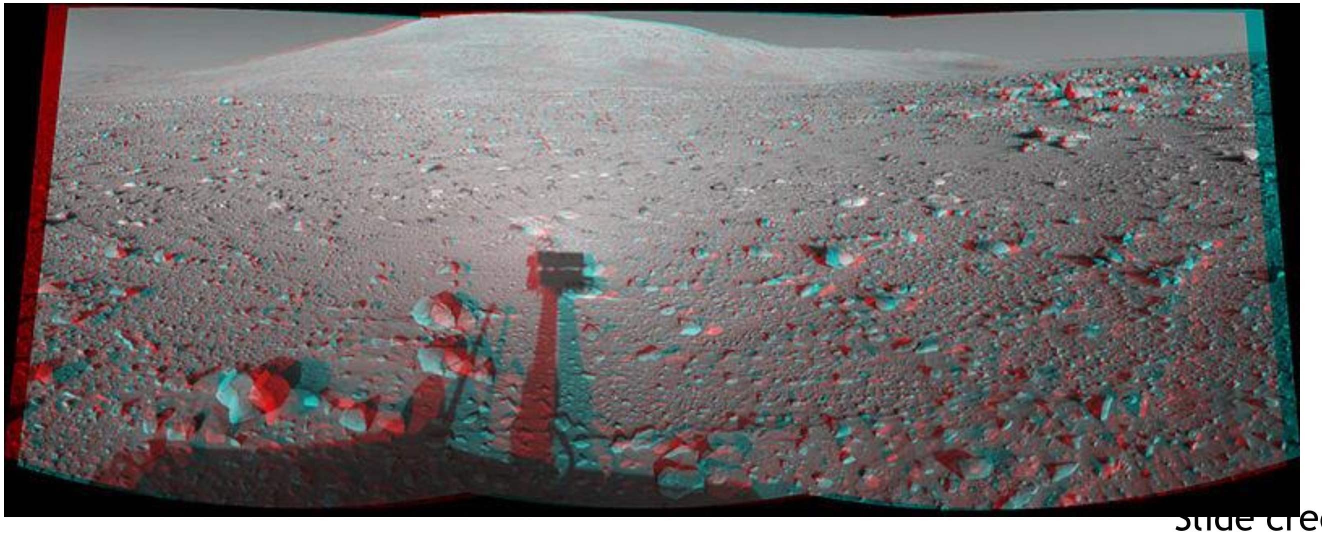
Autostereograms: http://www.magiceye.com

Slide credit: Svetlana Lazebnik, Steve Seitz

Application of Stereo: Robotic Exploration

Nomad robot searches for meteorites in Antartica

Real-time stereo on Mars



Slide credit: Steve Seitz

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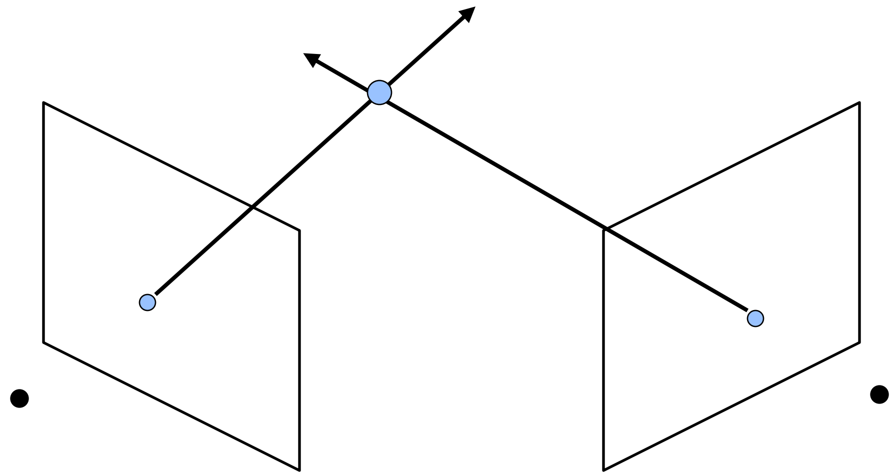
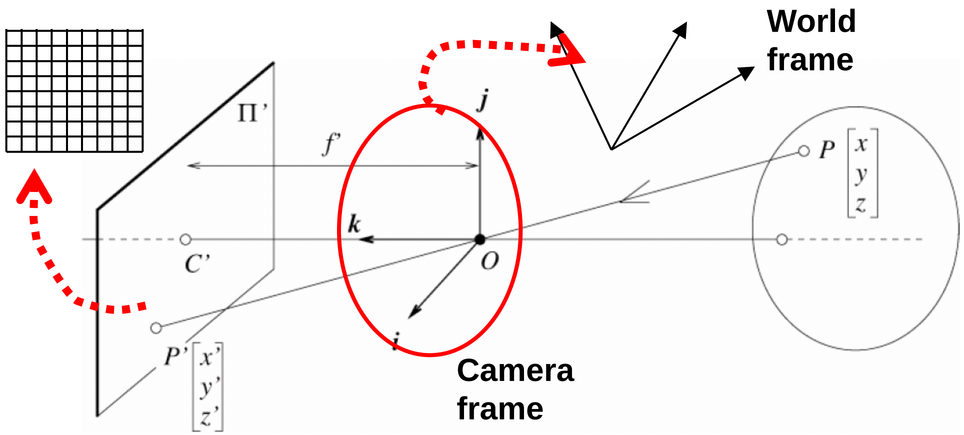
Depth with Stereo: Basic Idea



- **Basic Principle: Triangulation**
  - Gives reconstruction as intersection of two rays
  - Requires
    - Camera pose (calibration)
    - Point correspondence

Camera Calibration



Extrinsic parameters:
Camera frame ↔ Reference frame

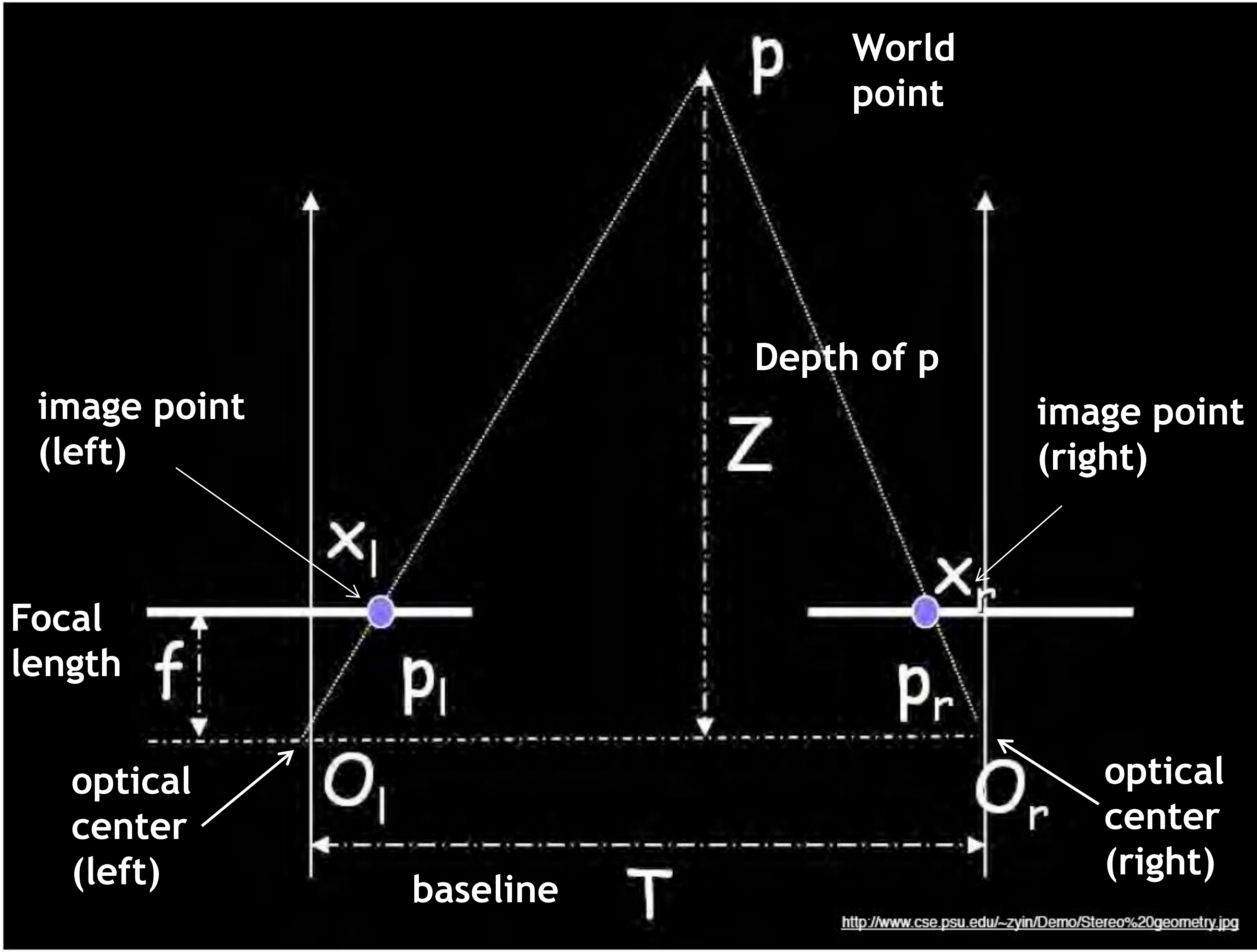
Intrinsic parameters:
Image coordinates relative to camera ↔ Pixel coordinates

- **Parameters**
  - *Extrinsic*: rotation matrix and translation vector
  - *Intrinsic*: focal length, pixel sizes (mm), image center point, radial distortion parameters

We'll assume for now that these parameters are given and fixed.

Geometry for a Simple Stereo System

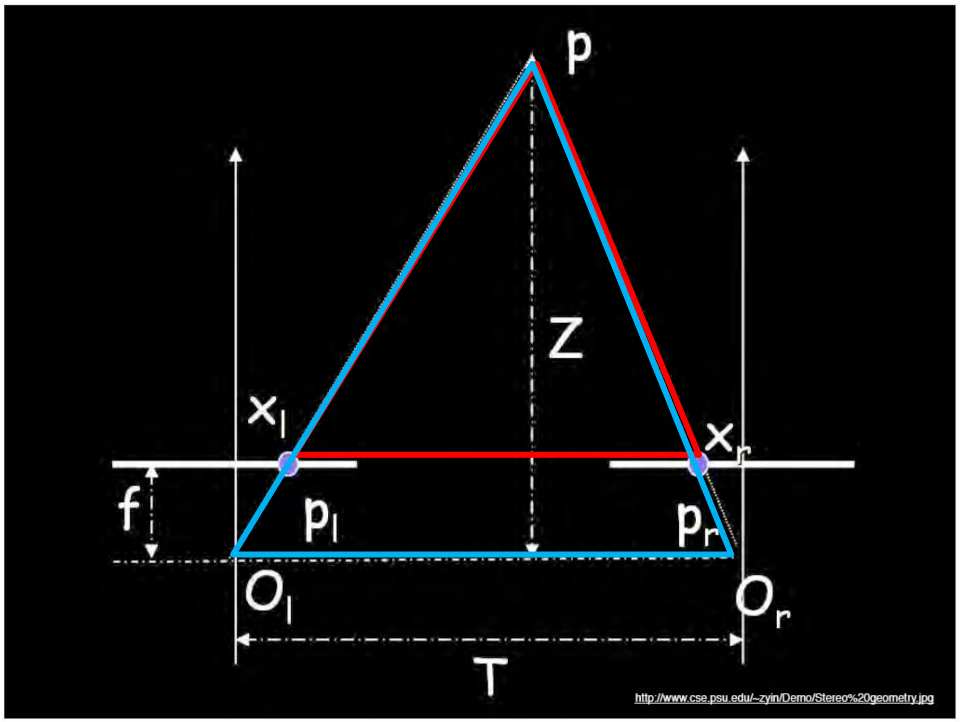
- First, assuming parallel optical axes, known camera parameters (i.e., calibrated cameras):



Slide credit: Kristen Grauman

Geometry for a Simple Stereo System

- Assume parallel optical axes, known camera parameters (i.e., calibrated cameras). We can triangulate via:



Similar triangles (p_l, P, p_r) and (O_l, P, O_r):

$$\frac{T - (x_r - x_l)}{Z - f} = \frac{T}{Z}$$

$$Z = f\frac{T}{x_r - x_l}$$

"disparity"

