Computer Vision – Lecture 9

Recognition with Global Representations II

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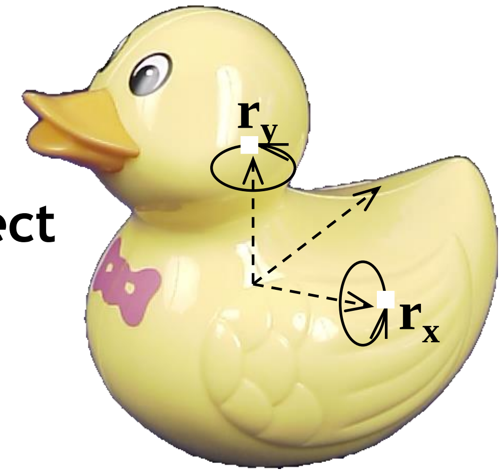
Course Outline

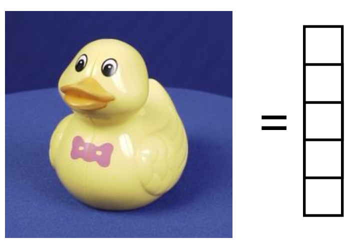
- Image Processing Basics
- Segmentation & Grouping
- Recognition
  - Global Representations
  - Subspace Representations
- Object Categorization I
  - Sliding Window based Object Detection
- Local Features & Matching
- Object Categorization II
  - Part based Approaches
- 3D Reconstruction
- Motion and Tracking

Recap: Appearance-Based Recognition

- Basic assumption
  - Objects can be represented by a set of images ("appearances").
  - For recognition, it is sufficient to just compare the 2D appearances.
  - No 3D model is needed.

3D object

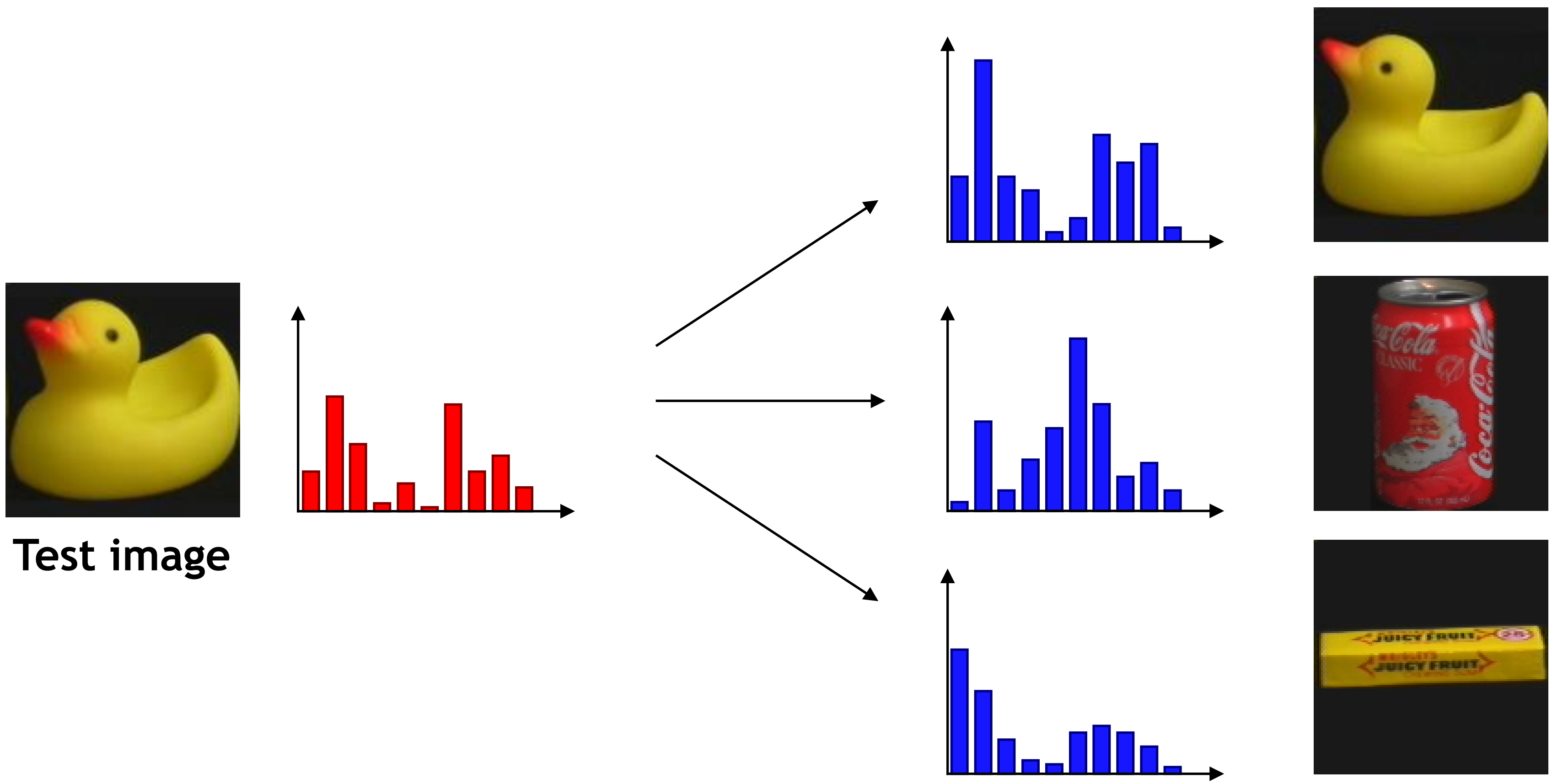




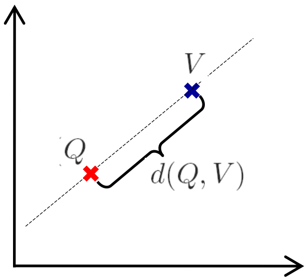
⇒ Fundamental paradigm shift in the 90's

Recap: Recognition Using Histograms

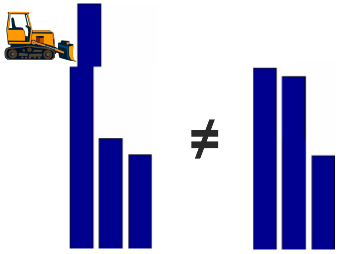
- Histogram comparison



Recap: Comparison Measures



- Vector space interpretation
  - Euclidean distance
  - Mahalanobis distance
- Statistical motivation
  - Chi-square
  - Bhattacharyya
- Information-theoretic motivation
  - Kullback-Leibler divergence, Jeffreys divergence
- Histogram motivation
  - Histogram intersection
- Ground distance
  - Earth Movers Distance (EMD)

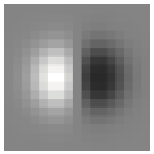
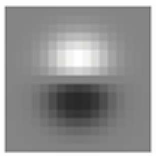
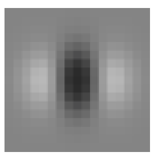
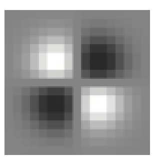
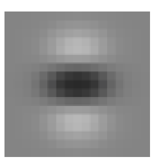


