

Computer Vision - Lecture 6

Segmentation

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Course Outline

- Image Processing Basics
 - Structure Extraction
- Segmentation
 - Segmentation as Clustering
 - Graph-theoretic Segmentation
- Recognition
 - Global Representations
 - Subspace representations
- Local Features & Matching
- Object Categorization
- 3D Reconstruction
- Motion and Tracking

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Recap: Chamfer Matching

• Chamfer Distance

- Average distance to nearest feature

$$D_{chamfer}(T, I) \equiv \frac{1}{|T|} \sum_{t \in T} d_T(t)$$

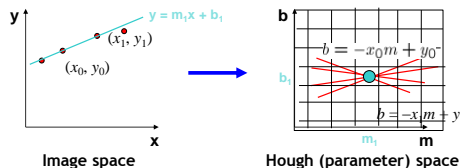
- This can be computed efficiently by correlating the edge template with the distance-transformed image



Edge image Distance transform image
B. Leibe [D. Gavrilu, DAGM'99]

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Recap: Hough Transform



- How can we use this to find the most likely parameters (m, b) for the most prominent line in the image space?
 - Let each edge point in image space vote for a set of possible parameters in Hough space
 - Accumulate votes in discrete set of bins; parameters with the most votes indicate line in image space.

see Exercise 3.1!

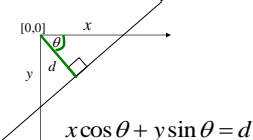
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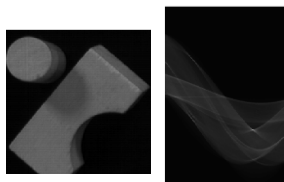
Recap: Hough Transf. Polar Parametrization

- Usual (m, b) parameter space problematic: can take on infinite values, undefined for vertical lines.



d : perpendicular distance from line to origin
 θ : angle the perpendicular makes with the x-axis

- Point in image space \Rightarrow sinusoid segment in Hough space

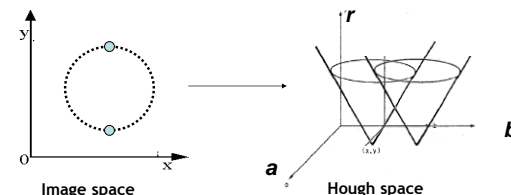


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Recap: Hough Transform for Circles

- Circle: center (a, b) and radius r
 $(x_i - a)^2 + (y_i - b)^2 = r^2$
- For an unknown radius r , unknown gradient direction



see Exercise 3.1!

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Hough Transform: Pros and Cons

Pros

- All points are processed independently, so can cope with occlusion
- Some robustness to noise: noise points unlikely to contribute consistently to any single bin
- Can detect multiple instances of a model in a single pass

Cons

- Complexity of search time increases exponentially with the number of model parameters
- Non-target shapes can produce spurious peaks in parameter space
- Quantization: hard to pick a good grid size

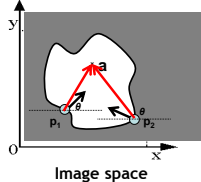
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Generalized Hough Transform

- What if we want to detect arbitrary shapes defined by boundary points and a reference point?



At each boundary point, compute displacement vector: $r = a - p_i$.

For a given model shape: store these vectors in a table indexed by gradient orientation θ .

[Dana H. Ballard, Generalizing the Hough Transform to Detect Arbitrary Shapes, 1980]

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Generalized Hough Transform

To detect the model shape in a new image:

- For each edge point
 - Index into table with its gradient orientation θ
 - Use retrieved r vectors to vote for position of reference point
- Peak in this Hough space is reference point with most supporting edges

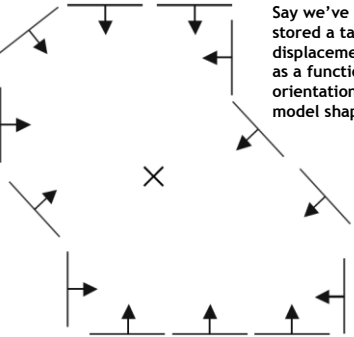
Assuming translation is the only transformation here, i.e., orientation and scale are fixed.

