Computer Vision - Lecture 17

Epipolar Geometry & Stereo Basics

19.01.2016

Bastian Leibe
RWTH Aachen
http://www.vision.rwth-aachen.de

leibe@vision.rwth-aachen.de

Announcements

- Exam Dates
  - 1st try: 29.02. 13:30 - 17:30h in AH I/II + AH VI, UMIC 025
  - 2nd try: 31.03. 09:40 - 12:40h in UMIC 025 + AH IV
  - *We will send around an email announcing the precise start/end times and your assigned exam rooms.*

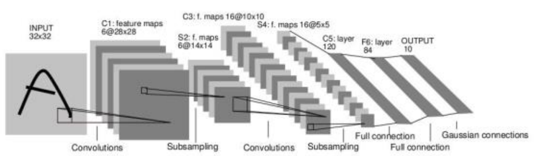
Announcements (2)

- Seminar in the summer semester
  - *"Current Topics in Computer Vision and Machine Learning"*
  - Block seminar, presentations at beginning of semester break
  - Registration period: 14.01.2016 – 27.01.2016
  - https://www.graphics.rwth-aachen.de/apse/check.php

Course Outline

- Image Processing Basics
- Segmentation & Grouping
- Object Recognition
- Local Features & Matching
- Object Categorization
- 3D Reconstruction
  - Epipolar Geometry and Stereo Basics
  - Camera calibration & Uncalibrated Reconstruction
  - Multi-view Stereo
- Optical Flow

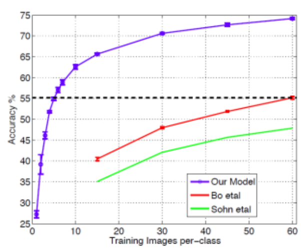
Recap: Convolutional Neural Networks



- Neural network with specialized connectivity structure
  - Stack multiple stages of feature extractors
  - Higher stages compute more global, more invariant features
  - Classification layer at the end

Y. LeCun, L. Bottou, Y. Bengio, and P. Haffner, Gradient-based learning applied to document recognition, Proceedings of the IEEE 86(11): 2278-2324, 1998.

The Learned Features are Generic



state of the art level (pre-CNN)

- Experiment: feature transfer
  - Train AlexNet-like network on ImageNet
  - Chop off last layer and train classification layer on CalTech256

$\Rightarrow$ State of the art accuracy already with only 6 training images!

Transfer Learning with CNNs

1. Train on ImageNet

2. If small dataset: fix all weights (treat CNN as fixed feature extractor), retrain only the classifier

I.e., swap the Softmax layer at the end

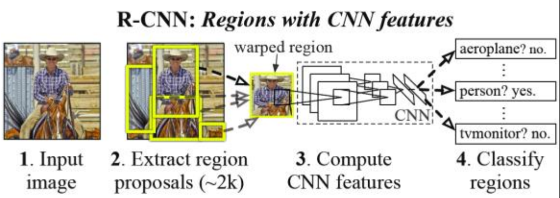
Transfer Learning with CNNs

1. Train on ImageNet

3. If you have medium sized dataset, "finetune" instead: use the old weights as initialization, train the full network or only some of the higher layers.

Retrain bigger portion of the network

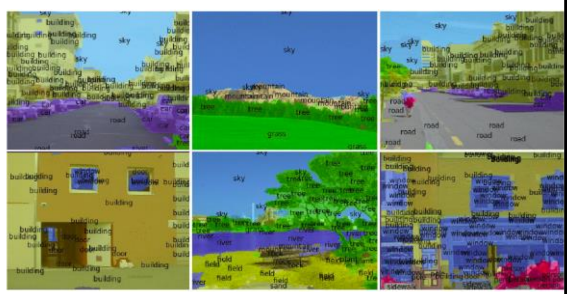
Other Tasks: Detection



- Results on PASCAL VOC Detection benchmark
  - Pre-CNN state of the art: 35.1% mAP [Uijlings et al., 2013]
    33.4% mAP DPM
  - R-CNN: 53.7% mAP

R. Girshick, J. Donahue, T. Darrell, and J. Malik, Rich Feature Hierarchies for Accurate Object Detection and Semantic Segmentation, CVPR 2014

Other Tasks: Semantic Segmentation

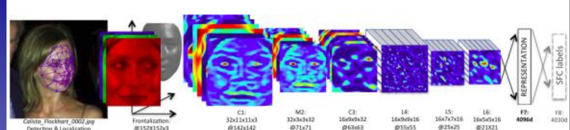


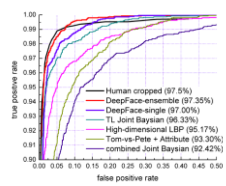
[Farabet et al. ICML 2012, PAMI 2013]