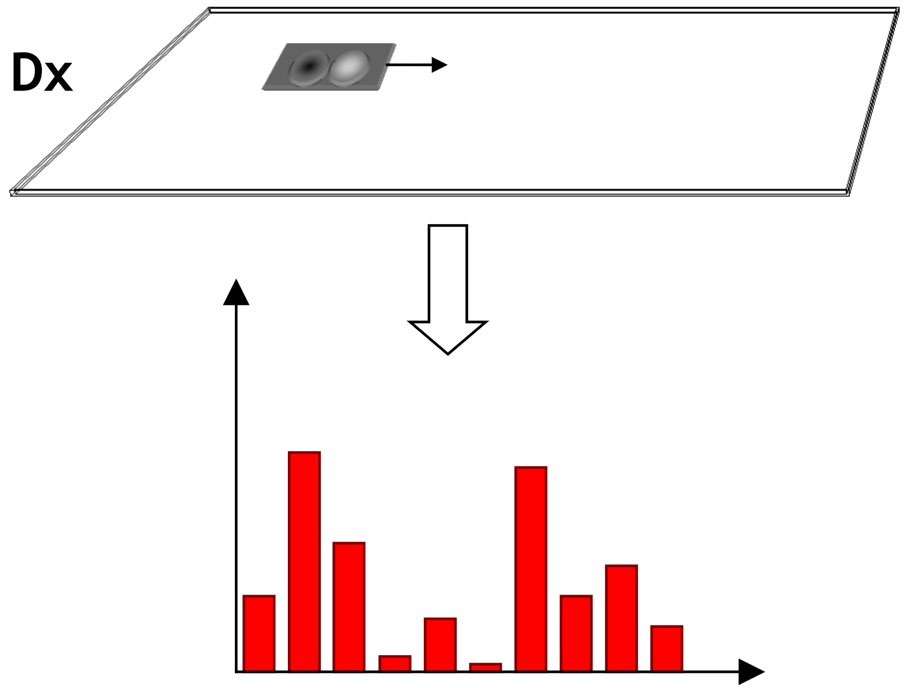
Recap: Recognition Using Histograms

- Simple algorithm
  1. Build a set of histograms $H=\{h_i\}$ for each known object
     - More exactly, for each *view* of each object
  2. Build a histogram h_t for the test image.
  3. Compare h_t to each $h_i \in H$
     - Using a suitable comparison measure
  4. Select the object with the best matching score
     - Or reject the test image if no object is similar enough.

“Nearest-Neighbor” strategy

Generalization of the Idea

- **Histograms of derivatives**
  - Dx 
  - Dy 
  - Dxx 
  - Dxy 
  - Dyy 



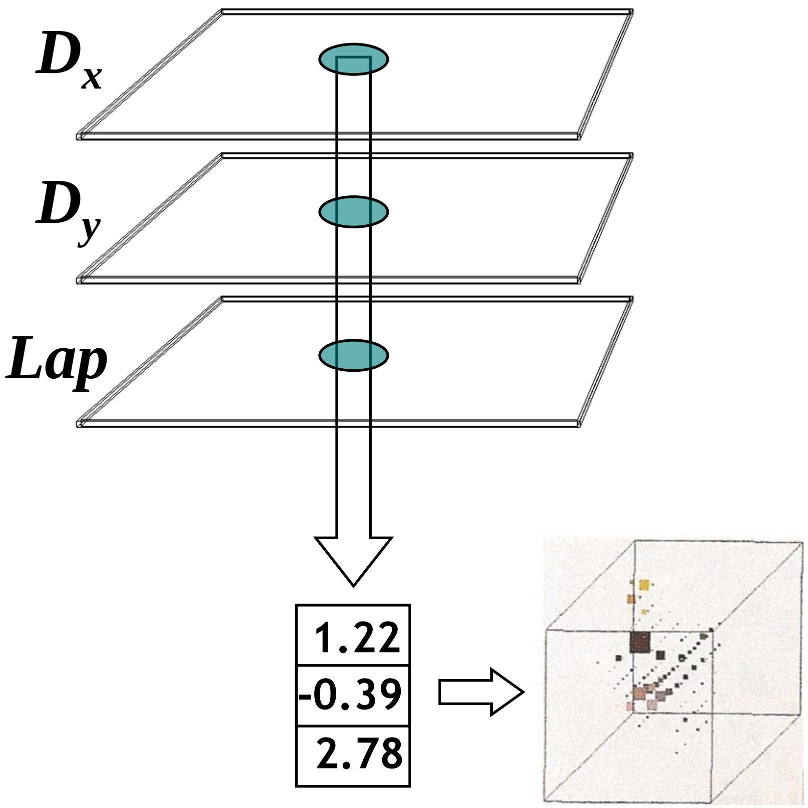
General Filter Response Histograms

- Any local descriptor (e.g. filter, filter combination) can be used to build a histogram.

- Examples:
  - Gradient magnitude $$Mag = \sqrt{D_x^2 + D_y^2}$$
  - Gradient direction $$Dir = \arctan \frac{D_y}{D_x}$$
  - Laplacian $$Lap = D_{xx} + D_{yy}$$

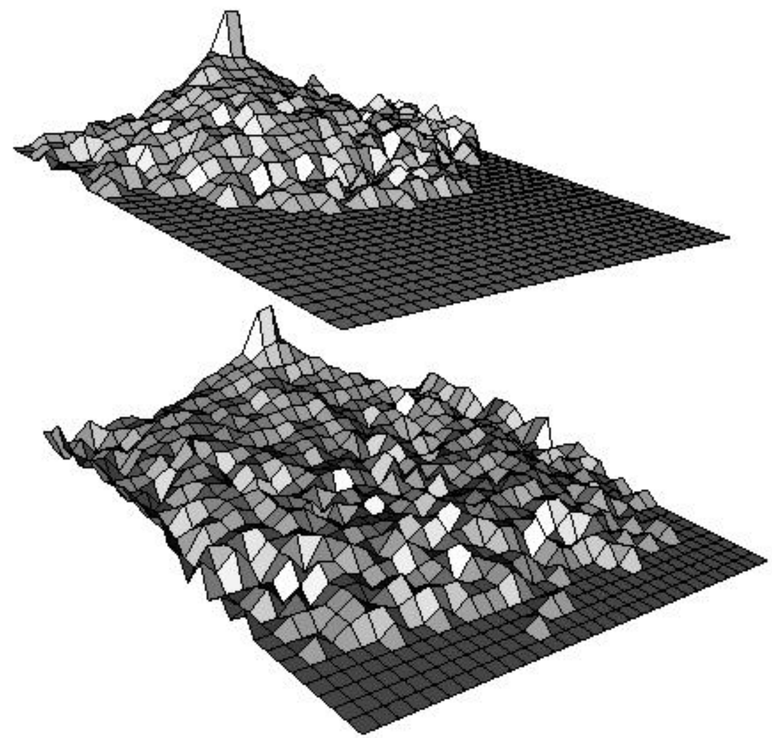
Multidimensional Representations

- Combination of several descriptors
  - Each descriptor is applied to the whole image.
  - Corresponding pixel values are combined into one feature vector.
  - Feature vectors are collected in multidimensional histogram.