Depth From Disparity

Image $I(x,y)$

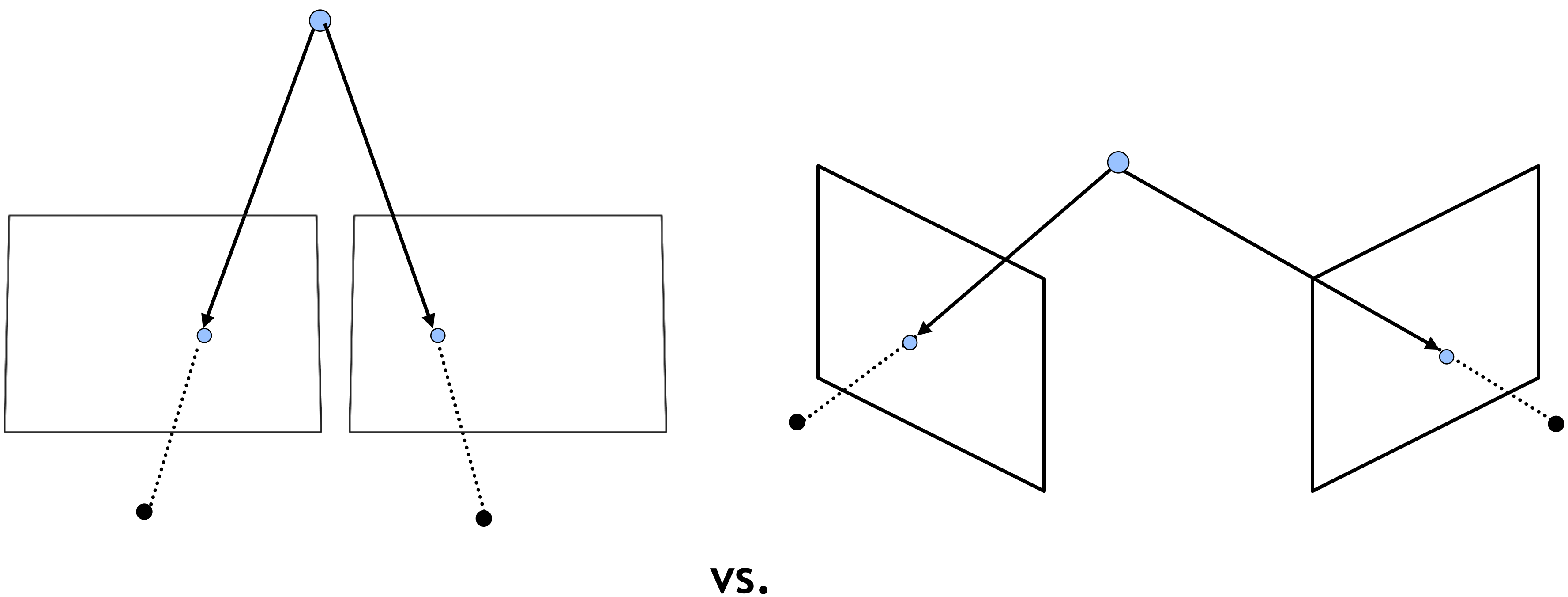
Disparity map $D(x,y)$

Image $I'(x',y')$

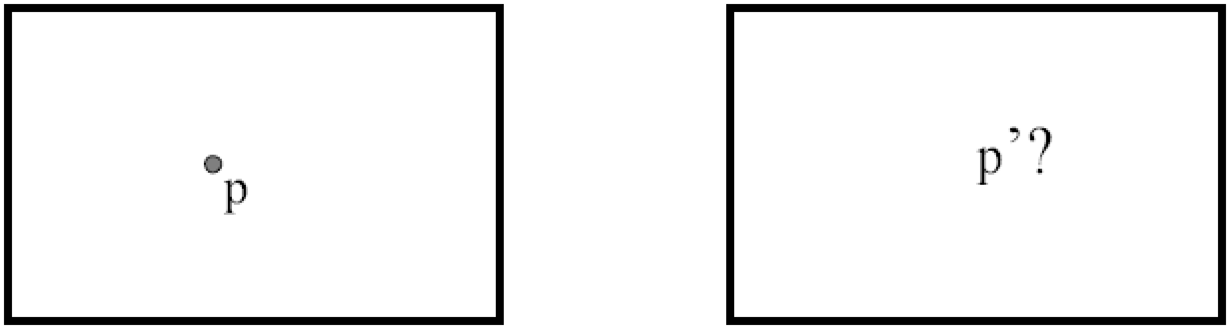
$$(x', y') = (x + D(x,y), y)$$

General Case With Calibrated Cameras

- The two cameras need not have parallel optical axes.

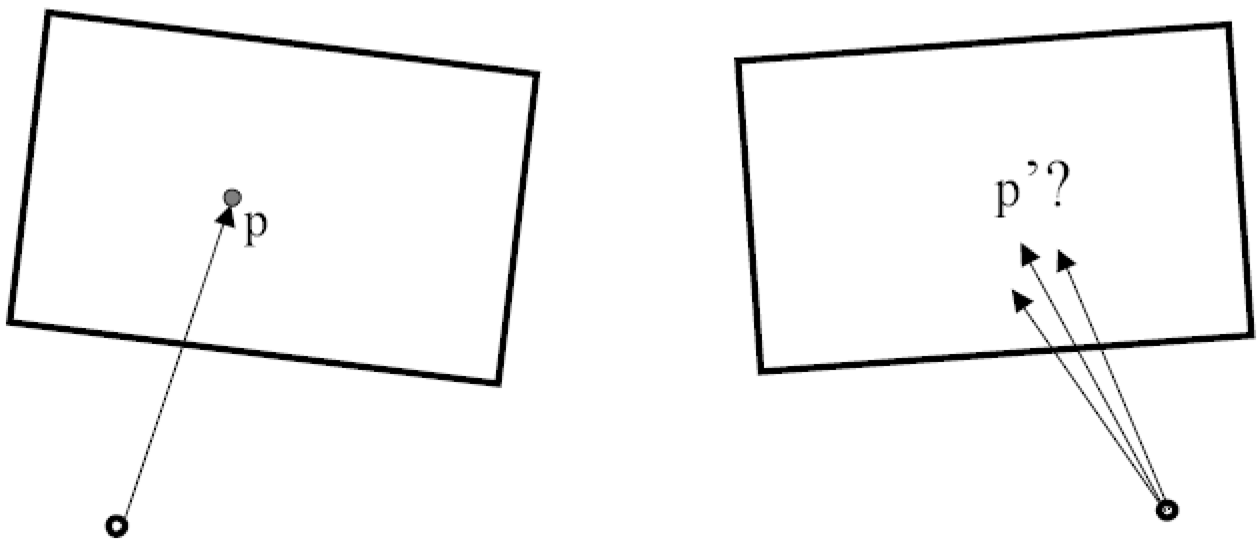


Stereo Correspondence Constraints



- **Given p in the left image, where can the corresponding point p' in the right image be?**

Stereo Correspondence Constraints



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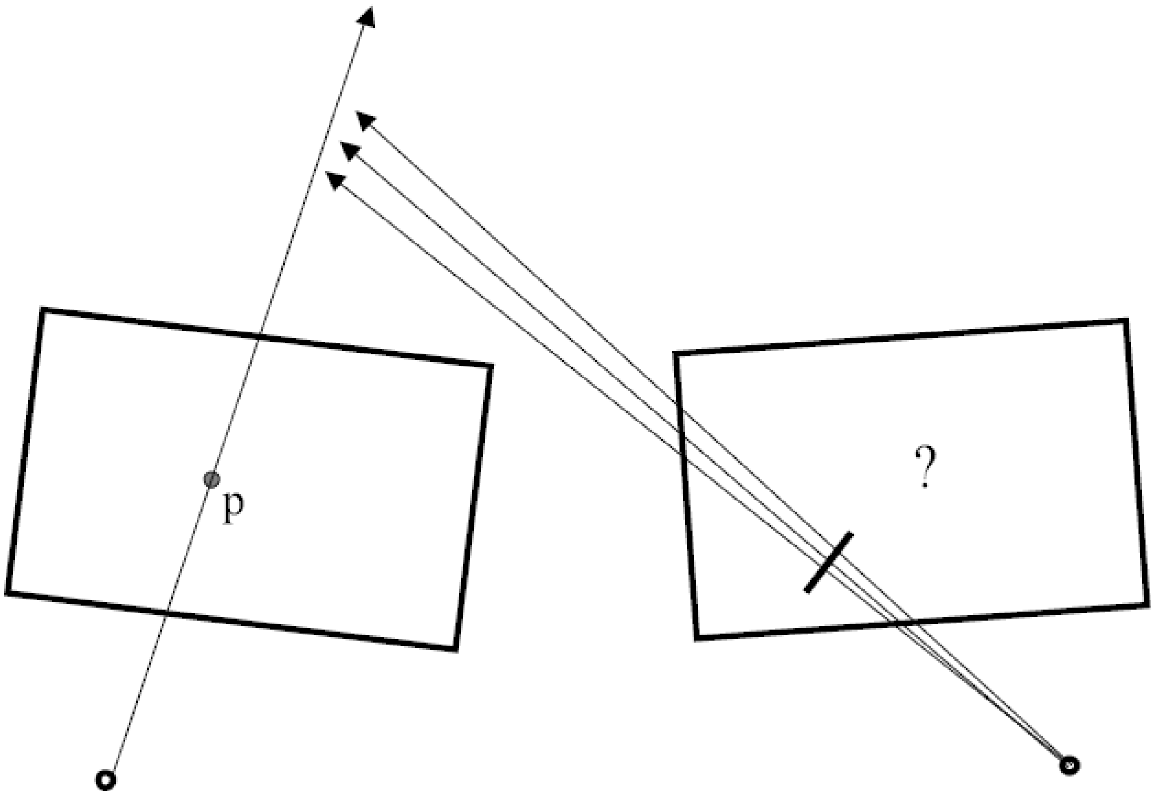
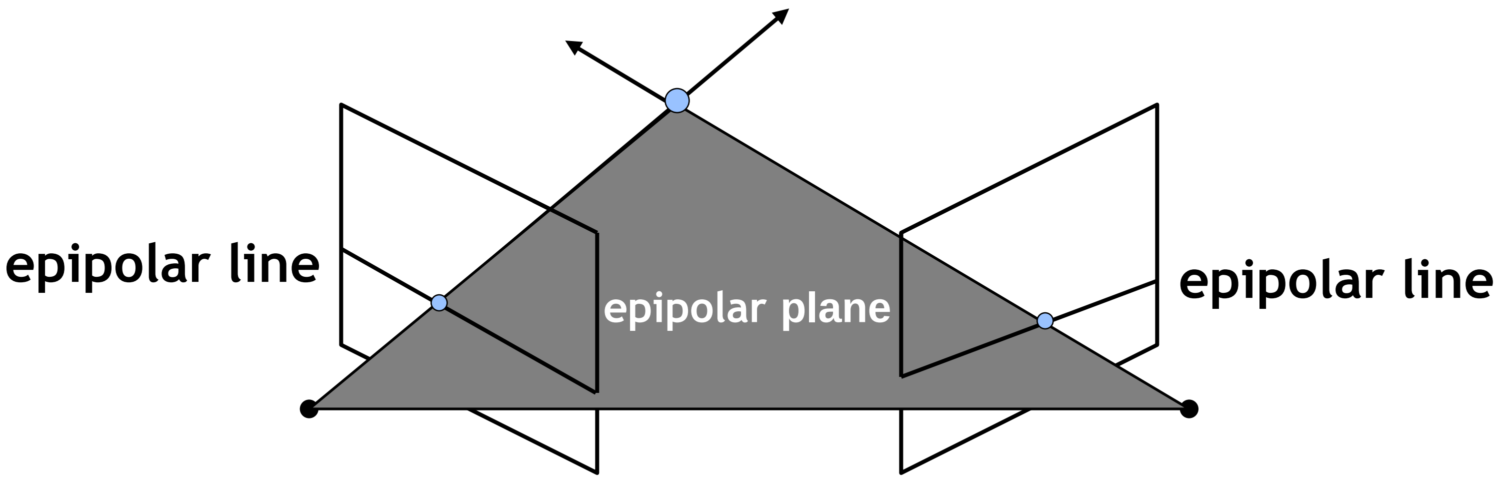
Slide credit: Kristen Grauman