Other Tasks: Semantic Segmentation

[Farabet et al. ICML 2012, PAMI 2013]

Other Tasks: Face Verification

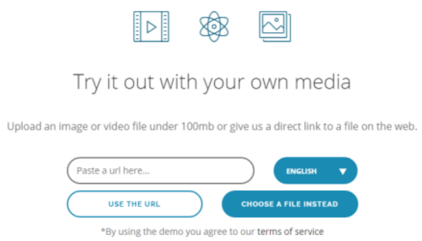




Y. Taigman, M. Yang, M. Ranzato, L. Wolf, DeepFace: Closing the Gap to Human-Level Performance in Face Verification, CVPR 2014

Commercial Recognition Services

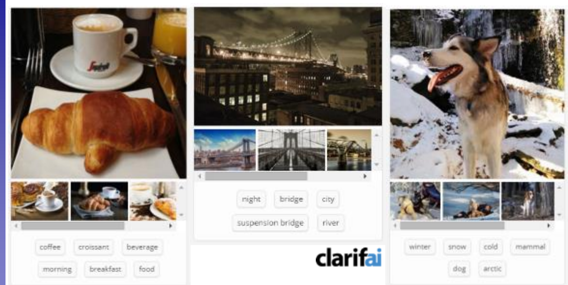
- E.g., clarifai



Computer Vision WS 15/16

Image source: clarifai.com

Commercial Recognition Services



- Be careful when testing with images from Google Search
  - Chances are they may have been seen in the training set...

Computer Vision WS 15/16

Image source: clarifai.com

Topics of This Lecture

- Geometric vision
  - Visual cues
  - Stereo vision
- Epipolar geometry
  - Depth with stereo
  - Geometry for a simple stereo system
  - Case example with parallel optical axes
  - General case with calibrated cameras
- Stereopsis & 3D Reconstruction
  - Correspondence search
  - Additional correspondence constraints
  - Possible sources of error
  - Applications

Computer Vision WS 14/15

Geometric vision

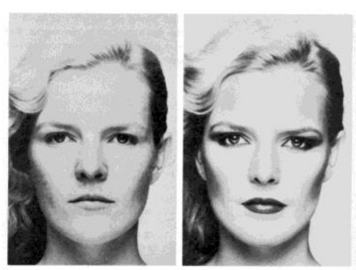
- Goal: Recovery of 3D structure
  - What cues in the image allow us to do this?

Computer Vision WS 14/15

Slide credit: Svetlana Lazebnik

Visual Cues

- Shading



Merle Norman Cosmetics, Los Angeles

Computer Vision WS 14/15

Slide credit: Steve Seitz

Visual Cues

- Shading
- Texture

The Visual Cliff, by William Vandivert, 1960

Computer Vision WS 14/15

Slide credit: Steve Seitz

Visual Cues

- Shading
- Texture
- **Focus**

From *The Art of Photography*, Canon

Slide credit: Steve Seitz | B. Leibe | 19

Visual Cues

- Shading
- Texture
- Focus
- **Perspective**

Slide credit: Steve Seitz | B. Leibe | 20

Visual Cues

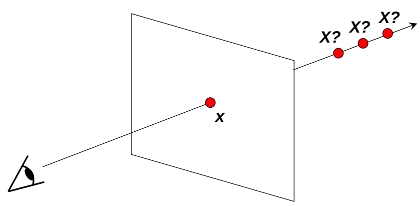
- Shading
- Texture
- Focus
- Perspective
- **Motion**

Figures from L. Zhang

Slide credit: Steve Seitz, Kristen Grauman | http://www.brainconnection.com/teasers/?main=illusion/motion-shape | 21

Our Goal: Recovery of 3D Structure

- We will focus on perspective and motion
- We need *multi-view geometry* because recovery of structure from one image is inherently ambiguous



Slide credit: Svetlana Lazebnik | B. Leibe | 22

To Illustrate This Point…

- Structure and depth are inherently ambiguous from single views.

Slide credit: Svetlana Lazebnik, Kristen Grauman | B. Leibe | 23

Stereo Vision

http://www.well.com/~jimg/stereo/stereo_list.html

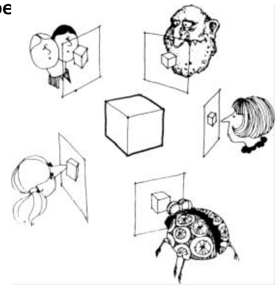
Slide credit: Kristen Grauman

What Is Stereo Vision?