Multidimensional Histograms

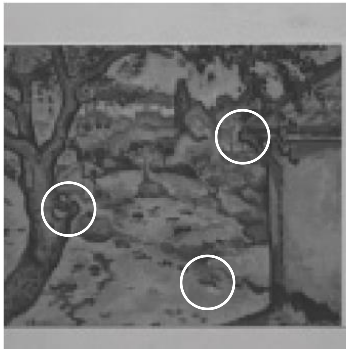
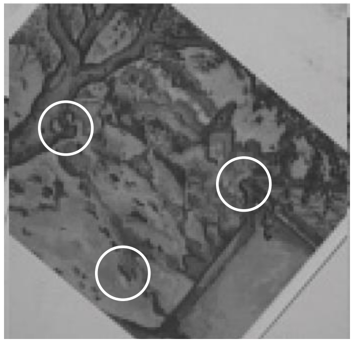
- **Examples**



[Schiele & Crowley, 2000]

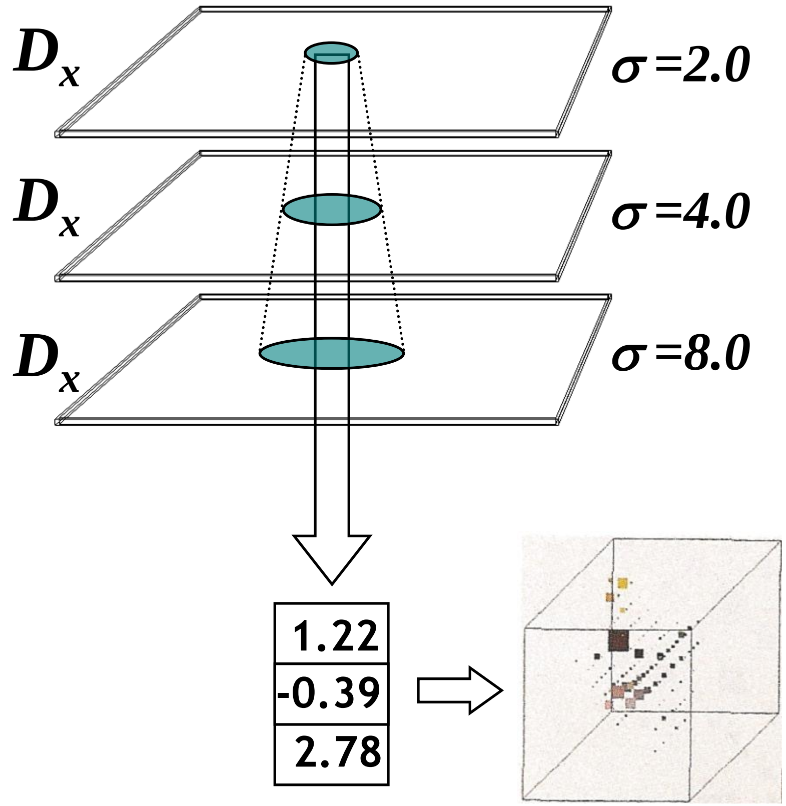
Multidimensional Representations

- **Useful simple combinations**
  - D_x-D_y **Rotation-variant**
    - Descriptor changes when image is rotated.
    - Useful for recognizing oriented structures (e.g. vertical lines)
  - *Mag-Lap* **Rotation-invariant**
    - Descriptor does *not* change when image is rotated.
    - Can be used to recognize rotated objects.
    - Less discriminant than rotation-variant descriptor.

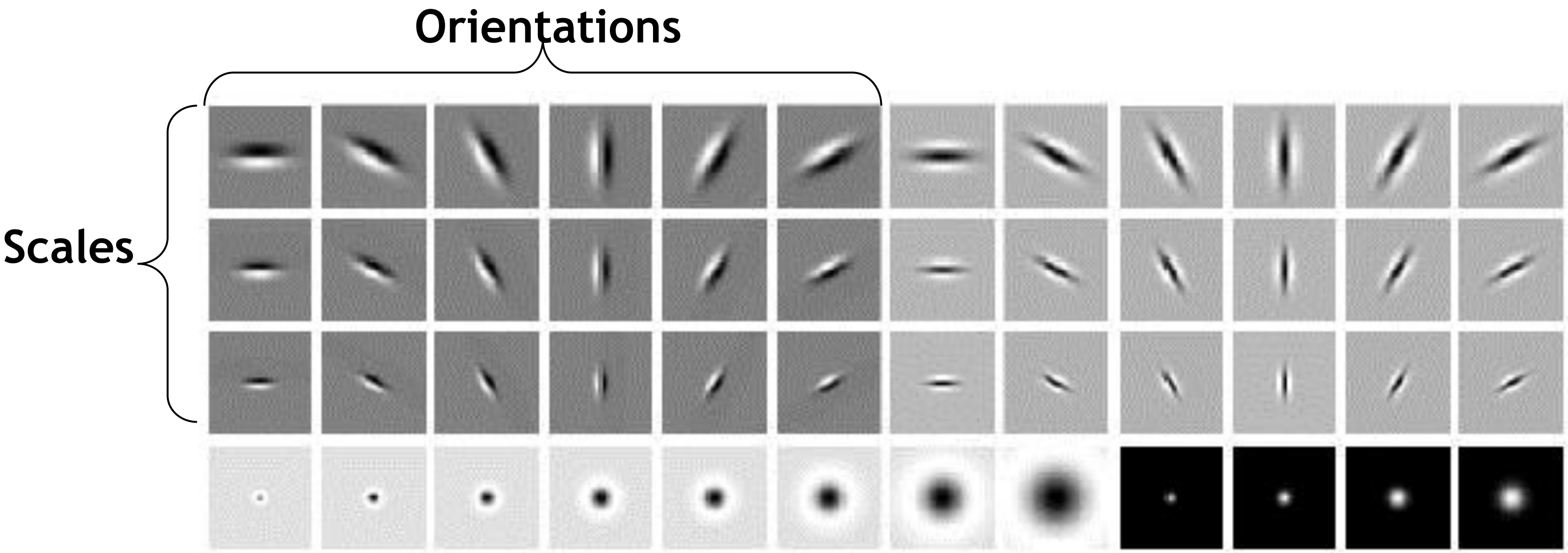




Special Case: Multiscale Representations

- Combination of several scales
  - Descriptors are computed at different scales.
  - Each scale captures different information about the object.
  - Size of the support region grows with increasing σ.
  - Feature vectors capture both local details and larger-scale structures.



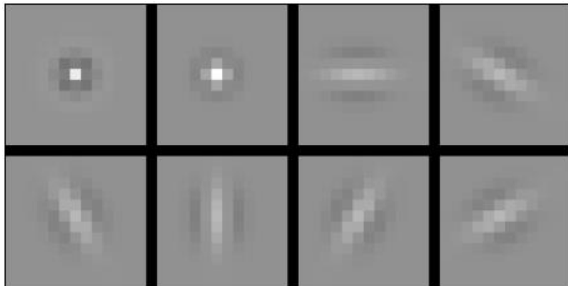
Generalization: Filter Banks



- What filters to put in the bank?
  - Typically we want a combination of scales and orientations, different types of patterns.