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Example: Generalized Hough Transform



Say we've already stored a table of displacement vectors as a function of edge orientation for this model shape.

Model shape

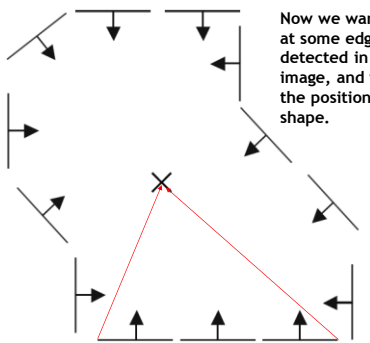
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Example: Generalized Hough Transform

Now we want to look at some edge points detected in a new image, and vote on the position of that shape.



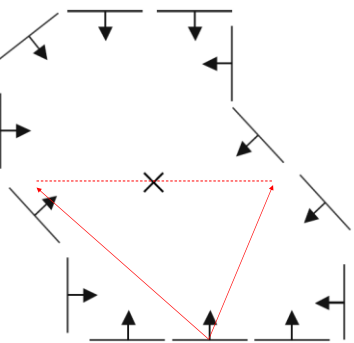
Displacement vectors for model points

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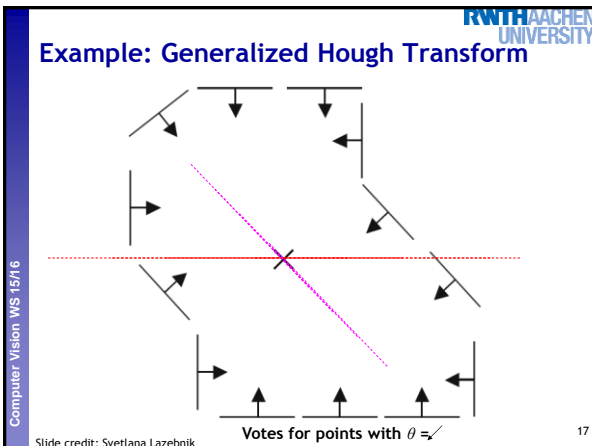
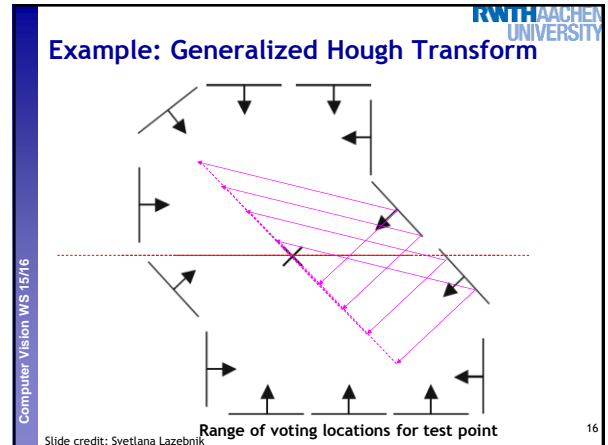
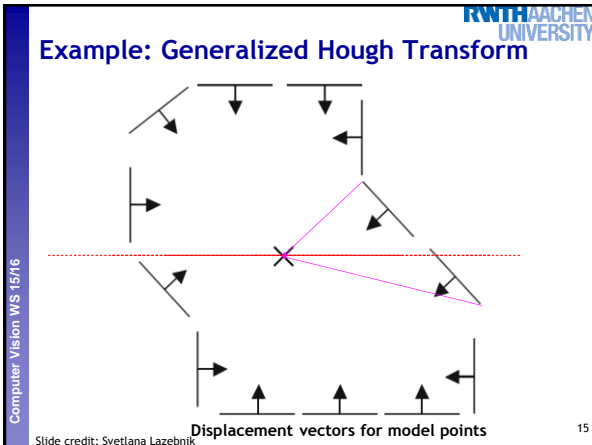
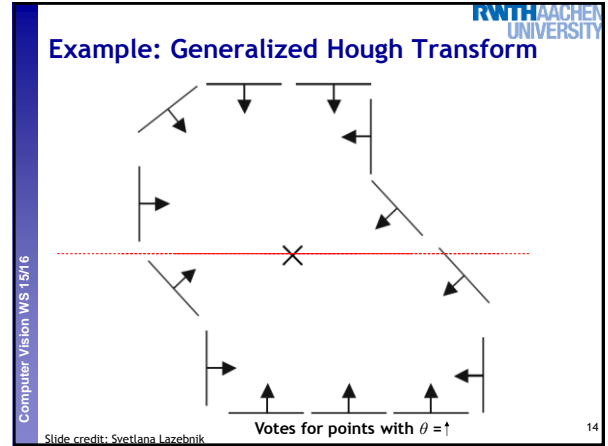
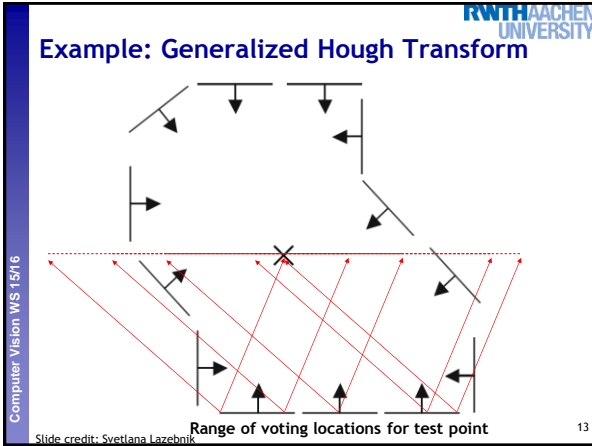
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Example: Generalized Hough Transform