- Generic problem formulation: given several images of the same object or scene, compute a representation of its 3D shape

What Is Stereo Vision?

- Generic problem formulation: given several images of the same object or scene, compute a representation of its 3D shape

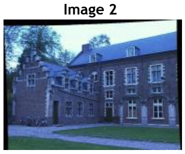


What Is Stereo Vision?

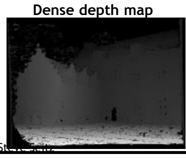
- Narrower formulation: given a calibrated binocular stereo pair, fuse it to produce a depth image

Image 1

Image 2



Dense depth map



What Is Stereo Vision?

- Narrower formulation: given a calibrated binocular stereo pair, fuse it to produce a depth image.
  - Humans can do it

Stereograms: Invented by Sir Charles Wheatstone, 1838

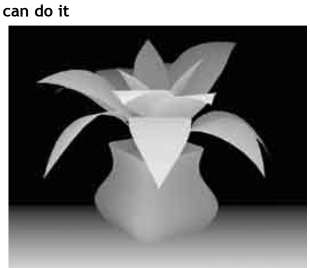
What Is Stereo Vision?

- Narrower formulation: given a calibrated binocular stereo pair, fuse it to produce a depth image.
  - Humans can do it

Autostereograms: http://www.magiceye.com

What Is Stereo Vision?

- Narrower formulation: given a calibrated binocular stereo pair, fuse it to produce a depth image.
  - Humans can do it

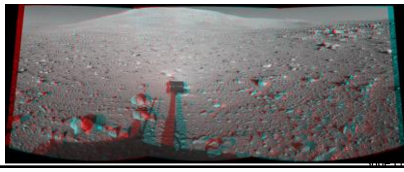


Autostereograms: http://www.magiceye.com

Application of Stereo: Robotic Exploration

Nomad robot searches for meteorites in Antartica

Real-time stereo on Mars

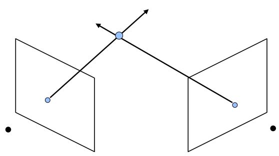


Slide credit: Steve Seitz

Topics of This Lecture

- Geometric vision
  - Visual cues
  - Stereo vision
- Epipolar geometry
  - Depth with stereo
  - Geometry for a simple stereo system
  - Case example with parallel optical axes
  - General case with calibrated cameras
- Stereopsis & 3D Reconstruction
  - Correspondence search
  - Additional correspondence constraints
  - Possible sources of error
  - Applications

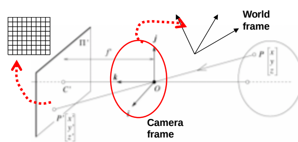
Depth with Stereo: Basic Idea



- Basic Principle: Triangulation
  - Gives reconstruction as intersection of two rays
  - Requires
    - Camera pose (calibration)
    - Point correspondence

Slide credit: Steve Seitz

Camera Calibration



Extrinsic parameters:
Camera frame ↔ Reference frame

Intrinsic parameters:
Image coordinates relative to camera ↔ Pixel coordinates

- Parameters
  - *Extrinsic*: rotation matrix and translation vector
  - *Intrinsic*: focal length, pixel sizes (mm), image center point, radial distortion parameters

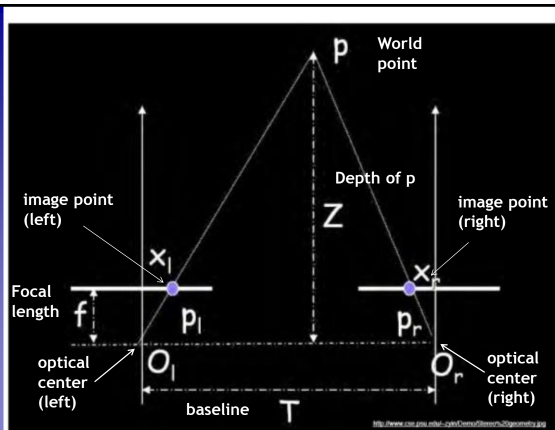
We'll assume for now that these parameters are given and fixed.