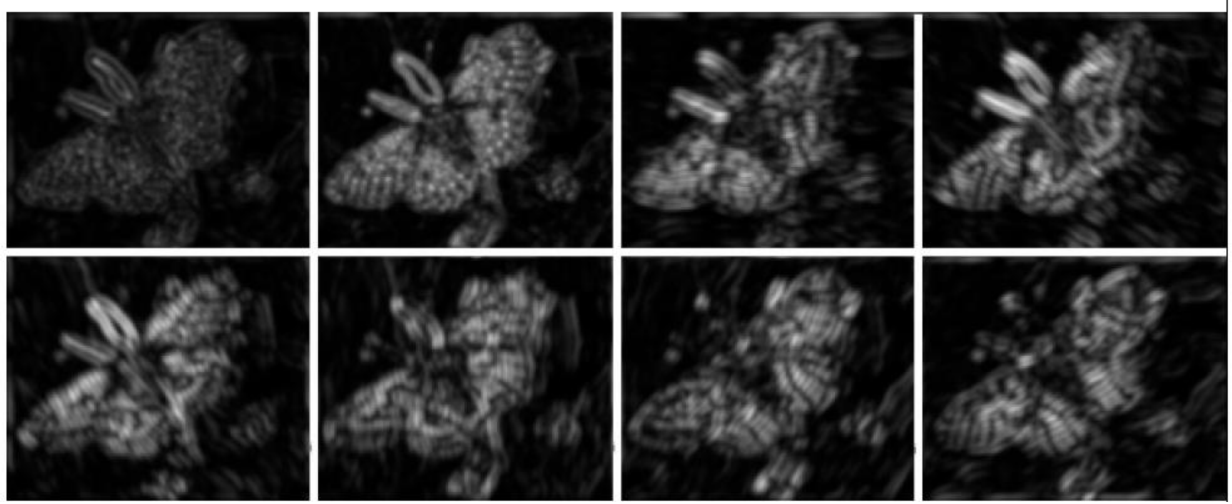
Matlab code available for these examples:
http://www.robots.ox.ac.uk/~vgg/research/texclass/filters.html

Example Application of a Filter Bank



Filter bank of 8 filters

Input image

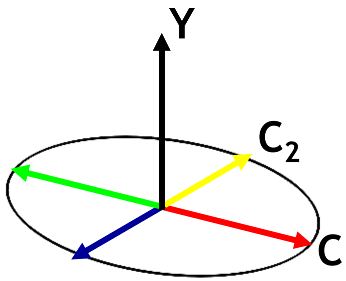


8 response images: magnitude of filtered outputs, per filter

Extension: Colored Derivatives

- YC_1C_2 color space

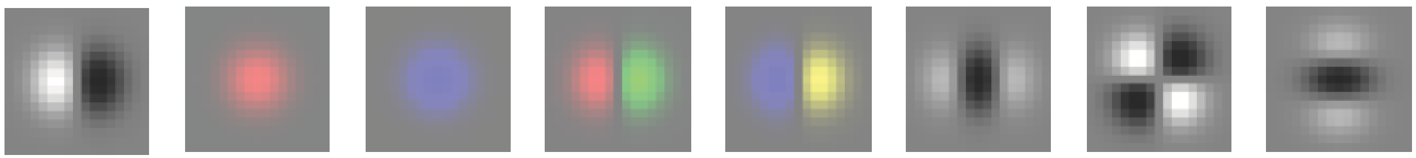
$$\begin{pmatrix} Y \\ C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} g_r & g_g & g_b \\ \frac{3g_g}{2} & -\frac{3g_r}{2} & 0 \\ \frac{g_b g_r}{g_r^2 + g_g^2} & \frac{g_b g_g}{g_r^2 + g_g^2} & -1 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$



- Color-opponent space
  - Inspired by models of the human visual system
  - Y ≡ intensity
  - C_1 ≡ red-green
  - C_2 ≡ blue-yellow

[Hall & Crowley, 2000]

Extension: Colored Derivatives



- Generalization: derivatives along
  - Y axis $\rightarrow$ intensity differences
  - C_1 axis $\rightarrow$ red-green differences
  - C_2 axis $\rightarrow$ blue-yellow differences

- Feature vector is rotated such that $D_y = 0$
  - Rotation-invariant descriptor

[Hall & Crowley, 2000]

Summary: Multidimensional Representations

- **Pros**
  - Work very well for recognition.
  - Usually, simple combinations are sufficient (e.g. D_x-D_y, Mag-Lap)
  - But multiple scales are very important!
  - Generalization: filter banks
- **Cons**
  - High-dimensional histograms $\Rightarrow$ lots of storage space
  - Global representation $\Rightarrow$ not robust to occlusion

You're Now Ready for First Applications…

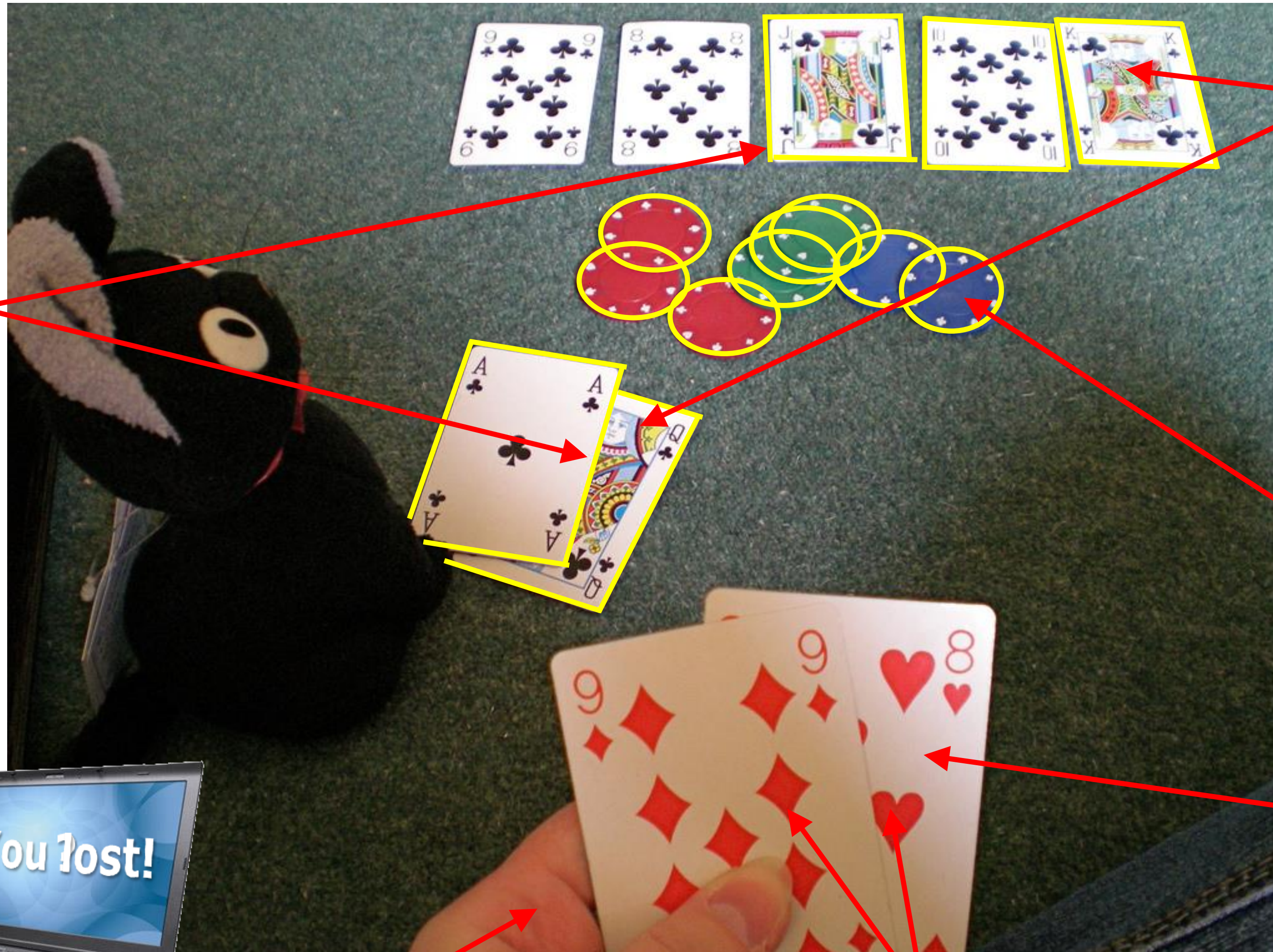


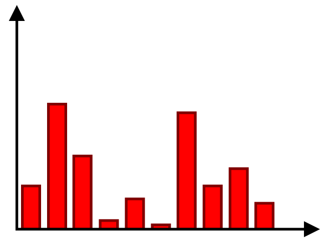
Image Source: http://www.flickr.com/photos/angelsk/2806412807/