Range of voting locations for test point

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Application in Recognition

- Instead of indexing displacements by gradient orientation, index by “visual codeword”.

Training image

Visual codeword with displacement vectors

B. Leibe, A. Leonardis, and B. Schiele, [Robust Object Detection with Interleaved Categorization and Segmentation](#), International Journal of Computer Vision, Vol. 77(1-3), 2008.

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Application in Recognition

- Instead of indexing displacements by gradient orientation, index by “visual codeword”.

Test image

- We'll hear more about this in later lectures...

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Topics of This Lecture

- Segmentation and grouping
 - Gestalt principles
 - Image Segmentation
- Segmentation as clustering
 - k-Means
 - Feature spaces
- Probabilistic clustering
 - Mixture of Gaussians, EM
- Model-free clustering
 - Mean-Shift clustering

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Examples of Grouping in Vision

Determining image regions

Grouping video frames into shots

Figure-ground

Object-level grouping

What things should be grouped?

What cues indicate groups?

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The Gestalt School

- Grouping is key to visual perception
- Elements in a collection can have properties that result from relationships
 - “The whole is greater than the sum of its parts”

Illusory/subjective contours

Occlusion

Familiar configuration

http://en.wikipedia.org/wiki/Gestalt_psychology

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Image source: Steve Lehar

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Gestalt Theory

- Gestalt: whole or group
 - Whole is greater than sum of its parts
 - Relationships among parts can yield new properties/features
- Psychologists identified series of factors that predispose set of elements to be grouped (by human visual system)

“I stand at the window and see a house, trees, sky. Theoretically I might say there were 327 brightnesses and nuances of colour. Do I have “327”? No. I have sky, house, and trees.”

Max Wertheimer (1880-1943)

Untersuchungen zur Lehre von der Gestalt, *Psychologische Forschung*, Vol. 4, pp. 301-350, 1923
<http://psy.ed.asu.edu/~classics/Wertheimer/Forms/forms.htm>

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Gestalt Factors

Not grouped

Proximity

Similarity

Similarity

Common Fate

Common Region

Parallelism

Symmetry

Continuity

Closure

- These factors make intuitive sense, but are very difficult to translate into algorithms.