Slide credit: Kristen Grauman

Geometry for a Simple Stereo System

- First, assuming parallel optical axes, known camera parameters (i.e., calibrated cameras):

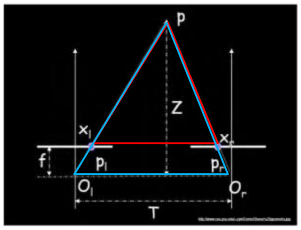
Slide credit: Kristen Grauman



Slide credit: Kristen Grauman

Geometry for a Simple Stereo System

- Assume parallel optical axes, known camera parameters (i.e., calibrated cameras). We can triangulate via:



Similar triangles (p_l, P, p_r) and (O_l, P, O_r):

$$\frac{T-(x_r-x_l)}{Z-f}=\frac{T}{Z}$$

$$Z=f\frac{T}{x_r-x_l}$$

"disparity"

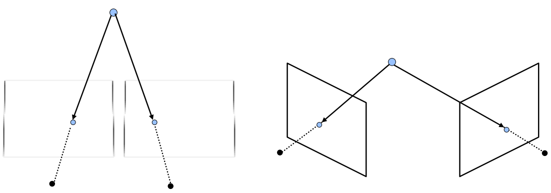
Depth From Disparity

Image $I(x,y)$ | Disparity map $D(x,y)$ | Image $I'(x',y')$

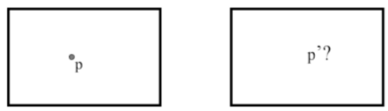
$$(x',y')=(x+D(x,y),y)$$

General Case With Calibrated Cameras

- The two cameras need not have parallel optical axes.

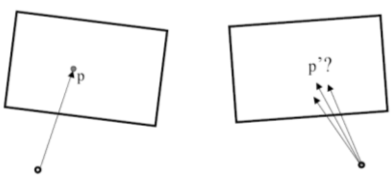


Stereo Correspondence Constraints



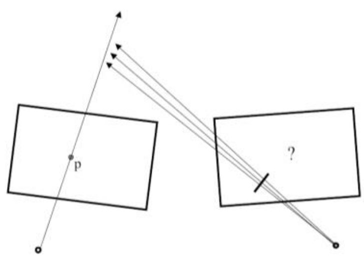
- Given p in the left image, where can the corresponding point p' in the right image be?

Stereo Correspondence Constraints



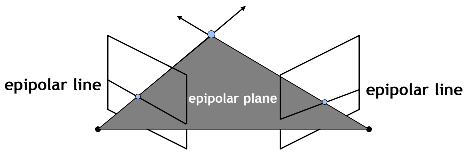
- Given p in the left image, where can the corresponding point p' in the right image be?

Stereo Correspondence Constraints



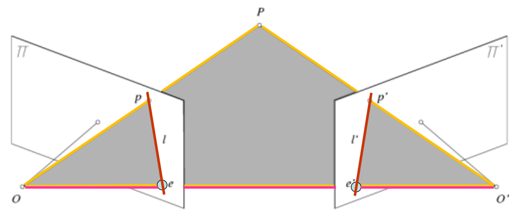
Stereo Correspondence Constraints

- Geometry of two views allows us to constrain where the corresponding pixel for some image point in the first view must occur in the second view.



- Epipolar constraint: Why is this useful?
  - Reduces correspondence problem to 1D search along conjugate epipolar lines.

Epipolar Geometry



• Epipolar Plane • Baseline

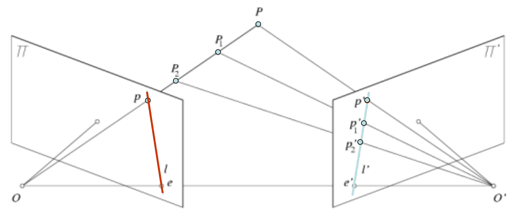
• Epipoles • Epipolar Lines

Epipolar Geometry: Terms

- ***Baseline***
  - Line joining the camera centers
- ***Epipole***
  - Point of intersection of baseline with the image plane
- ***Epipolar plane***
  - Plane containing baseline and world point
- ***Epipolar line***
  - Intersection of epipolar plane with the image plane

- Properties
  - All epipolar lines intersect at the epipole.
  - An epipolar plane intersects the left and right image planes in epipolar lines.

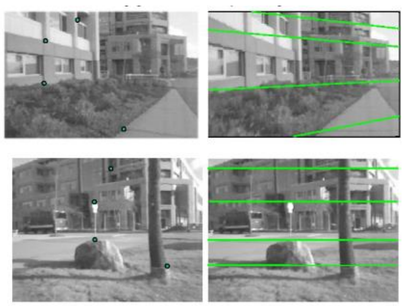
Epipolar Constraint



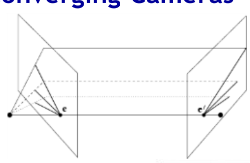
- Potential matches for p have to lie on the corresponding epipolar line l'.
- Potential matches for p' have to lie on the corresponding epipolar line l.