Topics of This Lecture

- Subspace Methods for Recognition
  - Motivation
- Principal Component Analysis (PCA)
  - Derivation
  - Object recognition with PCA
  - Eigenimages/Eigenfaces
  - Limitations
- Discussion: Global representations for recognition
  - Vectors of pixel intensities
  - Histograms
  - Localized Histograms
- Application: Image completion

Representations for Recognition

- Global object representations
  - We've seen histograms as one example
  - What could be other suitable representations?



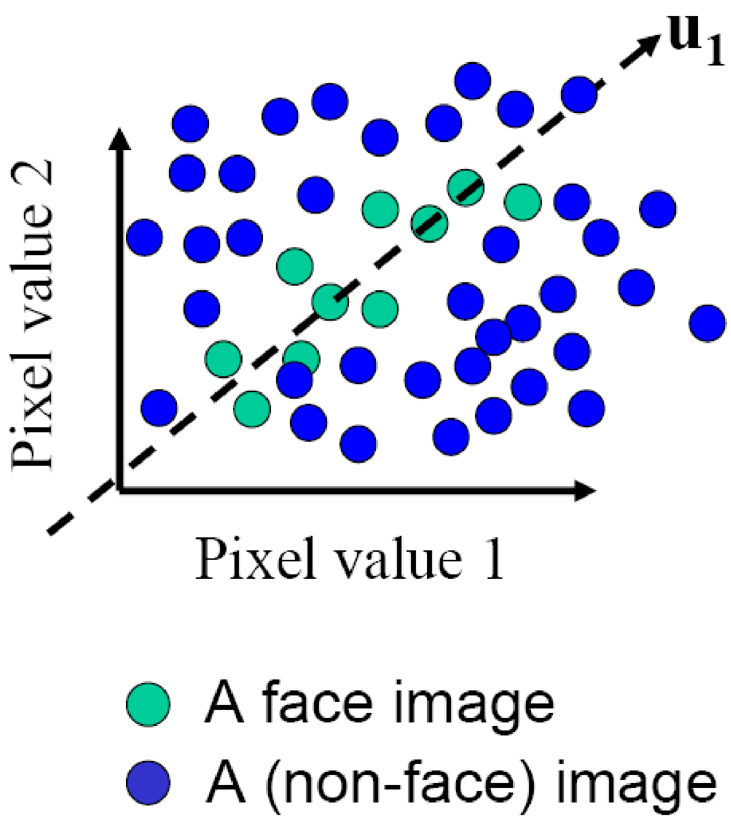
- More generally, we want to obtain representations that are well-suited for
  - Recognizing a certain class of objects
  - Identifying individuals from that class (identification)
- How can we arrive at such a representation?
- Approach 1:
  - Come up with a brilliant idea and tweak it until it works.
- *Can we do this more systematically?*

Example: The Space of All Face Images

- When viewed as vectors of pixel values, face images are extremely high-dimensional.
  - 100x100 image = 10,000 dimensions
- However, relatively few 10,000-dimensional vectors correspond to valid face images.
- We want to effectively model the subspace of face images.

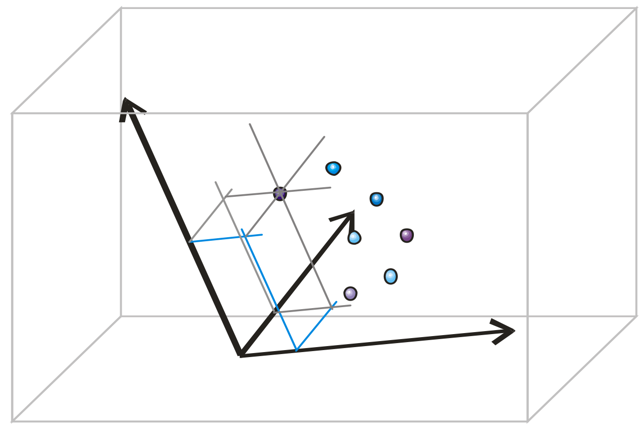
The Space of All Face Images

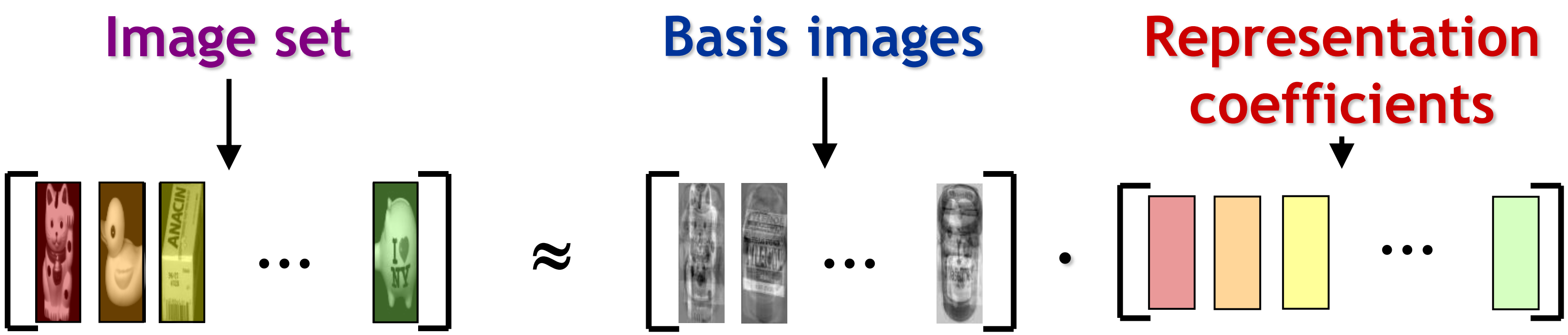
- **We want to construct a low-dimensional linear subspace that best explains the variation in the set of face images**