Stereo Correspondence Constraints



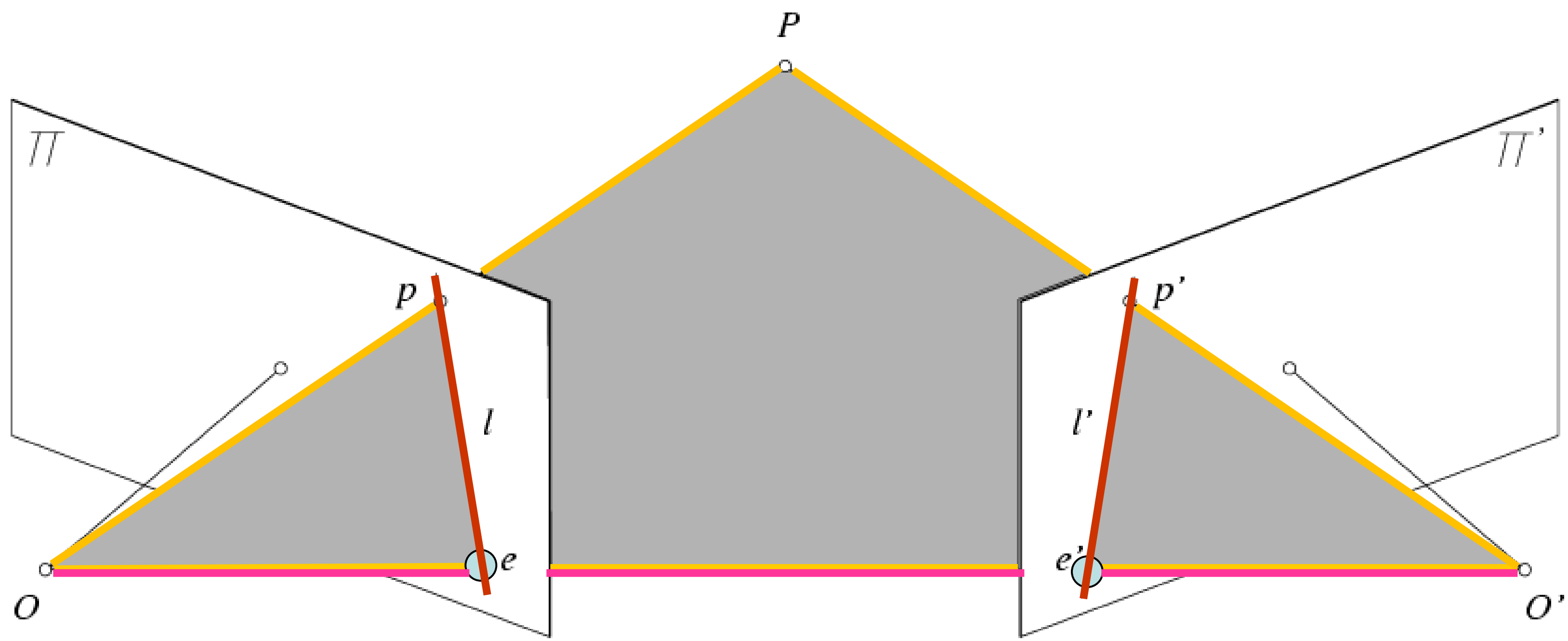
Stereo Correspondence Constraints

- Geometry of two views allows us to constrain where the corresponding pixel for some image point in the first view must occur in the second view.



- Epipolar constraint: Why is this useful?
  - Reduces correspondence problem to 1D search along conjugate epipolar lines.

Epipolar Geometry



- Epipolar Plane
- Baseline
- Epipoles
- Epipolar Lines

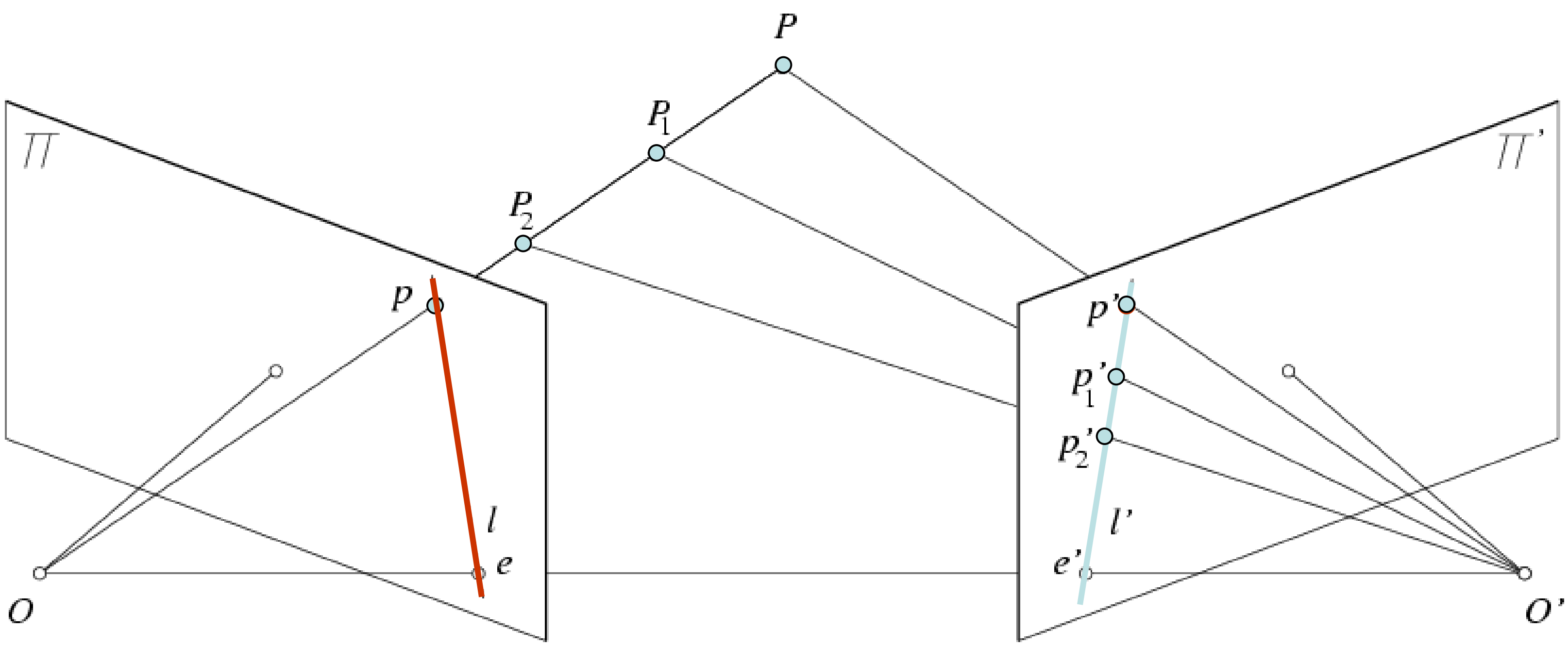
Slide adapted from Marc Pollefeys

Epipolar Geometry: Terms

- *Baseline*
  - Line joining the camera centers
- *Epipole*
  - Point of intersection of baseline with the image plane
- *Epipolar plane*
  - Plane containing baseline and world point
- *Epipolar line*
  - Intersection of epipolar plane with the image plane

- Properties
  - All epipolar lines intersect at the epipole.
  - An epipolar plane intersects the left and right image planes in epipolar lines.

Epipolar Constraint

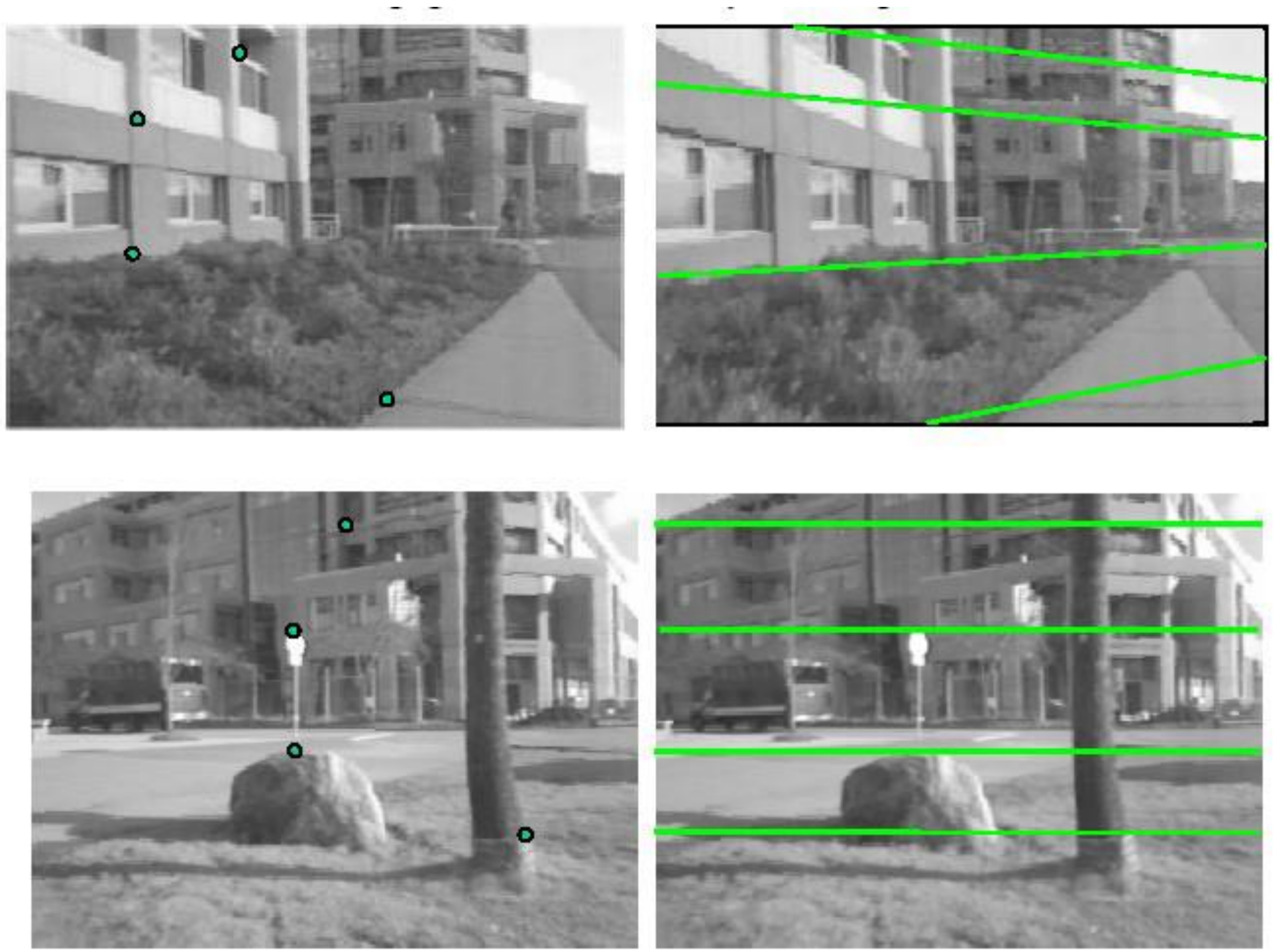


- Potential matches for p have to lie on the corresponding epipolar line l'.
- Potential matches for p' have to lie on the corresponding epipolar line l.

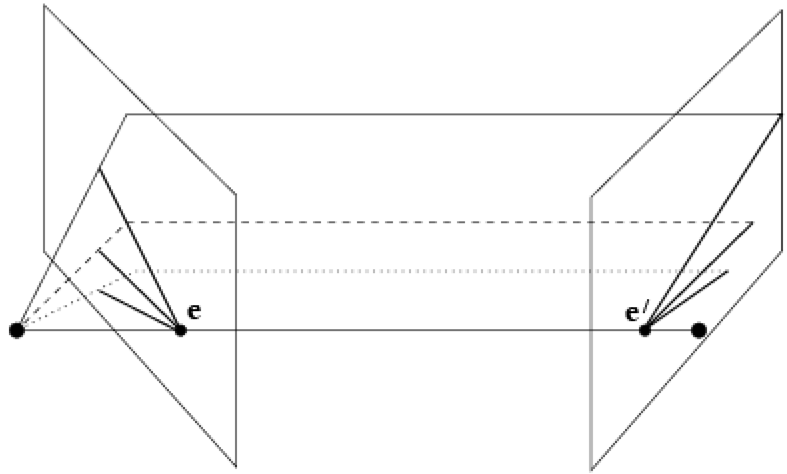
http://www.ai.sri.com/~luong/research/Meta3DViewer/EpipolarGeo.html