Image source: Forsyth & Ponce

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Continuity through Occlusion Cues

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Continuity through Occlusion Cues

Continuity, explanation by occlusion

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Continuity through Occlusion Cues

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Image source: Forsyth & Ponce

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Continuity through Occlusion Cues

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Image source: Forsyth & Ponce

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The Ultimate Gestalt?

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Image Segmentation

- Goal: identify groups of pixels that go together

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The Goals of Segmentation

- Separate image into coherent “objects”

Image

Human segmentation

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Image Segmentation: Toy Example

input image

intensity

- These intensities define the three groups.
- We could label every pixel in the image according to which of these primary intensities it is.
 - i.e., segment the image based on the intensity feature.
- What if the image isn't quite so simple?

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Intensity

Intensity

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Intensity

- Now how to determine the three main intensities that define our groups?
- We need to cluster.

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- Goal: choose three “centers” as the representative intensities, and label every pixel according to which of these centers it is nearest to.
- Best cluster centers are those that minimize SSD between all points and their nearest cluster center c_i :

$$\sum_{\text{clusters } i} \sum_{\text{points } p \text{ in cluster } i} \|p - c_i\|^2$$

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Clustering

- With this objective, it is a “chicken and egg” problem:
 - If we knew the *cluster centers*, we could allocate points to groups by assigning each to its closest center.

- If we knew the *group memberships*, we could get the centers by computing the mean per group.

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K-Means Clustering

- Basic idea: randomly initialize the k cluster centers, and iterate between the two steps we just saw.
 - Randomly initialize the cluster centers, c_1, \dots, c_k
 - Given cluster centers, determine points in each cluster
 - For each point p , find the closest c_i . Put p into cluster i
 - Given points in each cluster, solve for c_i
 - Set c_i to be the mean of points in cluster i
 - If c_i have changed, repeat Step 2

- Properties
 - Will always converge to *some* solution
 - Can be a “local minimum”
 - Does not always find the global minimum of objective function:

$$\sum_{\text{clusters } i} \sum_{\text{points } p \text{ in cluster } i} \|p - c_i\|^2$$

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Segmentation as Clustering

```

img_as_col = double(im(:));
cluster_membs = kmeans(img_as_col, K);
labelim = zeros(size(im));
for i=1:k
    inds = find(cluster_membs==i);
    meanval = mean(img_as_col(inds));
    labelim(inds) = meanval;
end
  
```

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K-Means++

- Can we prevent arbitrarily bad local minima?
 - Randomly choose first center.
 - Pick new center with prob. proportional to $\|p - c_i\|^2$
 - (Contribution of p to total error)
 - Repeat until k centers.
- Expected error = $O(\log k)$ * optimal

[Arthur & Vassilvitskii 2007](#)

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Feature Space

- Depending on what we choose as the *feature space*, we can group pixels in different ways.
- Grouping pixels based on **intensity** similarity

- Feature space: intensity value (1D)

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Feature Space

- Depending on what we choose as the *feature space*, we can group pixels in different ways.
- Grouping pixels based on **color** similarity

- Feature space: color value (3D)

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Segmentation as Clustering

- Depending on what we choose as the *feature space*, we can group pixels in different ways.
- Grouping pixels based on **texture similarity**

Filter bank of 24 filters

- Feature space:** filter bank responses (e.g., 24D)

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Smoothing Out Cluster Assignments

- Assigning a cluster label per pixel may yield outliers:

- How can we ensure they are spatially smooth?

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Segmentation as Clustering

- Depending on what we choose as the *feature space*, we can group pixels in different ways.
- Grouping pixels based on **intensity+position** similarity

⇒ Simple way to encode both *similarity* and *proximity*.