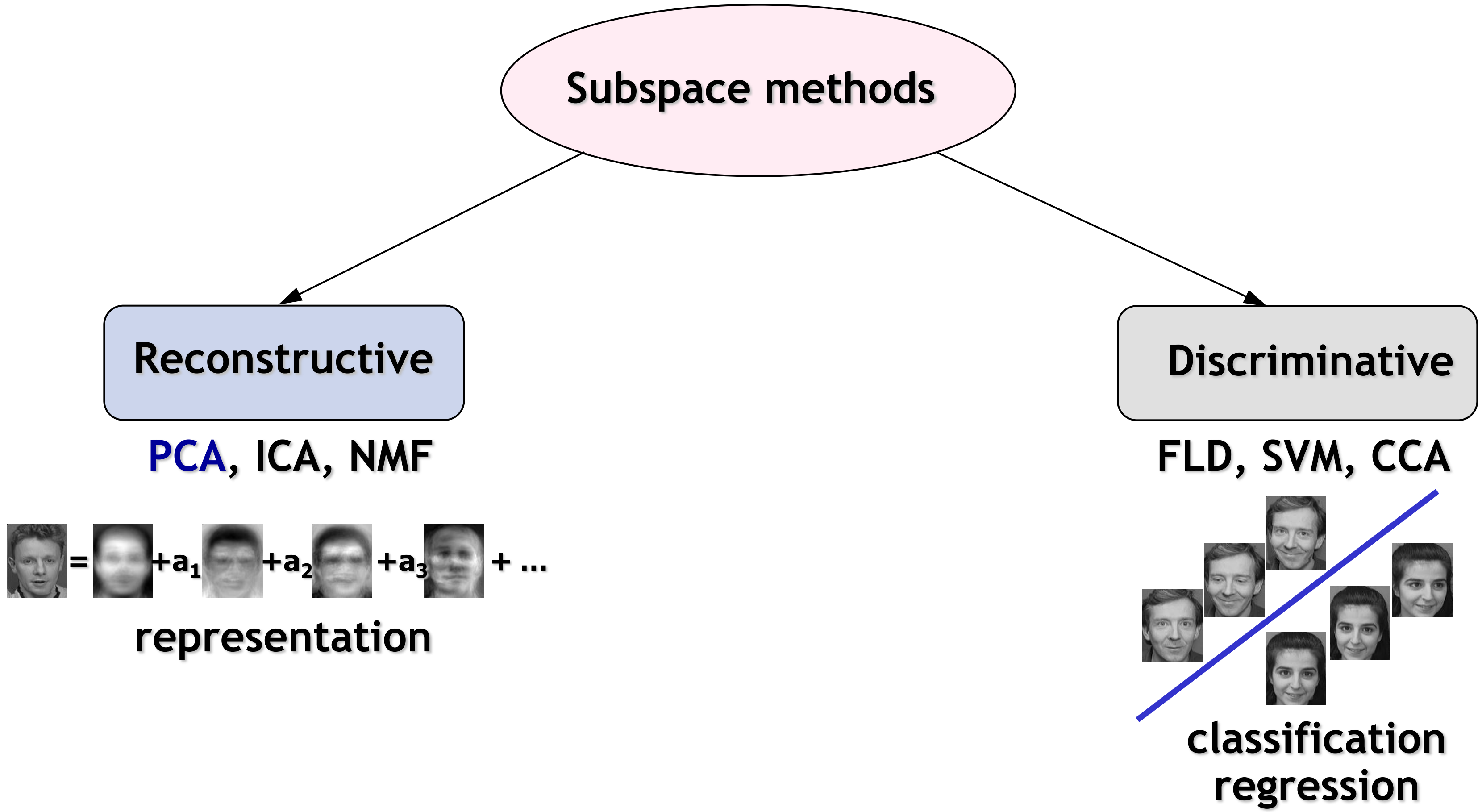
Subspace Methods

- Idea
  - Represent images as points in a high-dim. vector space
  - Valid images populate only a small fraction of the space
  - Characterize the subspace spanned by images





Subspace Methods



- **Today's topic: PCA**

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Principal Component Analysis

- Given: N data points x_1, … ,x_N in R^d
- We want to find a new set of features that are linear combinations of original ones:

$$u(\mathbf{x}_i) = \mathbf{u}^\top(\mathbf{x}_i - \boldsymbol{\mu})$$

(μ: mean of data points)

- What unit vector $\mathbf{u}$ in R^d captures the most variance of the data?

Principal Component Analysis

- Direction that maximizes the variance of the projected data:

$$var(u) = \frac{1}{N}\sum_{i=1}^{N} \underbrace{\mathbf{u}^{\mathrm{T}}(\mathbf{x}_i - \mu)}_{\text{Projection of data point}}(\mathbf{u}^{\mathrm{T}}(\mathbf{x}_i - \mu))^{\mathrm{T}}$$

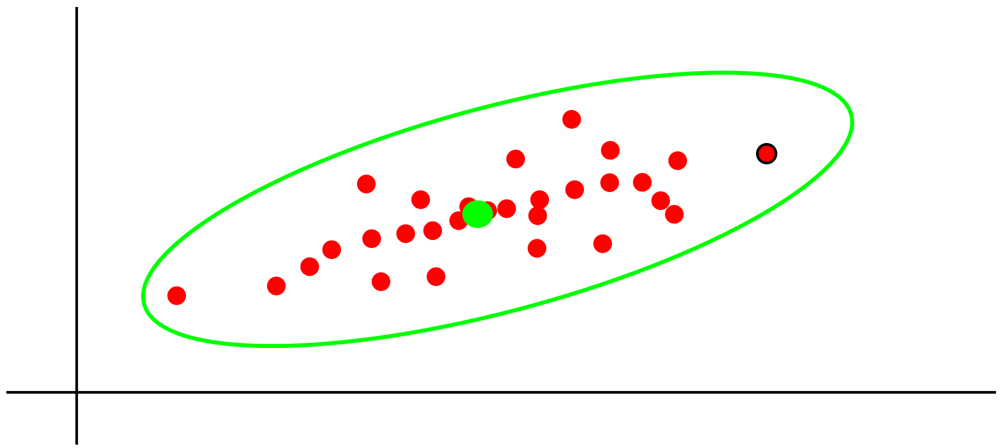
$$= \frac{1}{N}\mathbf{u}^{\mathrm{T}} \underbrace{\left[\sum_{i=1}^{N}(\mathbf{x}_i - \mu)(\mathbf{x}_i - \mu)^{\mathrm{T}}\right]}_{\text{Covariance matrix of data}} \mathbf{u}$$

$$= \frac{1}{N}\mathbf{u}^{\mathrm{T}}\Sigma\mathbf{u}$$

  - The direction that maximizes the variance is the eigenvector associated with the largest eigenvalue of Σ.