Slide credit: Marc Pollefeys

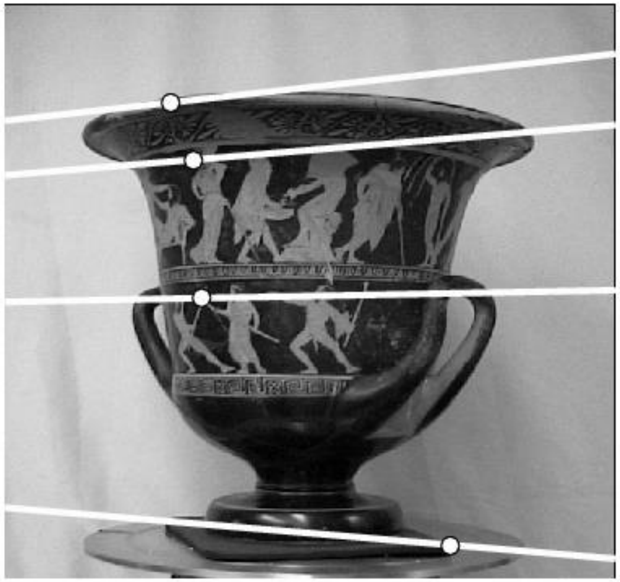
Example



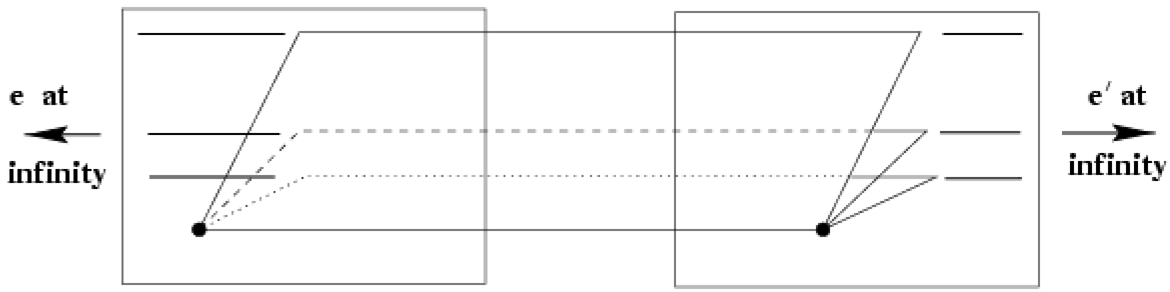
Example: Converging Cameras

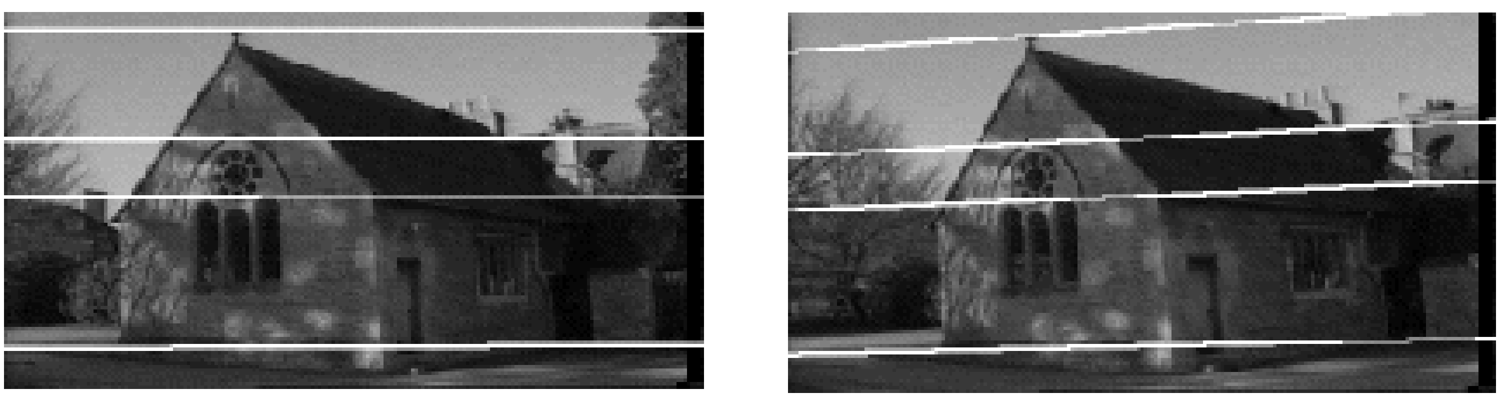


As position of 3D point varies, epipolar lines "rotate" about the baseline

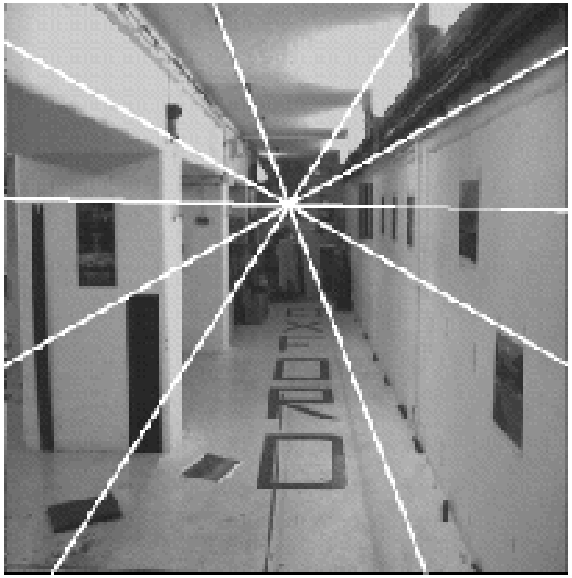


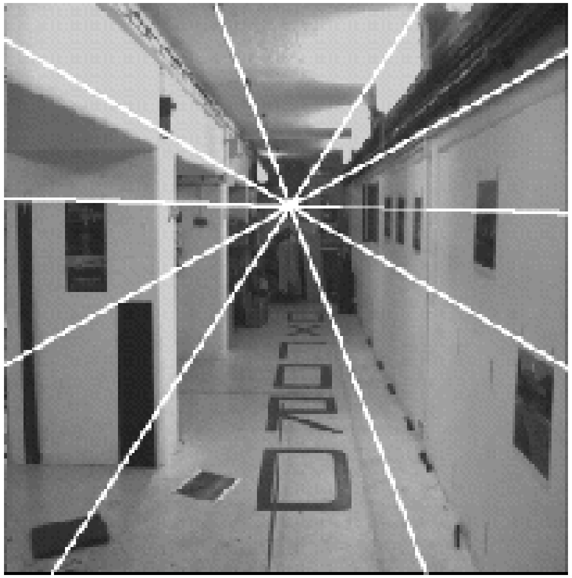
Example: Motion Parallel With Image Plane

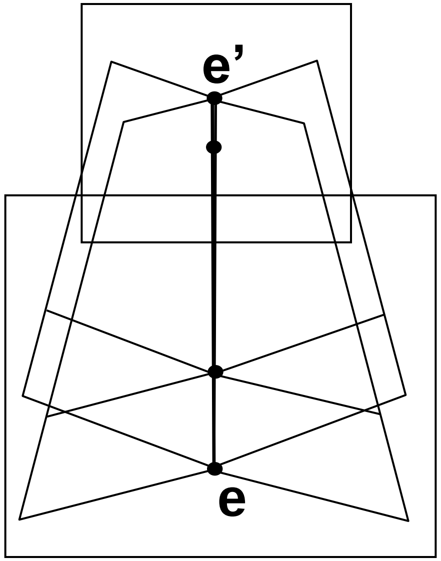




Example: Forward Motion







- Epipole has same coordinates in both images.
- Points move along lines radiating from e: “Focus of expansion”

Figure from Hartley & Zisserman

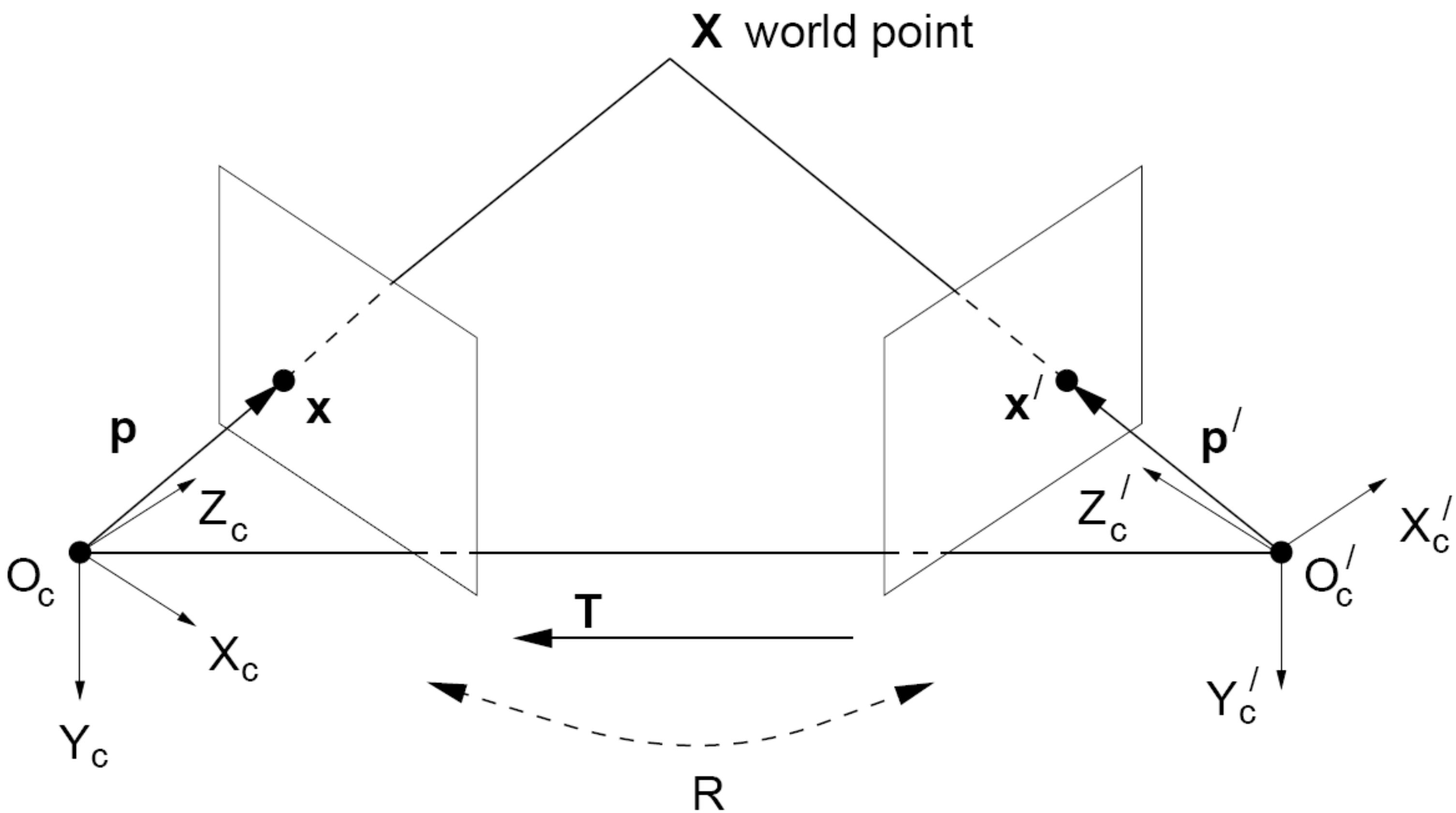
Let’s Formalize This!

- For a given stereo rig, how do we express the epipolar constraints algebraically?

- For this, we will need some linear algebra.

- But don’t worry! We’ll go through it step by step…

Stereo Geometry With Calibrated Cameras



- **If the rig is calibrated, we know:**
  - **How to rotate and translate camera reference frame 1 to get to camera reference frame 2.**
    - **Rotation: 3 x 3 matrix; translation: 3 vector.**

Rotation Matrix

$$\mathbf{R}_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

$$\mathbf{R}_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$\mathbf{R}_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Express 3D rotation as series of rotations around coordinate axes by angles α, β, γ

Overall rotation is product of these elementary rotations:

$$\mathbf{R} = \mathbf{R}_x \mathbf{R}_y \mathbf{R}_z$$

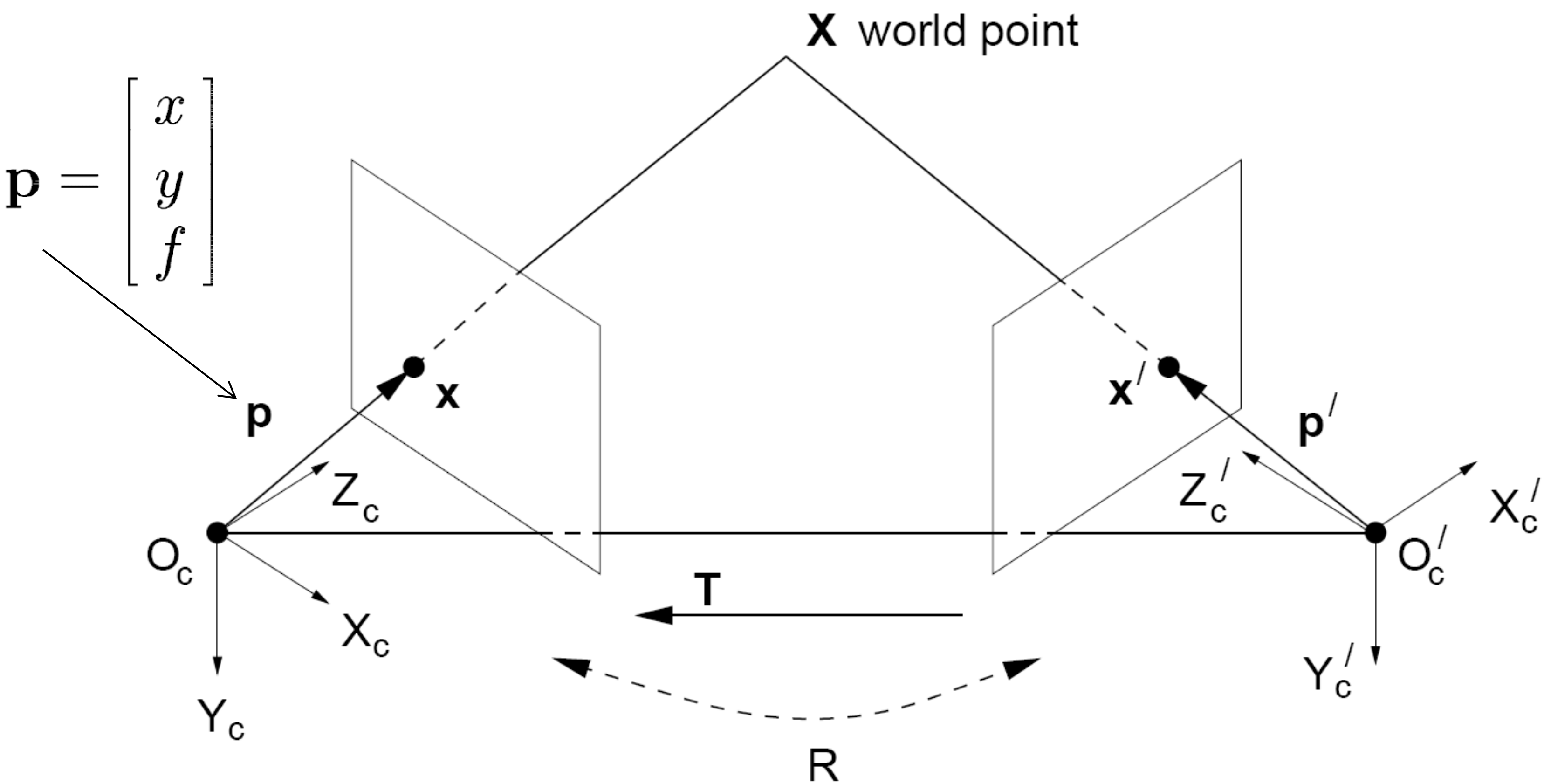
3D Rigid Transformation

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}$$

$$\mathbf{X}' = \mathbf{R}\mathbf{X} + \mathbf{T}$$

Slide credit: Kristen Grauman

Stereo Geometry With Calibrated Cameras



- Camera-centered coordinate systems are related by known rotation $\mathbf{R}$ and translation $\mathbf{T}$:

$$\mathbf{X}' = \mathbf{R}\mathbf{X} + \mathbf{T}$$

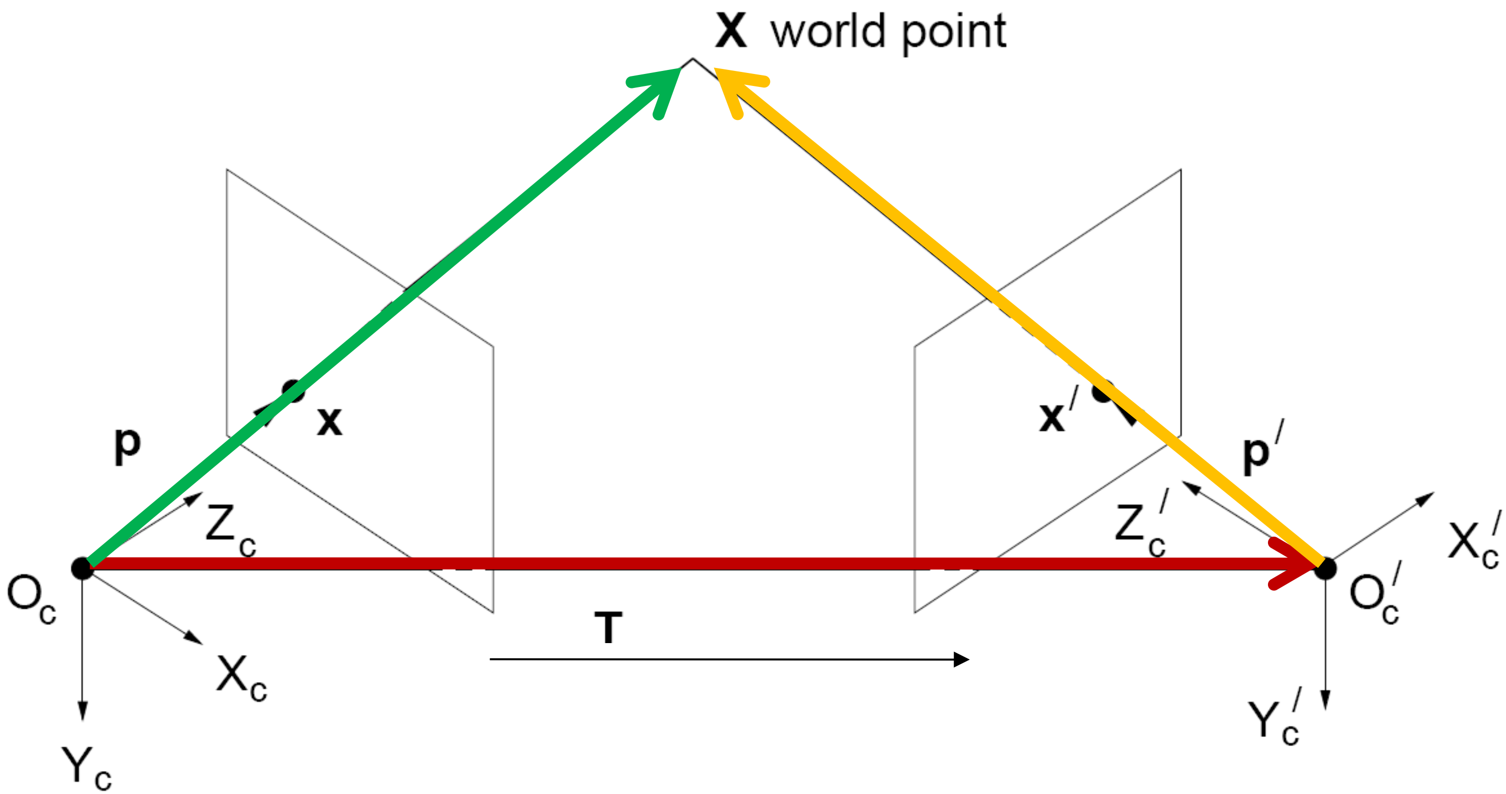
Excursion: Cross Product

$$\vec{a} \times \vec{b} = \vec{c}$$

$$\begin{aligned} \vec{a} \cdot \vec{c} &= 0 \\ \vec{b} \cdot \vec{c} &= 0 \end{aligned}$$

- **Vector cross product takes two vectors and returns a third vector that's perpendicular to both inputs.**
- **So here, c is perpendicular to both a and b, which means the dot product is 0.**

From Geometry to Algebra



$$\boxed{\mathbf{X}'} = \mathbf{R}\boxed{\mathbf{X}} + \boxed{\mathbf{T}}$$

$$\underbrace{\mathbf{T} \times \mathbf{X}'}_{\text{Normal to the plane}} = \mathbf{T} \times \mathbf{R}\mathbf{X} + \mathbf{T} \times \mathbf{T}$$

$$= \mathbf{T} \times \mathbf{R}\mathbf{X}$$

$$\mathbf{X}' \cdot (\mathbf{T} \times \mathbf{X}') = \mathbf{X}' \cdot (\mathbf{T} \times \mathbf{R}\mathbf{X})$$

$$0 = \mathbf{X}' \cdot (\mathbf{T} \times \mathbf{R}\mathbf{X})$$

Slide credit: Kristen Grauman

Matrix Form of Cross Product

$$\vec{a} \times \vec{b} \qquad = \vec{c} \qquad \begin{aligned} \vec{a} \cdot \vec{c} &= 0 \\ \vec{b} \cdot \vec{c} &= 0 \end{aligned}$$

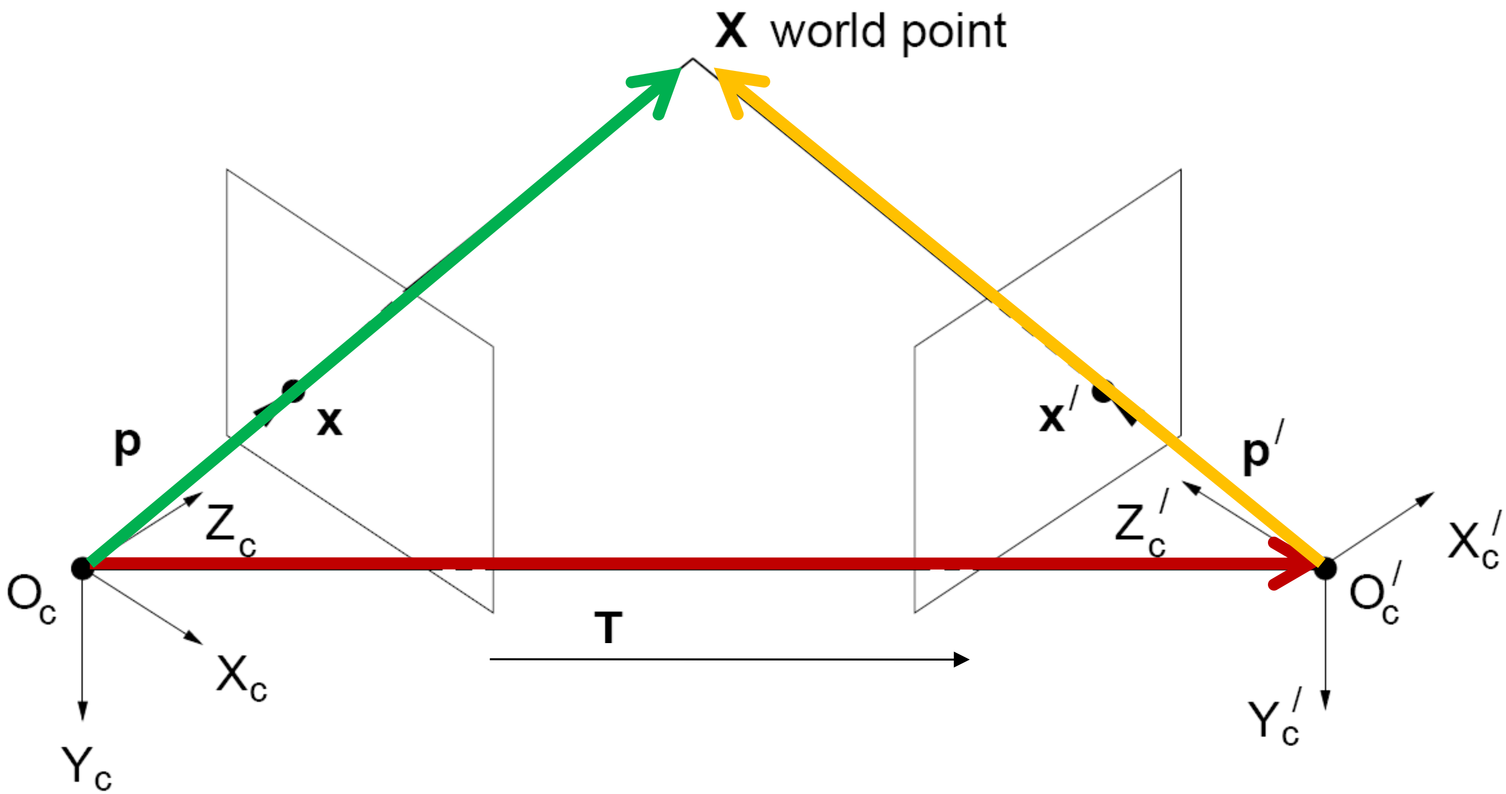
"skew symmetric" matrix

$$[a_\times] = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

$$\boxed{\vec{a} \times \vec{b} = [a_\times]\,\vec{b}}$$

Slide credit: Kristen Grauman

From Geometry to Algebra



$$\boxed{\mathbf{X}'} = \mathbf{R}\boxed{\mathbf{X}} + \boxed{\mathbf{T}}$$

$$\underbrace{\mathbf{T}\times\mathbf{X}'}_{\text{Normal to the plane}} = \mathbf{T}\times\mathbf{R}\mathbf{X} + \mathbf{T}\times\mathbf{T}$$

$$= \mathbf{T}\times\mathbf{R}\mathbf{X}$$

$$\mathbf{X}'\cdot(\mathbf{T}\times\mathbf{X}') = \mathbf{X}'\cdot(\mathbf{T}\times\mathbf{R}\mathbf{X})$$