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Summary K-Means

- Pros**
 - Simple, fast to compute
 - Converges to local minimum of within-cluster squared error
- Cons/issues**
 - Setting k ?
 - Sensitive to initial centers
 - Sensitive to outliers
 - Detects spherical clusters only
 - Assuming means can be computed

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Topics of This Lecture

- Segmentation and grouping
 - Gestalt principles
 - Image Segmentation
- Segmentation as clustering
 - k-Means
 - Feature spaces
- Probabilistic clustering**
 - Mixture of Gaussians, EM
- Model-free clustering**
 - Mean-Shift clustering

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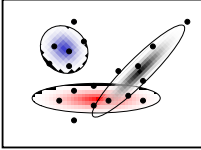
Probabilistic Clustering

- Basic questions**
 - What's the probability that a point x is in cluster m ?
 - What's the shape of each cluster?
- K-means doesn't answer these questions.
- Basic idea**
 - Instead of treating the data as a bunch of points, assume that they are all generated by sampling a continuous function.
 - This function is called a **generative model**.
 - Defined by a vector of parameters θ

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Mixture of Gaussians



- One generative model is a mixture of Gaussians (MoG)
 - K Gaussian blobs with means μ_j , cov. matrices Σ_j , dim. D

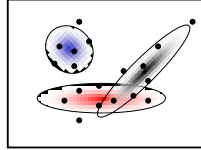
$$p(\mathbf{x}|\theta_j) = \frac{1}{(2\pi)^{D/2} |\Sigma_j|^{1/2}} \exp\left\{-\frac{1}{2}(\mathbf{x} - \mu_j)^T \Sigma_j^{-1} (\mathbf{x} - \mu_j)\right\}$$
 - Blob j is selected with probability π_j
 - The likelihood of observing \mathbf{x} is a weighted mixture of Gaussians

$$p(\mathbf{x}|\theta) = \sum_{j=1}^K \pi_j p(\mathbf{x}|\theta_j) \quad \theta = (\pi_1, \mu_1, \Sigma_1, \dots, \pi_M, \mu_M, \Sigma_M)$$

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Expectation Maximization (EM)



- Goal
 - Find blob parameters θ that maximize the likelihood function:

$$p(\text{data}|\theta) = \prod_{n=1}^N p(\mathbf{x}_n|\theta)$$
- Approach:
 1. E-step: given current guess of blobs, compute ownership of each point
 2. M-step: given ownership probabilities, update blobs to maximize likelihood function
 3. Repeat until convergence

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EM Algorithm

see lecture Machine Learning!

- Expectation-Maximization (EM) Algorithm
 - E-Step: softly assign samples to mixture components

$$\gamma_j(\mathbf{x}_n) \leftarrow \frac{\pi_j \mathcal{N}(\mathbf{x}_n|\mu_j, \Sigma_j)}{\sum_{k=1}^K \pi_k \mathcal{N}(\mathbf{x}_n|\mu_k, \Sigma_k)} \quad \forall j = 1, \dots, K, \quad n = 1, \dots, N$$
 - M-Step: re-estimate the parameters (separately for each mixture component) based on the soft assignments

$$\hat{N}_j \leftarrow \sum_{n=1}^N \gamma_j(\mathbf{x}_n) = \text{soft number of samples labeled } j$$

$$\hat{\pi}_j^{\text{new}} \leftarrow \frac{\hat{N}_j}{N}$$

$$\hat{\mu}_j^{\text{new}} \leftarrow \frac{1}{\hat{N}_j} \sum_{n=1}^N \gamma_j(\mathbf{x}_n) \mathbf{x}_n$$

$$\hat{\Sigma}_j^{\text{new}} \leftarrow \frac{1}{\hat{N}_j} \sum_{n=1}^N \gamma_j(\mathbf{x}_n) (\mathbf{x}_n - \hat{\mu}_j^{\text{new}})(\mathbf{x}_n - \hat{\mu}_j^{\text{new}})^T$$

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Applications of EM


- Turns out this is useful for all sorts of problems
 - Any clustering problem
 - Any model estimation problem
 - Missing data problems
 - Finding outliers
 - Segmentation problems
 - Segmentation based on color
 - Segmentation based on motion
 - Foreground/background separation
 - ...
- EM demo
 - <http://lcn.epfl.ch/tutorial/english/gaussian/html/index.html>