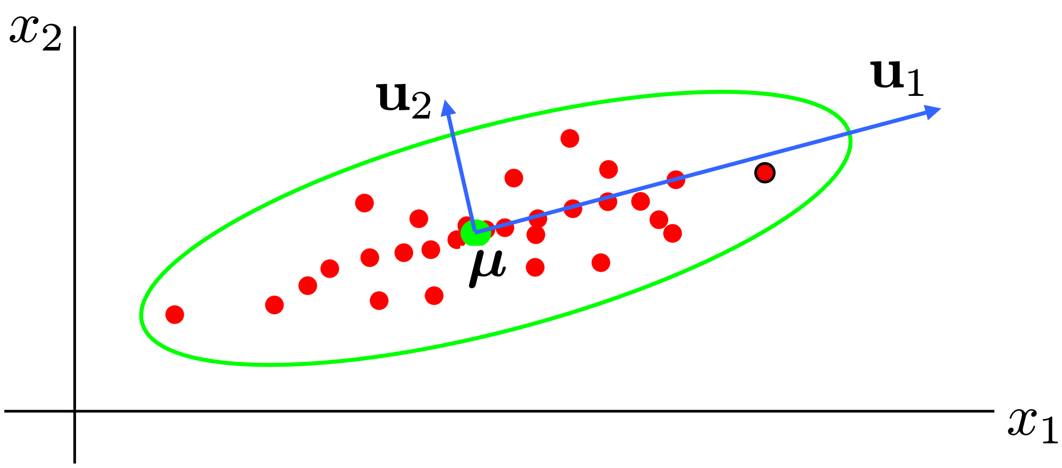
Remember: Fitting a Gaussian

- Mean and covariance matrix of data define a Gaussian model



Interpretation of PCA

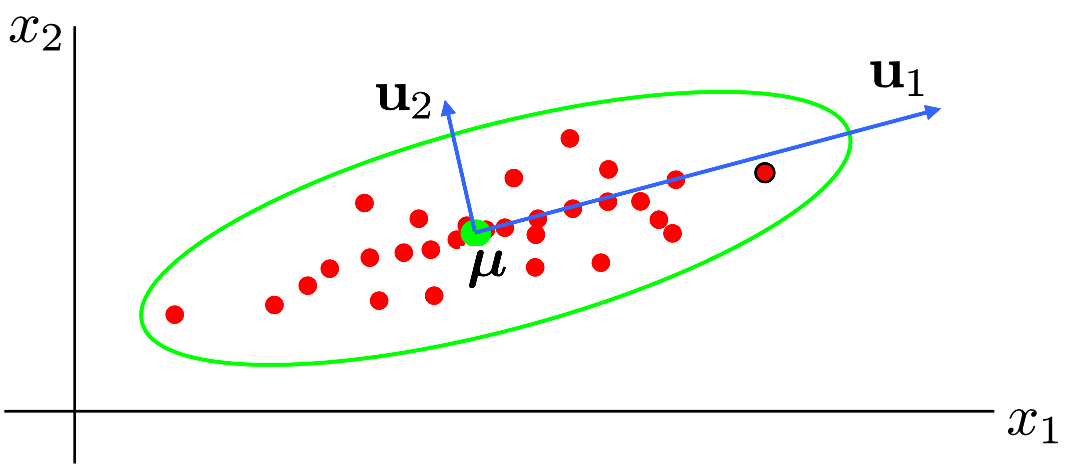
- **Compute eigenvectors of covariance Σ.**
  - **Eigenvectors: main directions**
  - **Eigenvalues: variances along eigenvector**



- **Result: coordinate transform to best represent the variance of the data**

Interpretation of PCA

- Now, suppose we want to represent the data using just a single dimension.
  - I.e., project it onto a single axis
  - What would be the best choice for this axis?



Interpretation of PCA

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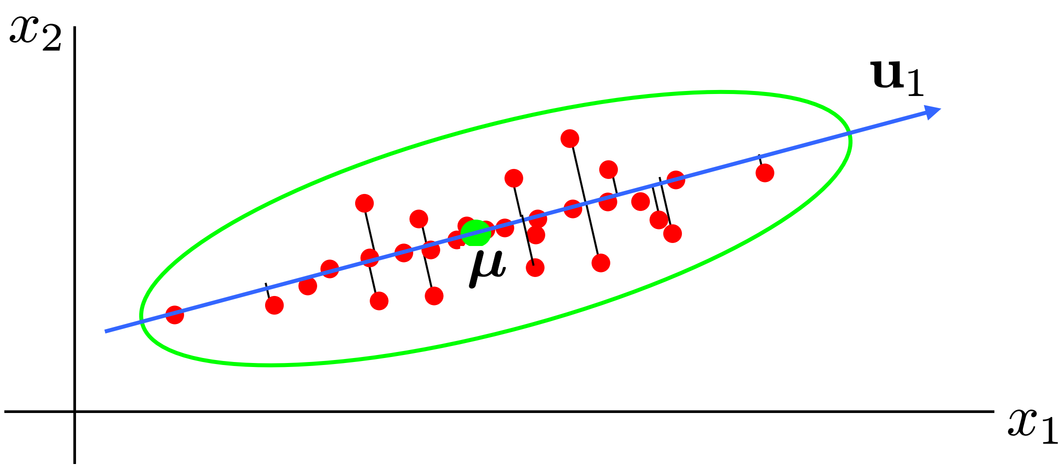


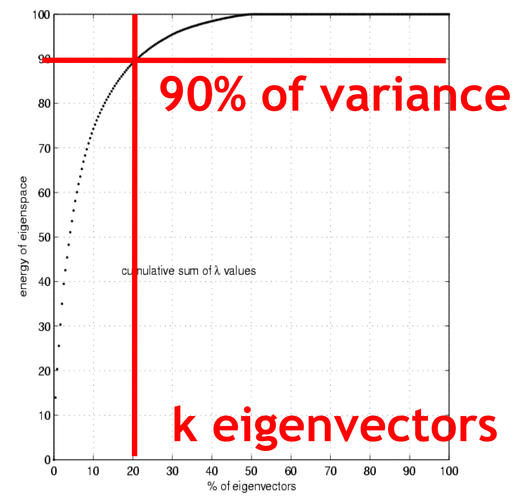
- The first eigenvector gives us the best reconstruction.
  - Direction that retains most of the variance of the data.

Properties of PCA

- It can be shown that the mean square error between x_i and its reconstruction using only m principle eigenvectors is given by the expression:

$$\sum_{j=1}^{N} \lambda_j - \sum_{j=1}^{m} \lambda_j = \sum_{j=m+1}^{N} \lambda_j$$

  - where λ_j are the eigenvalues



Cumulative influence of eigenvectors

- **Interpretation**
  - PCA minimizes reconstruction error
  - PCA maximizes variance of projection
  - Finds a more "natural" coordinate system for the sample data.

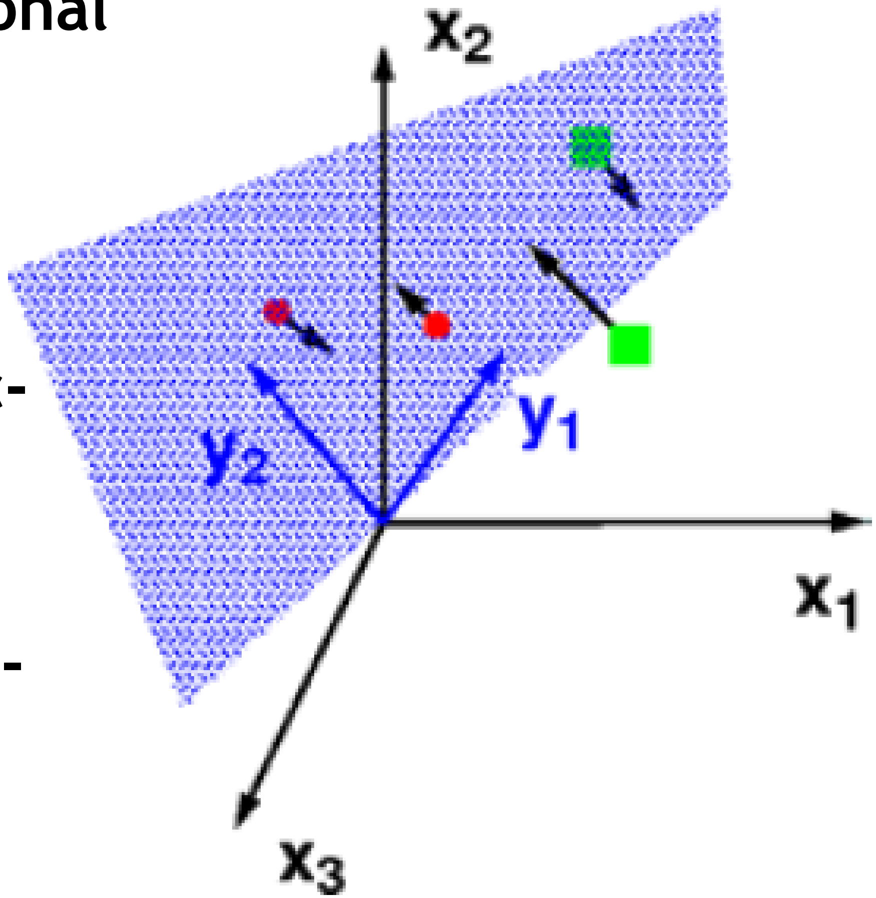
Projection and Reconstruction

- An n-pixel image $x \in \mathrm{R}^n$ can be projected to a low-dimensional feature space $y \in \mathrm{R}^m$ by