http://www.ai.sri.com/~luong/research/Meta3DViewer/EpipolarGeo.html

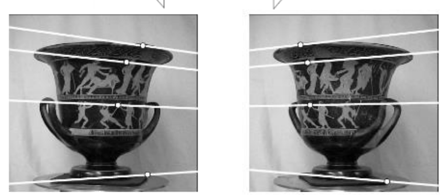
Example



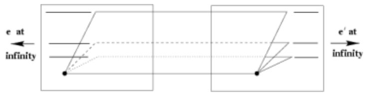
Example: Converging Cameras



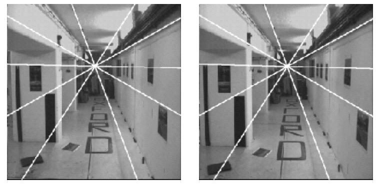
As position of 3D point varies, epipolar lines "rotate" about the baseline

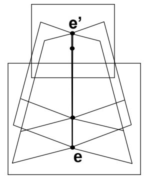


Example: Motion Parallel With Image Plane



Example: Forward Motion



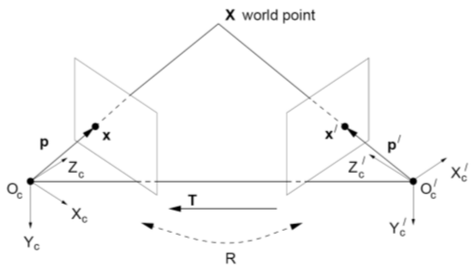


- Epipole has same coordinates in both images.
- Points move along lines radiating from e: "Focus of expansion"

Let's Formalize This!

- For a given stereo rig, how do we express the epipolar constraints algebraically?
- For this, we will need some linear algebra.
- But don't worry! We'll go through it step by step…

Stereo Geometry With Calibrated Cameras



- If the rig is calibrated, we know:
  - How to rotate and translate camera reference frame 1 to get to camera reference frame 2.
    - Rotation: 3 x 3 matrix; translation: 3 vector.

Rotation Matrix

$$\mathbf{R}_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

$$\mathbf{R}_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$\mathbf{R}_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Express 3D rotation as series of rotations around coordinate axes by angles α, β, γ

Overall rotation is product of these elementary rotations:

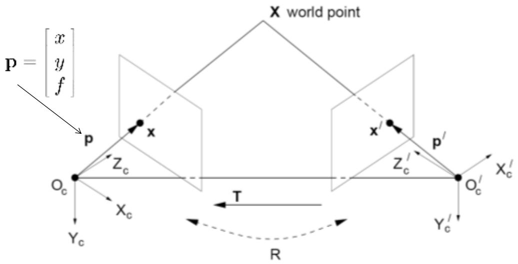
$$\mathbf{R} = \mathbf{R}_x \mathbf{R}_y \mathbf{R}_z$$

3D Rigid Transformation

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}$$

$$\mathbf{X}' = \mathbf{R}\mathbf{X} + \mathbf{T}$$

Stereo Geometry With Calibrated Cameras



- Camera-centered coordinate systems are related by known rotation $\mathbf{R}$ and translation $\mathbf{T}$:

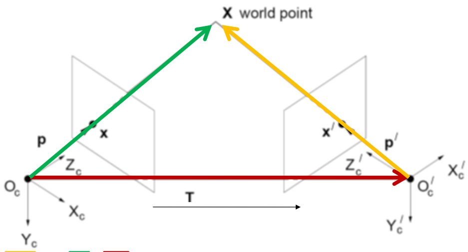
$$\mathbf{X}' = \mathbf{R}\mathbf{X} + \mathbf{T}$$

Excursion: Cross Product

$$\vec{a} \times \vec{b} = \vec{c} \qquad \begin{aligned} \vec{a} \cdot \vec{c} &= 0 \\ \vec{b} \cdot \vec{c} &= 0 \end{aligned}$$

- Vector cross product takes two vectors and returns a third vector that's perpendicular to both inputs.
- So here, c is perpendicular to both a and b, which means the dot product is 0.

From Geometry to Algebra



$$\boxed{\mathbf{X}'} = \mathbf{R}\boxed{\mathbf{X}} + \boxed{\mathbf{T}}$$

$$\underbrace{\mathbf{T} \times \mathbf{X}'}_{\text{Normal to the plane}} = \mathbf{T} \times \mathbf{R}\mathbf{X} + \mathbf{T} \times \mathbf{T}$$

$$= \mathbf{T} \times \mathbf{R}\mathbf{X}$$

$$\mathbf{X}' \cdot (\mathbf{T} \times \mathbf{X}') = \mathbf{X}' \cdot (\mathbf{T} \times \mathbf{R}\mathbf{X})$$