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Segmentation with EM

Original image



EM segmentation results



k=2 k=3 k=4 k=5

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Summary: Mixtures of Gaussians, EM

- Pros
 - Probabilistic interpretation
 - Soft assignments between data points and clusters
 - Generative model, can predict novel data points
 - Relatively compact storage
- Cons
 - Local minima
 - k-means is NP-hard even with k=2
 - Initialization
 - Often a good idea to start with some k-means iterations.
 - Need to know number of components
 - Solutions: model selection (AIC, BIC), Dirichlet process mixture
 - Need to choose generative model
 - Numerical problems are often a nuisance

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 - Feature spaces
- Probabilistic clustering
 - Mixture of Gaussians, EM
- **Model-free clustering**
 - Mean-Shift clustering

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Finding Modes in a Histogram

- How many modes are there?
 - *Mode* = local maximum of the density of a given distribution
 - Easy to see, hard to compute

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Mean-Shift Segmentation

- An advanced and versatile technique for clustering-based segmentation

Segmented "landscape 1"

Segmented "landscape 2"

<http://www.caip.rutgers.edu/~comanici/MSPAMI/msPamiResults.html>

D. Comanici and P. Meer, *Mean Shift: A Robust Approach toward Feature Space Analysis*, PAMI 2002.

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Mean-Shift Algorithm

- Iterative Mode Search
 1. Initialize random seed, and window W
 2. Calculate center of gravity (the "mean") of W : $\sum_{x \in W} xH(x)$
 3. Shift the search window to the mean
 4. Repeat Step 2 until convergence