$$\boxed{0 = \mathbf{X}'\cdot(\mathbf{T}\times\mathbf{R}\mathbf{X})}$$

Slide credit: Kristen Grauman

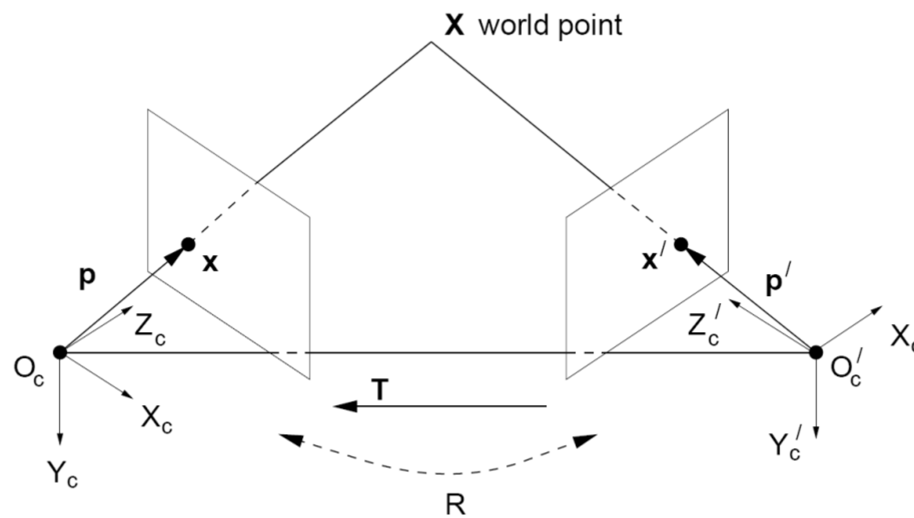
Essential Matrix

$$\boxed{\mathbf{X}'\cdot\left(\mathbf{T}\times\mathbf{R}\mathbf{X}\right)=0}$$

$$\mathbf{X}'\cdot\left(\mathbf{T}_x\ \mathbf{R}\mathbf{X}\right)=0$$

Let $\mathbf{E}=\mathbf{T}_x\mathbf{R}$

$$\mathbf{X}'^T\mathbf{E}\mathbf{X}=0$$



- This holds for the rays p and p' that are parallel to the camera-centered position vectors X and X', so we have: $\boxed{\mathbf{p}'^T\,\mathbf{E}\mathbf{p}=0}$
- $\mathbf{E}$ is called the essential matrix, which relates corresponding image points [Longuet-Higgins 1981]

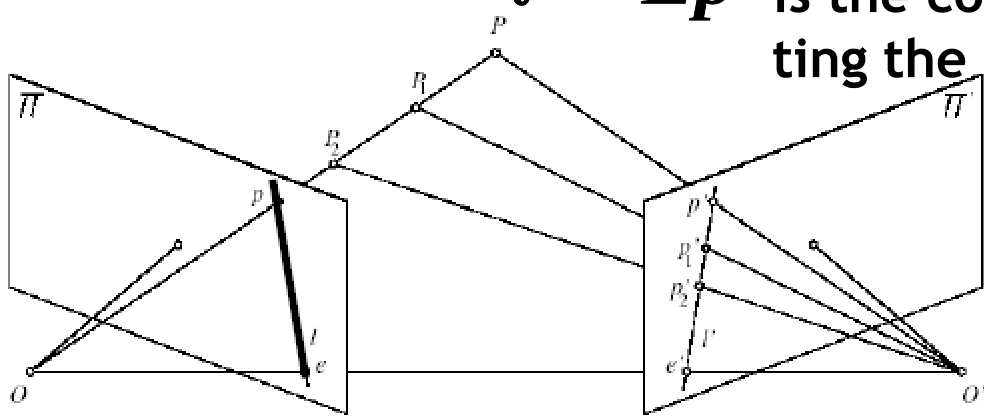
Essential Matrix and Epipolar Lines

$$\boxed{\mathbf{p}'^{\mathrm{T}}\mathbf{E}\mathbf{p} = 0}$$

Epipolar constraint: if we observe point p in one image, then its position p' in second image must satisfy this equation.

$\boldsymbol{l}' = \boldsymbol{E}\boldsymbol{p}$ is the coordinate vector representing the epipolar line for point p

(i.e., the line is given by: $\boldsymbol{l}'^{\top}\mathbf{x} = 0$)



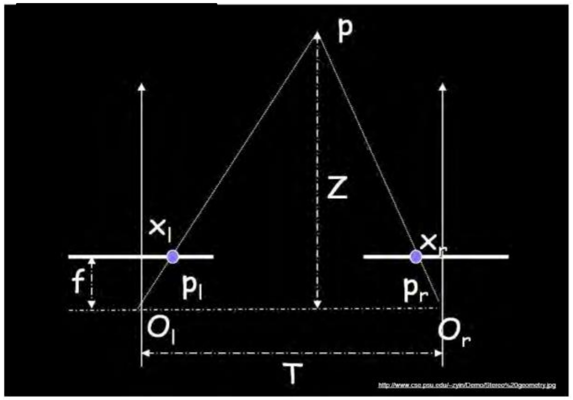
$\boldsymbol{l} = \boldsymbol{E}^{T}\boldsymbol{p}'$ is the coordinate vector representing the epipolar line for point p'

Essential Matrix: Properties

- Relates image of corresponding points in both cameras, given rotation and translation.
- Assuming intrinsic parameters are known

$$\mathbf{E} = \mathbf{T}_x \mathbf{R}$$

Essential Matrix Example: Parallel Cameras



$$\mathbf{R} =$$

$$\mathbf{T} =$$

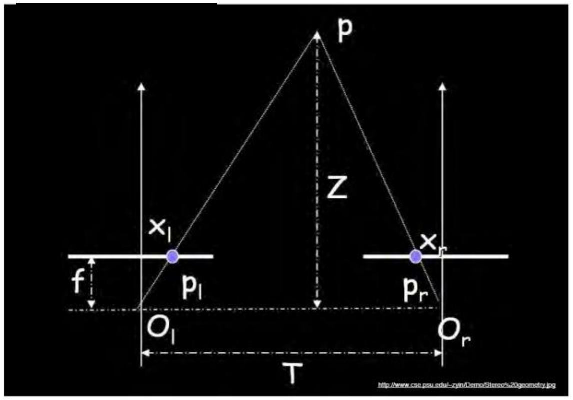
$$\mathbf{E} = [\mathbf{T}_{\mathbf{x}}]\mathbf{R} =$$

$$\mathbf{p}'^{\mathrm{T}}\mathbf{E}\mathbf{p} = 0$$

For the parallel cameras, image of any point must lie on same horizontal line in each image plane.

Slide credit: Kristen Grauman

Essential Matrix Example: Parallel Cameras



$$\mathbf{R} = \mathbf{I}$$

$$\mathbf{T} = [-d, 0, 0]^{\mathrm{T}}$$

$$\mathbf{E} = [\mathbf{T_x}]\mathbf{R} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & d \\ 0 & -d & 0 \end{pmatrix}$$

$$\mathbf{p}'^{\mathrm{T}}\mathbf{E}\mathbf{p} = 0$$

$$\begin{bmatrix} x' & y' & f \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & d \\ 0 & -d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ f \end{bmatrix} = 0$$

$$\Leftrightarrow \begin{bmatrix} x' & y' & f \end{bmatrix} \begin{bmatrix} 0 \\ df \\ -dy \end{bmatrix} = 0$$

$$\boxed{\Leftrightarrow y = y'}$$

For the parallel cameras, image of any point must lie on same horizontal line in each image plane.

Slide credit: Kristen Grauman

More General Case

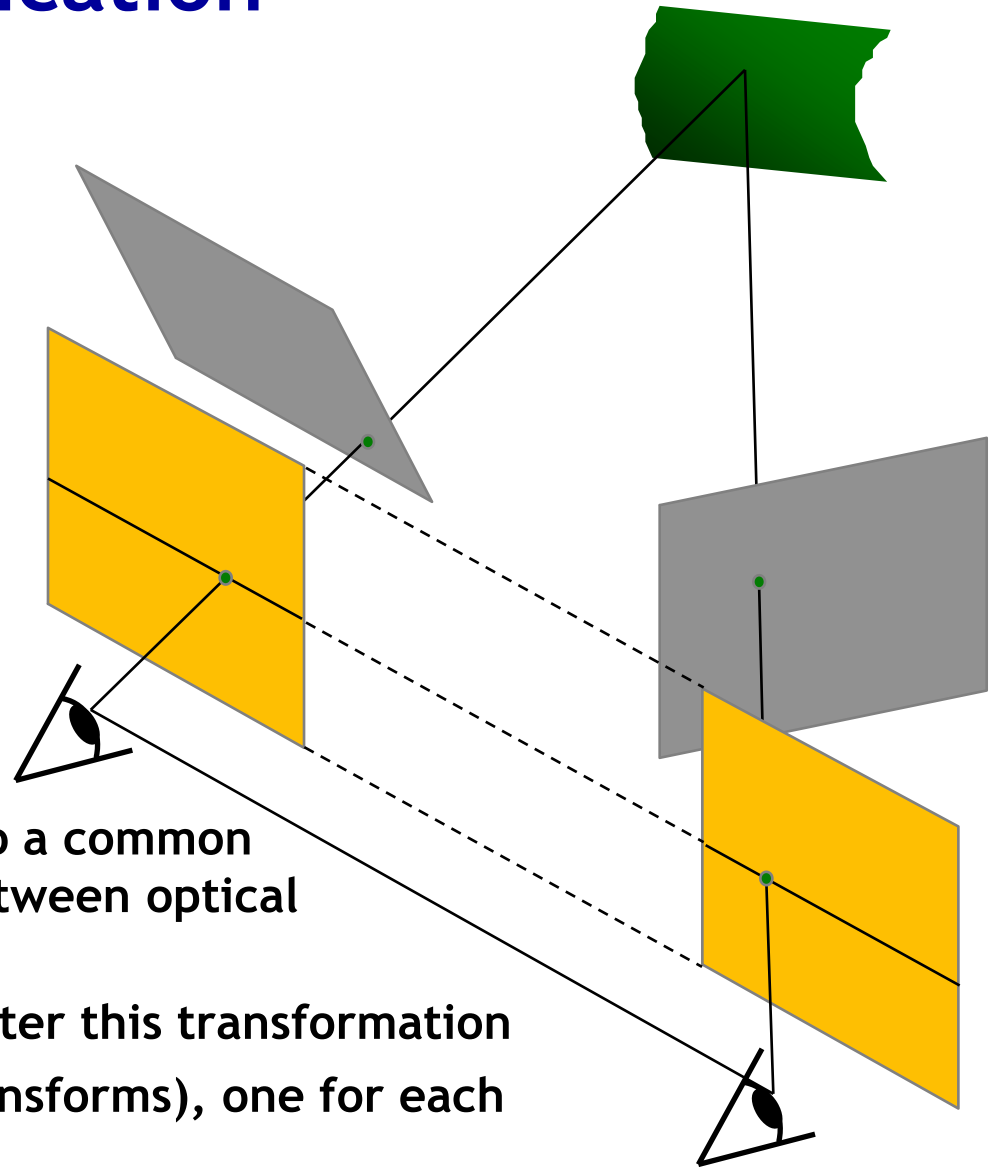
Image $I(x,y)$ Disparity map $D(x,y)$ Image $I'(x',y')$



$$(x', y') = (x + D(x,y), y)$$

What about when cameras' optical axes are not parallel?