$$0 = \mathbf{X}' \cdot (\mathbf{T} \times \mathbf{R}\mathbf{X})$$

Matrix Form of Cross Product

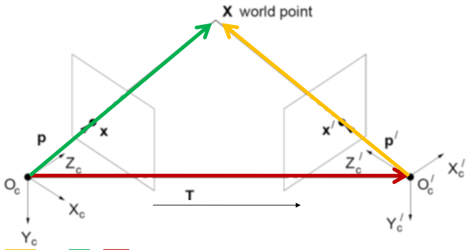
$$\vec{a} \times \vec{b} \qquad = \vec{c} \qquad \begin{aligned} \vec{a} \cdot \vec{c} &= 0 \\ \vec{b} \cdot \vec{c} &= 0 \end{aligned}$$

"skew symmetric" matrix

$$[a_\times] = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

$$\boxed{\vec{a} \times \vec{b} = [a_\times]\,\vec{b}}$$

From Geometry to Algebra



$$\boxed{\mathbf{X}'} = \mathbf{R}\boxed{\mathbf{X}} + \boxed{\mathbf{T}}$$

$$\underbrace{\mathbf{T} \times \mathbf{X}'}_{\text{Normal to the plane}} = \mathbf{T} \times \mathbf{R}\mathbf{X} + \mathbf{T} \times \mathbf{T}$$

$$= \mathbf{T} \times \mathbf{R}\mathbf{X}$$

$$\mathbf{X}' \cdot (\mathbf{T} \times \mathbf{X}') = \mathbf{X}' \cdot (\mathbf{T} \times \mathbf{R}\mathbf{X})$$

$$\boxed{0 = \mathbf{X}' \cdot (\mathbf{T} \times \mathbf{R}\mathbf{X})}$$

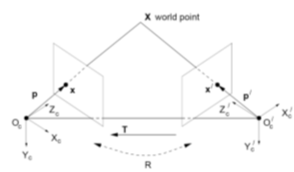
Essential Matrix

$$\boxed{\mathbf{X}' \cdot (\mathbf{T} \times \mathbf{R}\mathbf{X}) = 0}$$

$$\mathbf{X}' \cdot (\mathbf{T}_x \ \mathbf{R}\mathbf{X}) = 0$$

Let $\mathbf{E} = \mathbf{T}_x\mathbf{R}$

$$\mathbf{X}'^T \mathbf{E}\mathbf{X} = 0$$



- This holds for the rays p and p' that are parallel to the camera-centered position vectors X and X', so we have: $\boxed{\mathbf{p}'^T \mathbf{E}\mathbf{p} = 0}$
- $\mathbf{E}$ is called the essential matrix, which relates corresponding image points [Longuet-Higgins 1981]

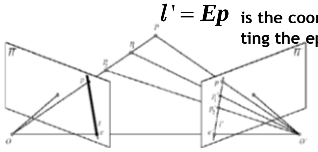
Essential Matrix and Epipolar Lines

$\boxed{\mathbf{p}'^{\mathrm{T}}\mathbf{E}\mathbf{p} = 0}$

Epipolar constraint: if we observe point p in one image, then its position p' in second image must satisfy this equation.

$l' = Ep$ is the coordinate vector representing the epipolar line for point p

(i.e., the line is given by: $l'^{\mathrm{T}}\mathbf{x} = 0$)



$l = E^T p'$ is the coordinate vector representing the epipolar line for point p'

Computer Vision WS 14/15

Slide credit: Kristen Grauman

B. Leibe

Essential Matrix: Properties

- Relates image of corresponding points in both cameras, given rotation and translation.
- Assuming intrinsic parameters are known

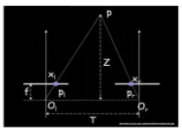
$$\mathbf{E} = \mathbf{T}_x\mathbf{R}$$

Computer Vision WS 14/15

Slide credit: Kristen Grauman

B. Leibe

Essential Matrix Example: Parallel Cameras



$\mathbf{R} =$

$\mathbf{T} =$

$\mathbf{E} = [\mathbf{T}_x]\mathbf{R} =$

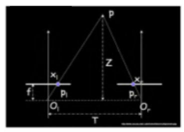
$\mathbf{p}'^{\mathrm{T}}\mathbf{E}\mathbf{p} = 0$

For the parallel cameras, image of any point must lie on same horizontal line in each image plane.