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Mean-Shift

Region of interest

Center of mass

Mean Shift vector

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Mean-Shift

Region of interest

Center of mass

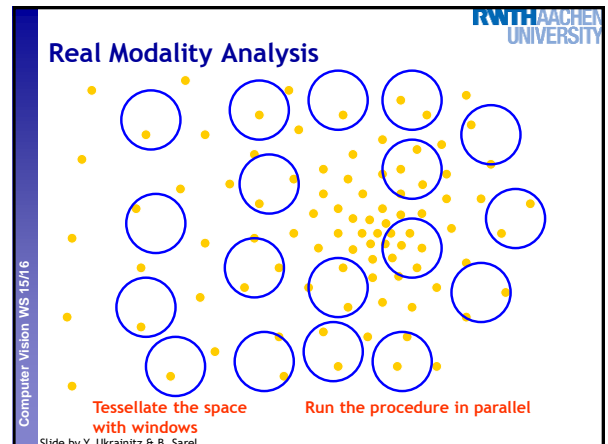
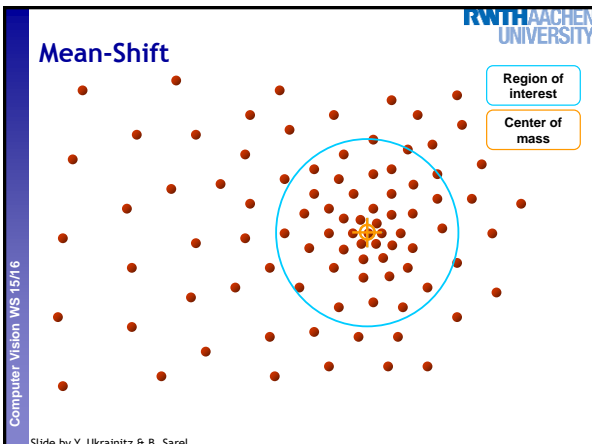
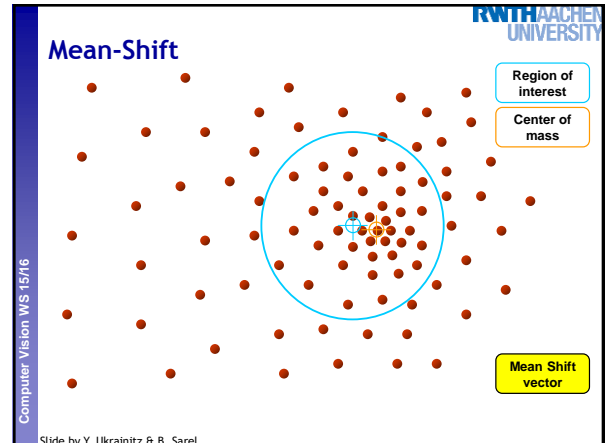
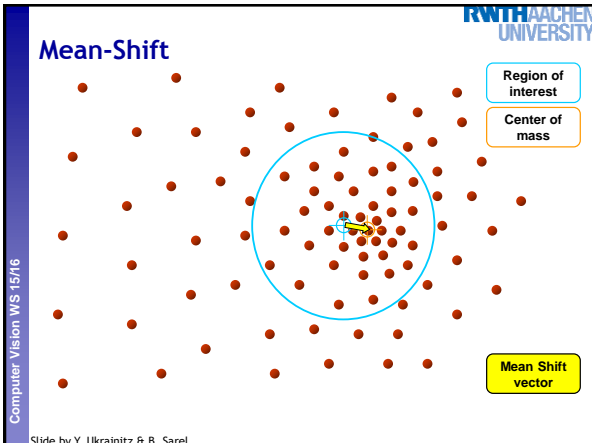
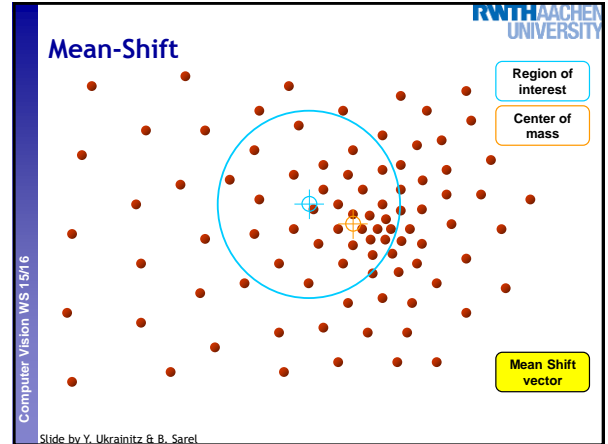
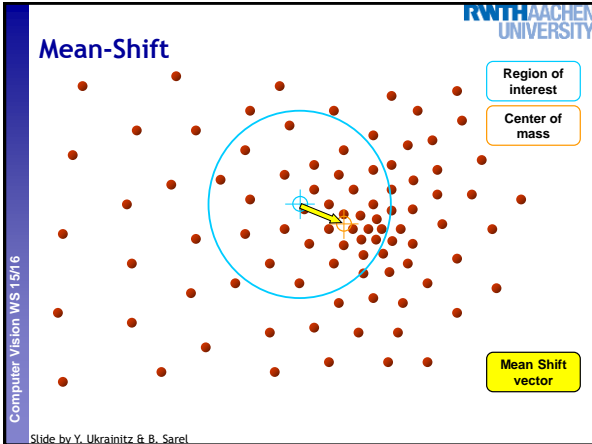
Mean Shift vector

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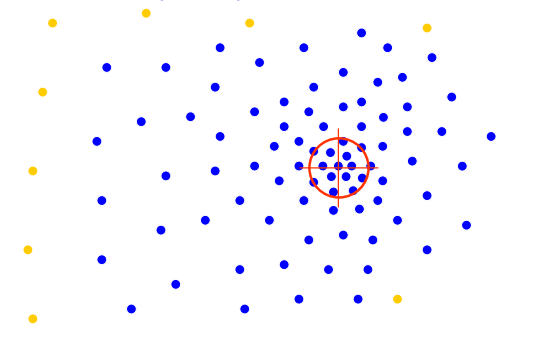
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Real Modality Analysis



The blue data points were traversed by the windows towards the mode.