Stereo Image Rectification



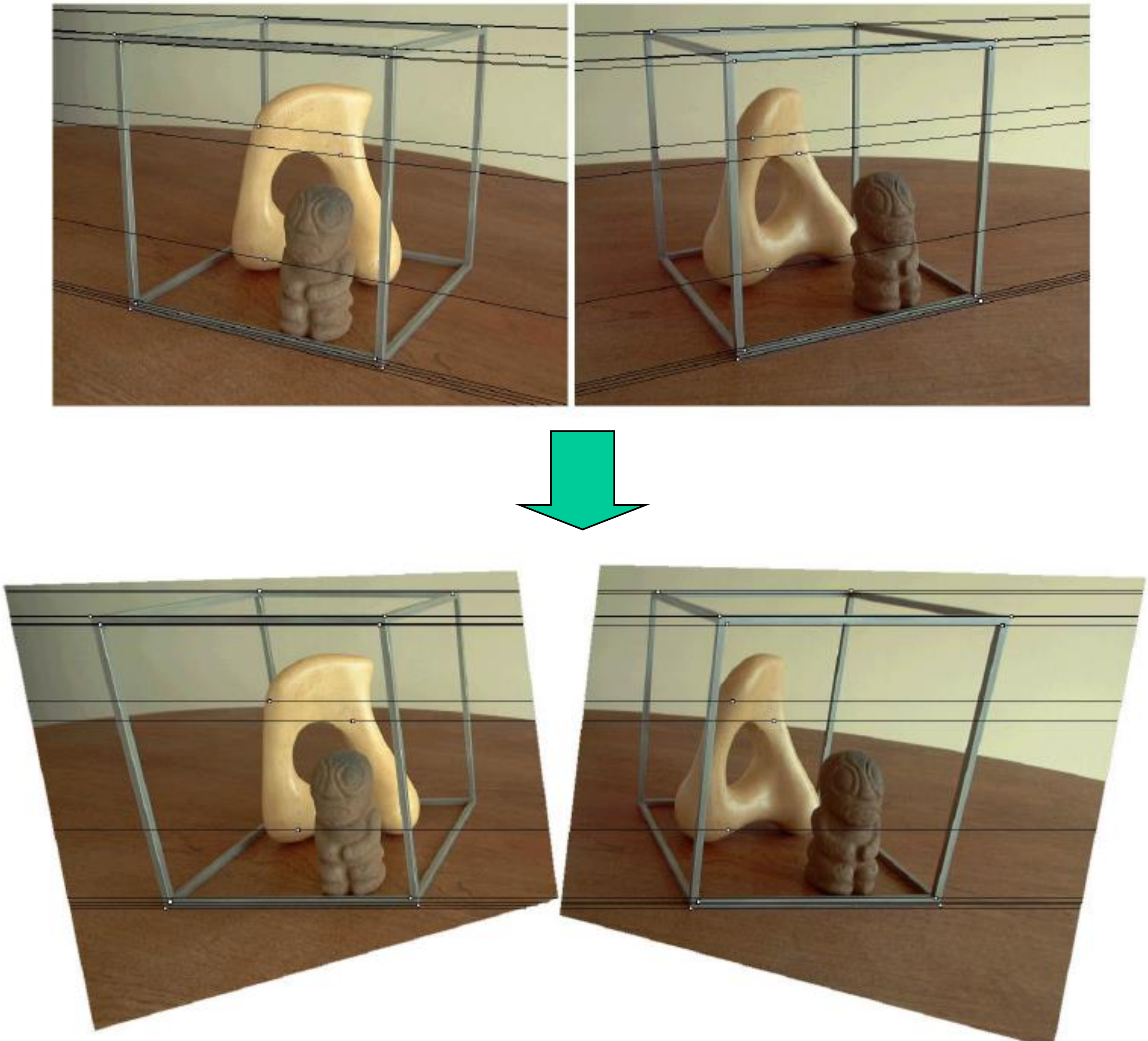
- In practice, it is convenient if image scanlines are the epipolar lines.

- Algorithm
  - Reproject image planes onto a common plane parallel to the line between optical centers
  - Pixel motion is horizontal after this transformation
  - Two homographies (3×3 transforms), one for each input image reprojection

Slide adapted from Li Zhang

C. Loop & Z. Zhang, Computing Rectifying Homographies for Stereo Vision. CVPR'99

Stereo Image Rectification: Example



Source: Alyosha Efros

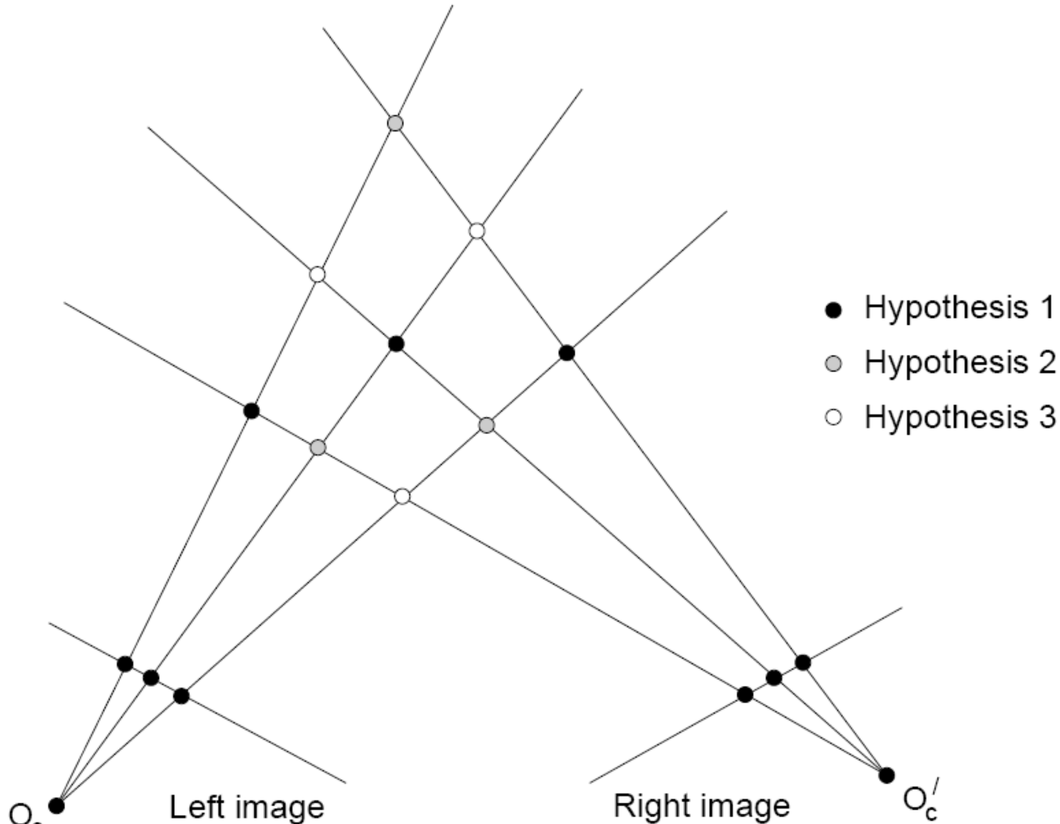
Topics of This Lecture

- Geometric vision
  - Visual cues
  - Stereo vision
- Epipolar geometry
  - Depth with stereo
  - Geometry for a simple stereo system
  - Case example with parallel optical axes
  - General case with calibrated cameras
- **Stereopsis & 3D Reconstruction**
  - Correspondence search
  - Additional correspondence constraints
  - Possible sources of error
  - Applications

Stereo Reconstruction

- Main Steps
  - Calibrate cameras
  - Rectify images
  - Compute disparity
  - Estimate depth

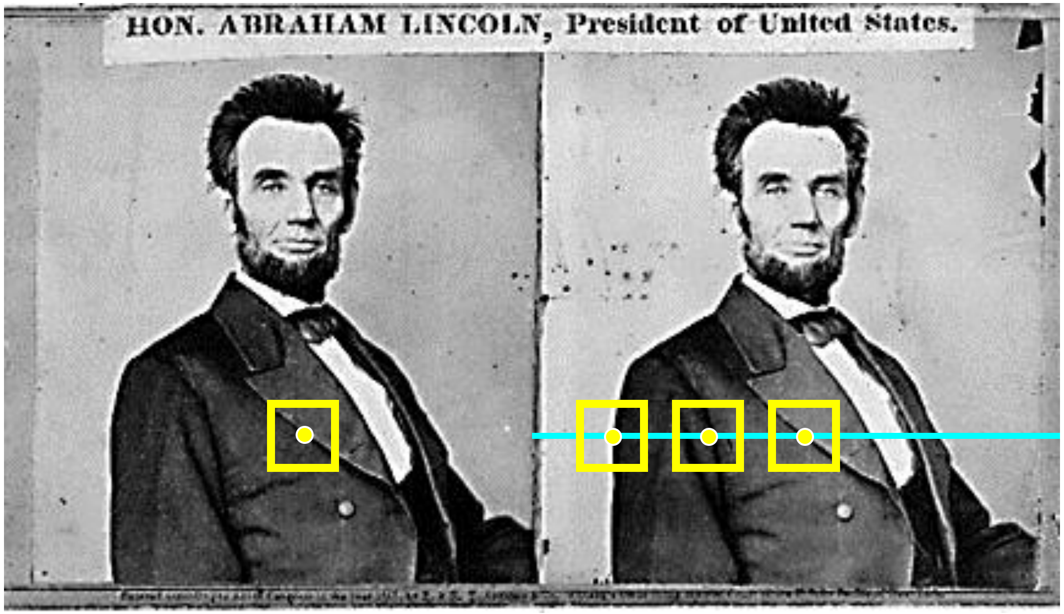
Correspondence Problem



Multiple match hypotheses satisfy epipolar constraint, but which is correct?

Figure from Gee & Cipolla 1999

Dense Correspondence Search



- For each pixel in the first image
  - Find corresponding epipolar line in the right image
  - Examine all pixels on the epipolar line and pick the best match (e.g. SSD, correlation)
  - Triangulate the matches to get depth information
- This is easiest when epipolar lines are scanlines

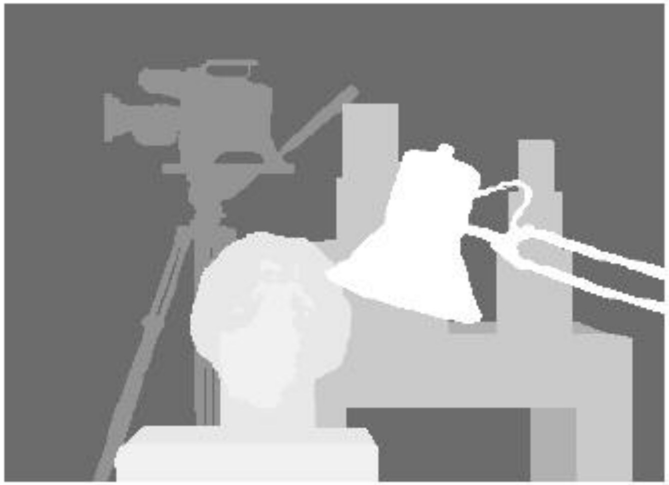
  ⇒ Rectify images first

adapted from Svetlana Lazebnik, Li Zhang

Example: Window Search

- **Data from University of Tsukuba**

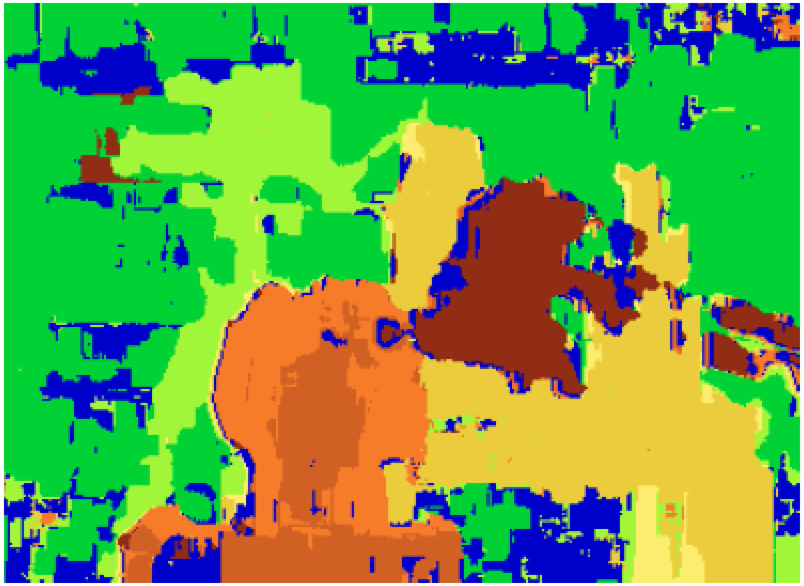
Scene



Ground truth

Example: Window Search

- **Data from University of Tsukuba**

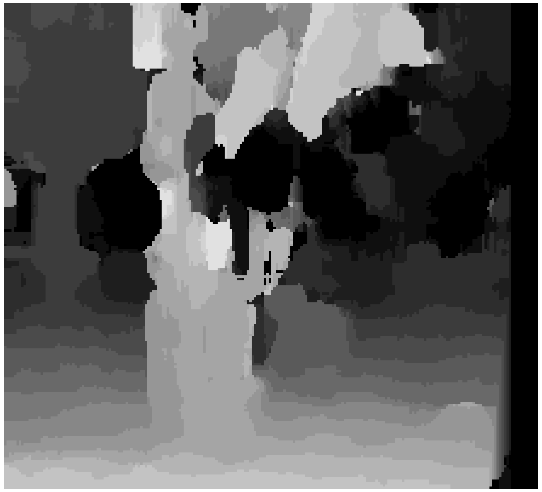


Window-based matching
(best window size)

Ground truth

Effect of Window Size

$$W = 3$$



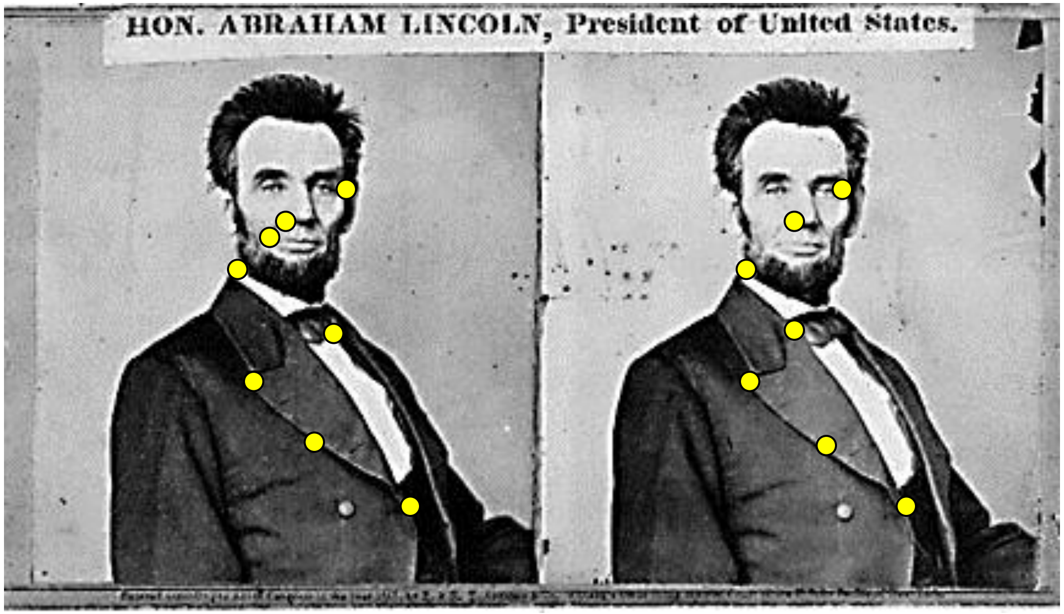
$$W = 20$$

Want window large enough to have sufficient intensity variation, yet small enough to contain only pixels with about the same disparity.