Computer Vision WS 14/15

Slide credit: Kristen Grauman

Essential Matrix Example: Parallel Cameras



$\mathbf{R} = \mathbf{I}$

$\mathbf{T} = [-d, 0, 0]^{\mathrm{T}}$

$\mathbf{E} = [\mathbf{T}_x]\mathbf{R} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & d \\ 0 & -d & 0 \end{pmatrix}$

$\mathbf{p}'^{\mathrm{T}}\mathbf{E}\mathbf{p} = 0$

$$\begin{bmatrix} x' & y' & f \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & d \\ 0 & -d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ f \end{bmatrix} = 0$$

$$\Leftrightarrow \begin{bmatrix} x' & y' & f \end{bmatrix} \begin{bmatrix} 0 \\ df \\ -dy \end{bmatrix} = 0$$

$$\boxed{\Leftrightarrow y = y'}$$

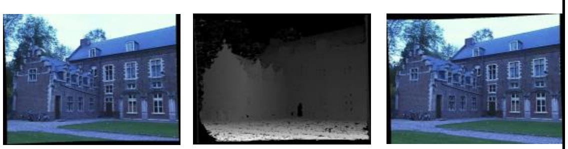
For the parallel cameras, image of any point must lie on same horizontal line in each image plane.

Computer Vision WS 14/15

Slide credit: Kristen Grauman

More General Case

Image $I(x,y)$ | Disparity map $D(x,y)$ | Image $I'(x',y')$



$$(x', y') = (x + D(x,y), y)$$

What about when cameras' optical axes are not parallel?

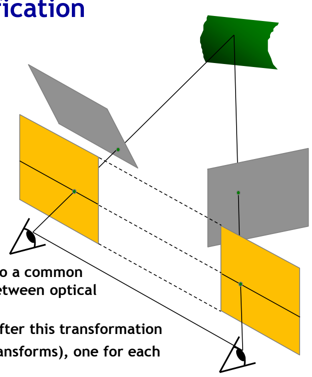
Computer Vision WS 14/15

Slide credit: Kristen Grauman

B. Leibe

Stereo Image Rectification

- In practice, it is convenient if image scanlines are the epipolar lines.

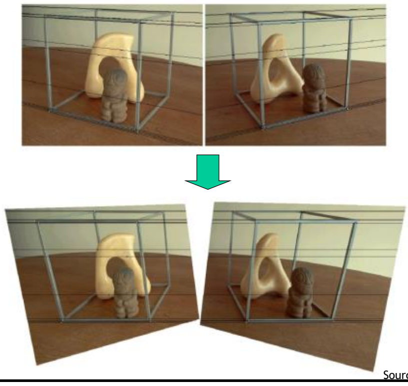


- Algorithm
  - Reproject image planes onto a common plane parallel to the line between optical centers
  - Pixel motion is horizontal after this transformation
  - Two homographies (3×3 transforms), one for each input image reprojection

Computer Vision WS 14/15

Slide adapted from Li Zhang   C. Loop & Z. Zhang, Computing Rectifying Homographies for Stereo Vision, CVPR'99

Stereo Image Rectification: Example



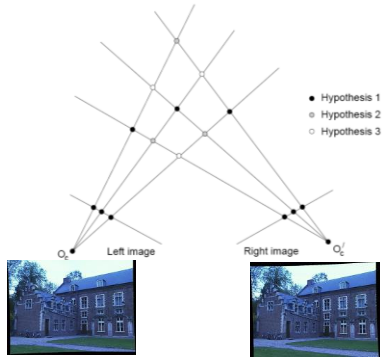
Topics of This Lecture

- Geometric vision
  - Visual cues
  - Stereo vision
- Epipolar geometry
  - Depth with stereo
  - Geometry for a simple stereo system
  - Case example with parallel optical axes
  - General case with calibrated cameras
- Stereopsis & 3D Reconstruction
  - Correspondence search
  - Additional correspondence constraints
  - Possible sources of error
  - Applications

Stereo Reconstruction

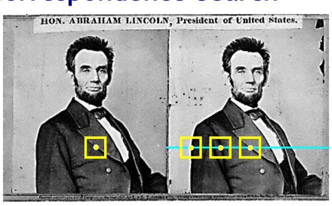
- Main Steps
  - Calibrate cameras
  - Rectify images
  - Compute disparity
  - Estimate depth

Correspondence Problem



Multiple match hypotheses satisfy epipolar constraint, but which is correct?