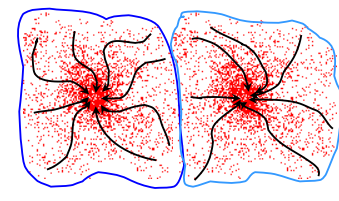
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Mean-Shift Clustering

- Cluster: all data points in the attraction basin of a mode
- Attraction basin: the region for which all trajectories lead to the same mode



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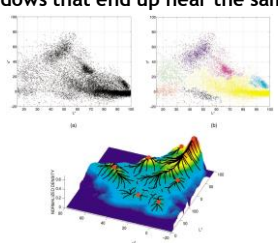
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Mean-Shift Clustering/Segmentation

- Find features (color, gradients, texture, etc)
- Initialize windows at individual pixel locations
- Perform mean shift for each window until convergence
- Merge windows that end up near the same "peak" or mode



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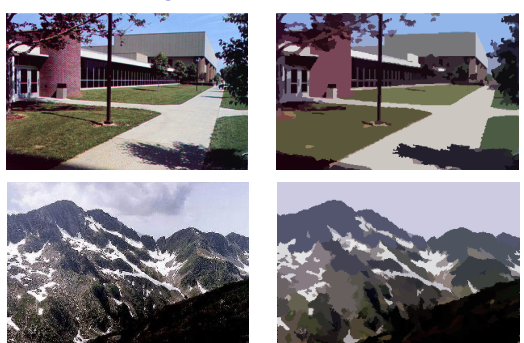
Slide credit: Svetlana Lazebnik

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Mean-Shift Segmentation Results



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<http://www.caip.rutgers.edu/~comanici/MSPAMI/msPamiResults.html>

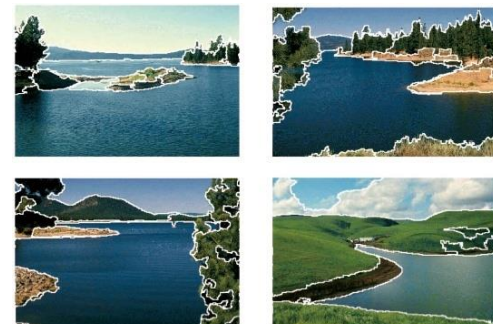
Slide credit: Svetlana Lazebnik

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More Results



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More Results



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Problem: Computational Complexity

- Need to shift many windows...
- Many computations will be redundant.

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Speedups: Basin of Attraction

1. Assign all points within radius r of end point to the mode.

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Speedups

2. Assign all points within radius r/c of the search path to the mode.

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Summary Mean-Shift

- **Pros**
 - General, application-independent tool
 - Model-free, does not assume any prior shape (spherical, elliptical, etc.) on data clusters
 - Just a single parameter (window size h)
 - h has a physical meaning (unlike k -means)
 - Finds variable number of modes
 - Robust to outliers
- **Cons**
 - Output depends on window size
 - Window size (bandwidth) selection is not trivial
 - Computationally (relatively) expensive ($\sim 2s/\text{image}$)
 - Does not scale well with dimension of feature space

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Segmentation: Caveats

- We've looked at *bottom-up* ways to segment an image into regions, yet finding meaningful segments is intertwined with the recognition problem.
- Often want to avoid making hard decisions too soon
- Difficult to evaluate; when is a segmentation successful?

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Generic Clustering

- We have focused on ways to group pixels into image segments based on their appearance
 - Find groups; "quantize" feature space
- In general, we can use clustering techniques to find groups of similar "tokens", provided we know how to compare the tokens.
 - E.g., segment an image into the types of motions present
 - E.g., segment a video into the types of scenes (shots) present

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References and Further Reading

- Background information on segmentation by clustering can be found in Chapter 14 of
 - D. Forsyth, J. Ponce,
Computer Vision - A Modern Approach.
Prentice Hall, 2003
- More on the EM algorithm can be found in Chapter 16.1.2.
- Try the k-means and EM demos at
 - http://home.dei.polimi.it/matteucc/Clustering/tutorial_html/AppletKM.html
 - <http://icn.epfl.ch/tutorial/english/gaussian/html/index.html>