Figures from Li Zhang

RWTH AACHEN UNIVERSITY

Alternative: Sparse Correspondence Search



- **Idea: Restrict search to sparse set of detected features**
- **Rather than pixel values (or lists of pixel values) use *feature descriptor* and an associated *feature distance***
- **Still narrow search further by epipolar geometry**

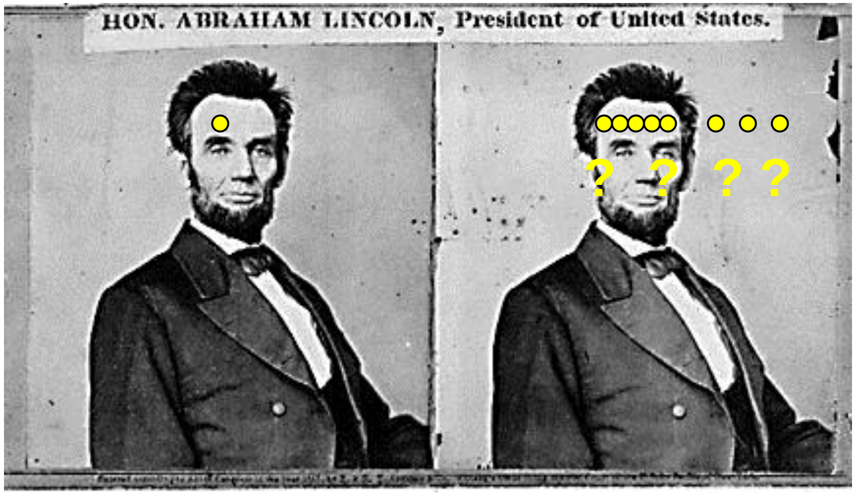
What would make good features?

Dense vs. Sparse

- Sparse
  - Efficiency
  - Can have more reliable feature matches, less sensitive to illumination than raw pixels
  - But…
    - Have to know enough to pick good features
    - Sparse information
- Dense
  - Simple process
  - More depth estimates, can be useful for surface reconstruction
  - But…
    - Breaks down in textureless regions anyway
    - Raw pixel distances can be brittle
    - Not good with very different viewpoints

Slide credit: Kristen Grauman

Difficulties in Similarity Constraint



Untextured surfaces

Occlusions

Possible Sources of Error?

- Low-contrast / textureless image regions
- Occlusions
- Camera calibration errors
- Violations of *brightness constancy* (e.g., specular reflections)
- Large motions

Application: View Interpolation

Right Image

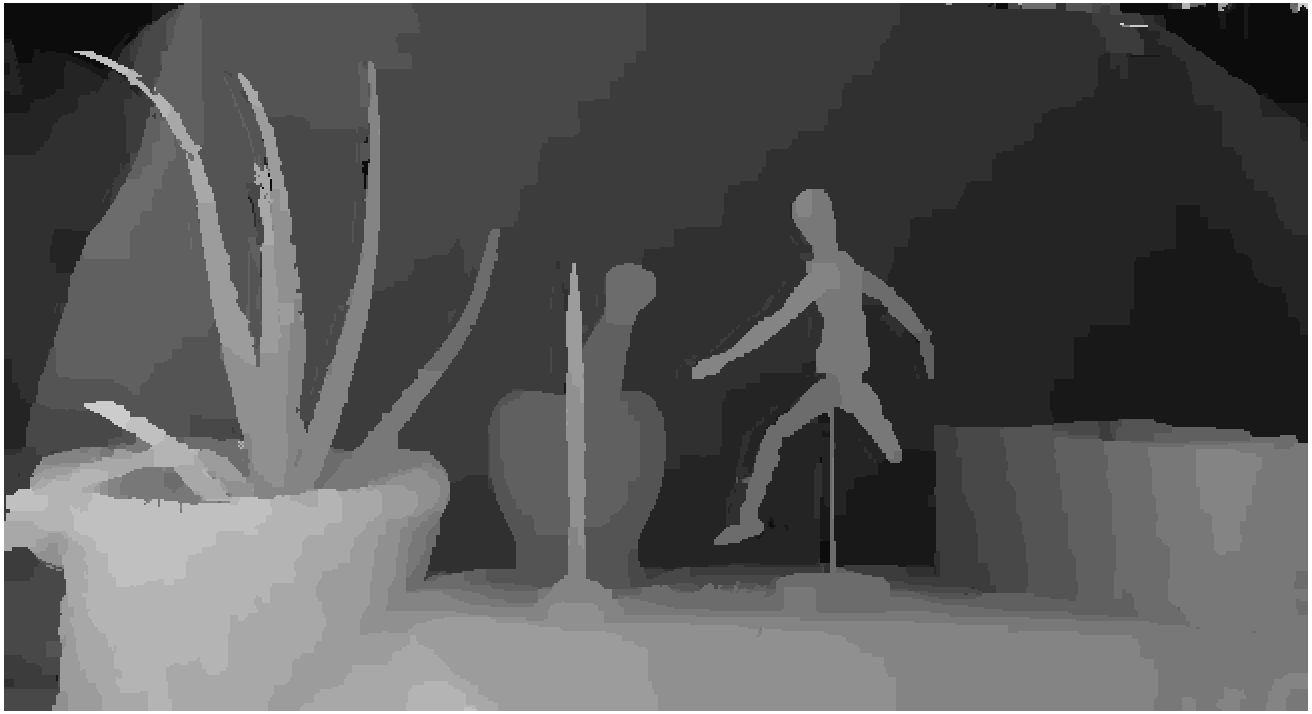
Slide credit: Svetlana Lazebnik

Application: View Interpolation

Left Image

Slide credit: Svetlana Lazebnik

Application: View Interpolation



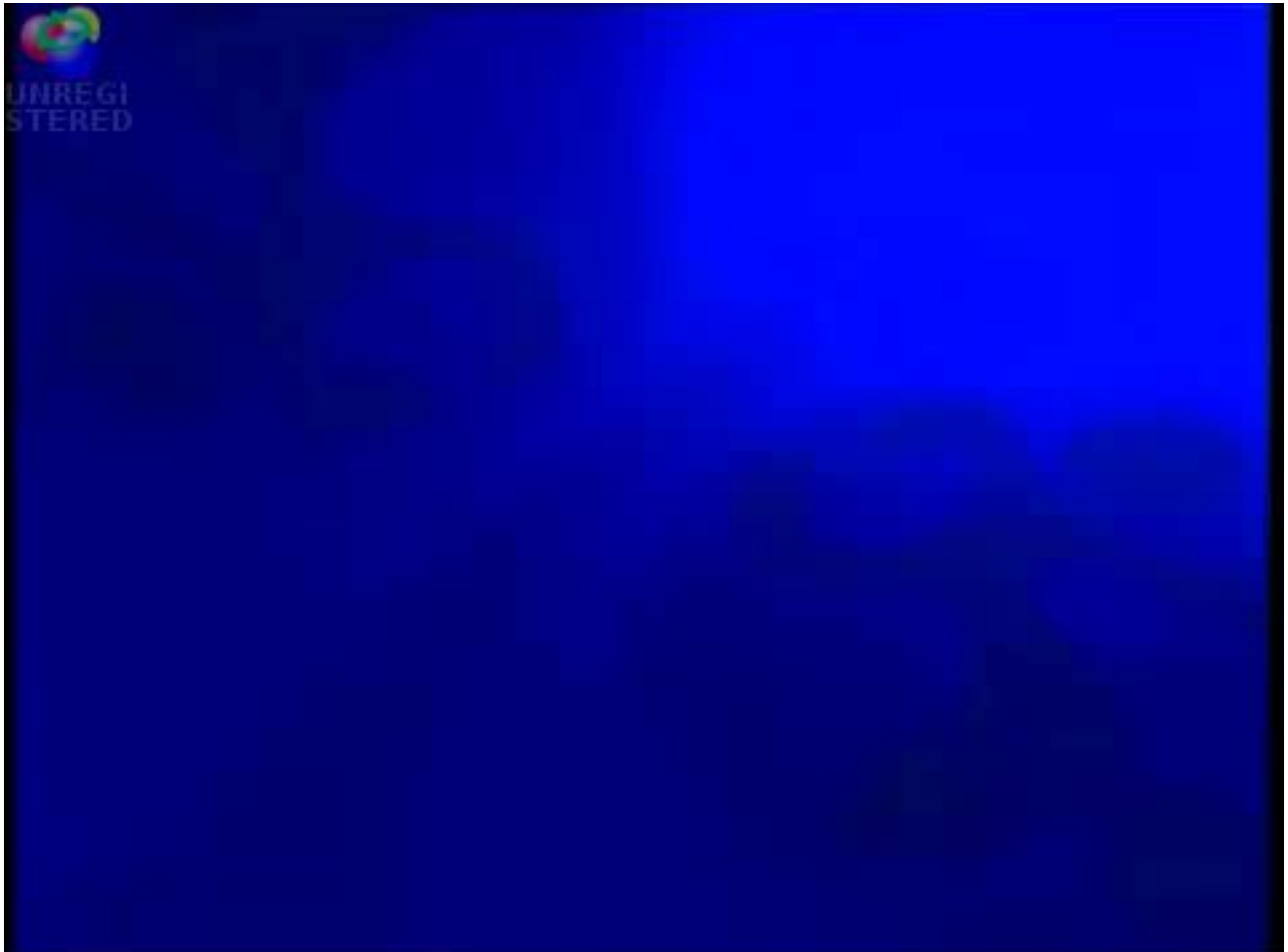
Disparity

Slide credit: Svetlana Lazebnik

Application: View Interpolation

Slide credit: Svetlana Lazebnik

Application: Free-Viewpoint Video



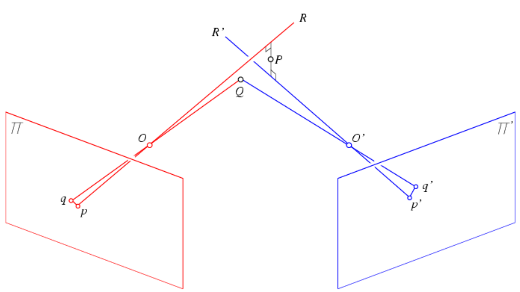
http://www.liberovision.com

Summary: Stereo Reconstruction

- Main Steps
  - Calibrate cameras
  - Rectify images
  - Compute disparity
  - Estimate depth

- So far, we have only considered calibrated cameras…

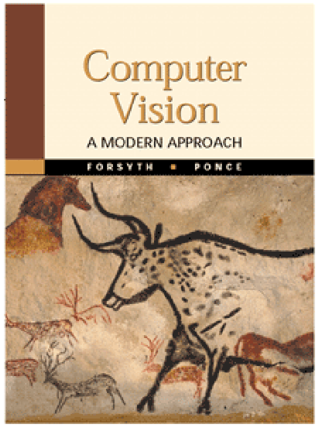
- Next lecture
  - Uncalibrated cameras
  - Camera parameters
  - Revisiting epipolar geometry
  - Robust fitting



References and Further Reading

- **Background information on epipolar geometry and stereopsis can be found in Chapters 10.1-10.2 and 11.1-11.3 of**

  D. Forsyth, J. Ponce,
  Computer Vision – A Modern Approach.
  Prentice Hall, 2003



- **More detailed information (if you really want to implement 3D reconstruction algorithms) can be found in Chapters 9 and 10 of**

  R. Hartley, A. Zisserman
  Multiple View Geometry in Computer Vision
  2nd Ed., Cambridge Univ. Press, 2004