Dense Correspondence Search

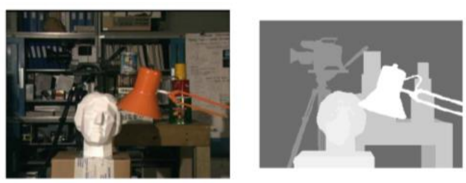


- For each pixel in the first image
  - Find corresponding epipolar line in the right image
  - Examine all pixels on the epipolar line and pick the best match (e.g. SSD, correlation)
  - Triangulate the matches to get depth information
- This is easiest when epipolar lines are scanlines

  $\Rightarrow$ Rectify images first

Example: Window Search

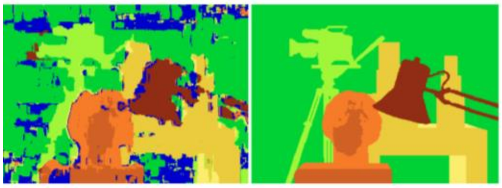
- Data from University of Tsukuba



Scene | Ground truth

Example: Window Search

- Data from University of Tsukuba



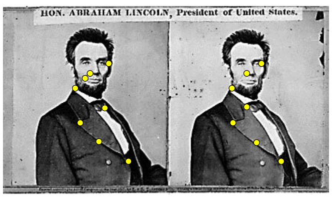
Window-based matching (best window size) | Ground truth

Effect of Window Size

$W = 3$ | $W = 20$

Want window large enough to have sufficient intensity variation, yet small enough to contain only pixels with about the same disparity.

Alternative: Sparse Correspondence Search



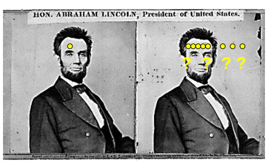
- Idea: Restrict search to sparse set of detected features
- Rather than pixel values (or lists of pixel values) use *feature descriptor* and an associated *feature distance*
- Still narrow search further by epipolar geometry

What would make good features?

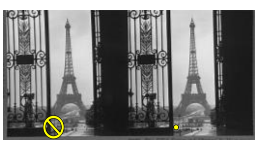
Dense vs. Sparse

- Sparse
  - Efficiency
  - Can have more reliable feature matches, less sensitive to illumination than raw pixels
  - But…
    - Have to know enough to pick good features
    - Sparse information
- Dense
  - Simple process
  - More depth estimates, can be useful for surface reconstruction
  - But…
    - Breaks down in textureless regions anyway
    - Raw pixel distances can be brittle
    - Not good with very different viewpoints

Difficulties in Similarity Constraint



Untextured surfaces



Occlusions

Possible Sources of Error?

- Low-contrast / textureless image regions
- Occlusions
- Camera calibration errors
- Violations of *brightness constancy* (e.g., specular reflections)
- Large motions

Application: View Interpolation

Computer Vision WS 14/15

Right Image

Slide credit: Svetlana Lazebnik

Application: View Interpolation

Computer Vision WS 14/15

Left Image

Slide credit: Svetlana Lazebnik

Application: View Interpolation

Computer Vision WS 14/15

Disparity

Slide credit: Svetlana Lazebnik

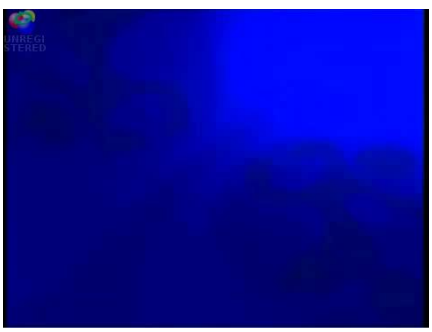
Application: View Interpolation

Computer Vision WS 14/15

Slide credit: Svetlana Lazebnik

Application: Free-Viewpoint Video

Computer Vision WS 14/15



http://www.liberovision.com

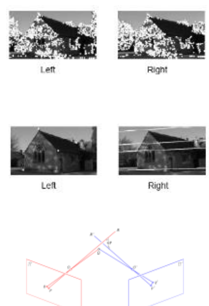
B. Leibe

84

Summary: Stereo Reconstruction

Computer Vision WS 14/15

- Main Steps
  - Calibrate cameras
  - Rectify images
  - Compute disparity
  - Estimate depth
- So far, we have only considered calibrated cameras…
- Next lecture
  - Uncalibrated cameras
  - Camera parameters
  - Revisiting epipolar geometry
  - Robust fitting



Slide credit: Kristen Grauman

B. Leibe

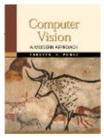
85

RWTHAACHEN UNIVERSITY

References and Further Reading

- Background information on epipolar geometry and stereopsis can be found in Chapters 10.1-10.2 and 11.1-11.3 of

  D. Forsyth, J. Ponce,
  Computer Vision – A Modern Approach.
  Prentice Hall, 2003



- More detailed information (if you really want to implement 3D reconstruction algorithms) can be found in Chapters 9 and 10 of

  R. Hartley, A. Zisserman
  Multiple View Geometry in Computer Vision
  2nd Ed., Cambridge Univ. Press, 2004