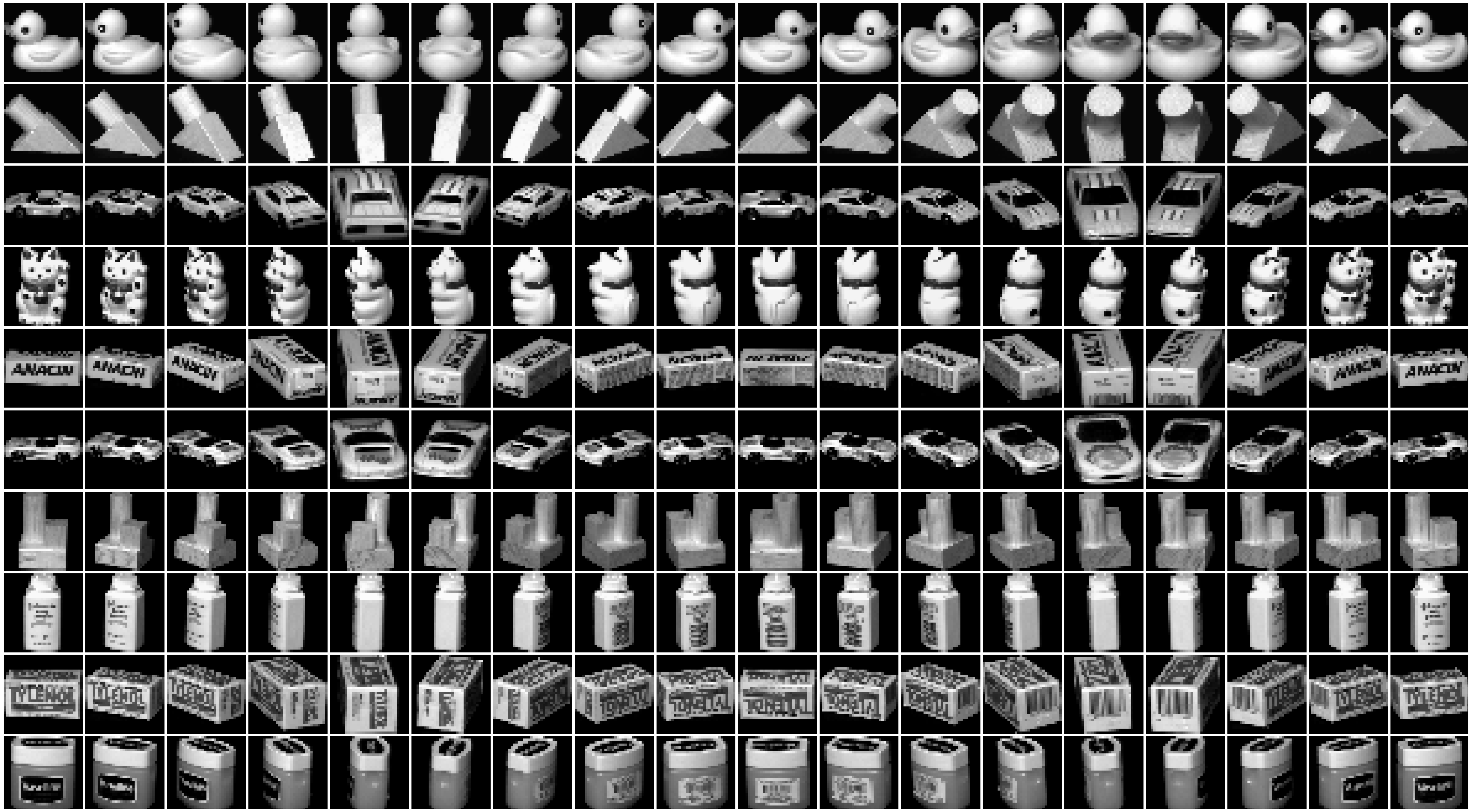
$$y = Ux$$

- From $y \in \mathrm{R}^m$, the reconstruction of the point is $U^T y$

- The error of the reconstruction is $\left\| x - U^T U x \right\|$

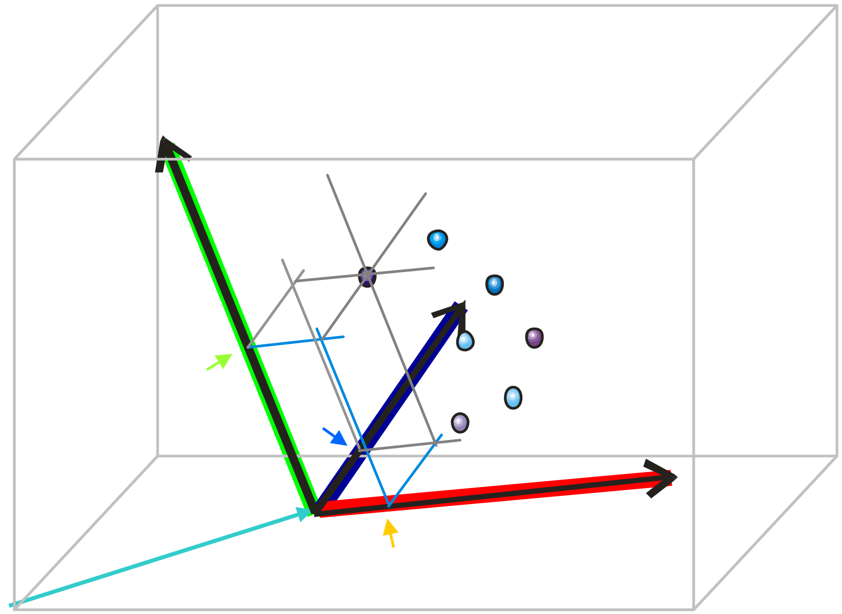


Example: Object Representation

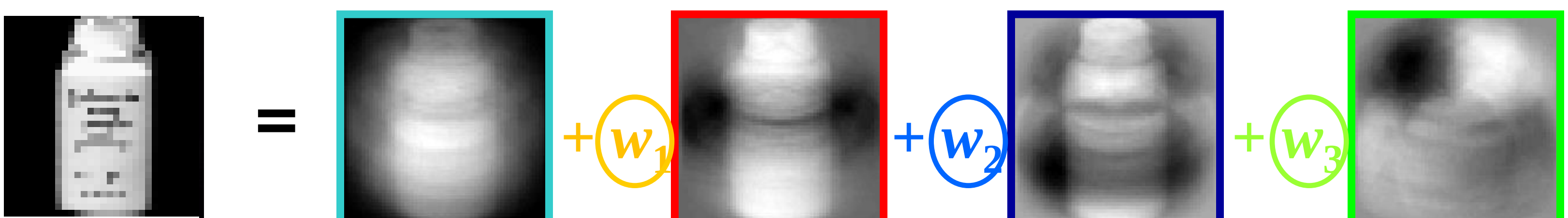


Slide credit: Ales Leonardis

Principal Component Analysis

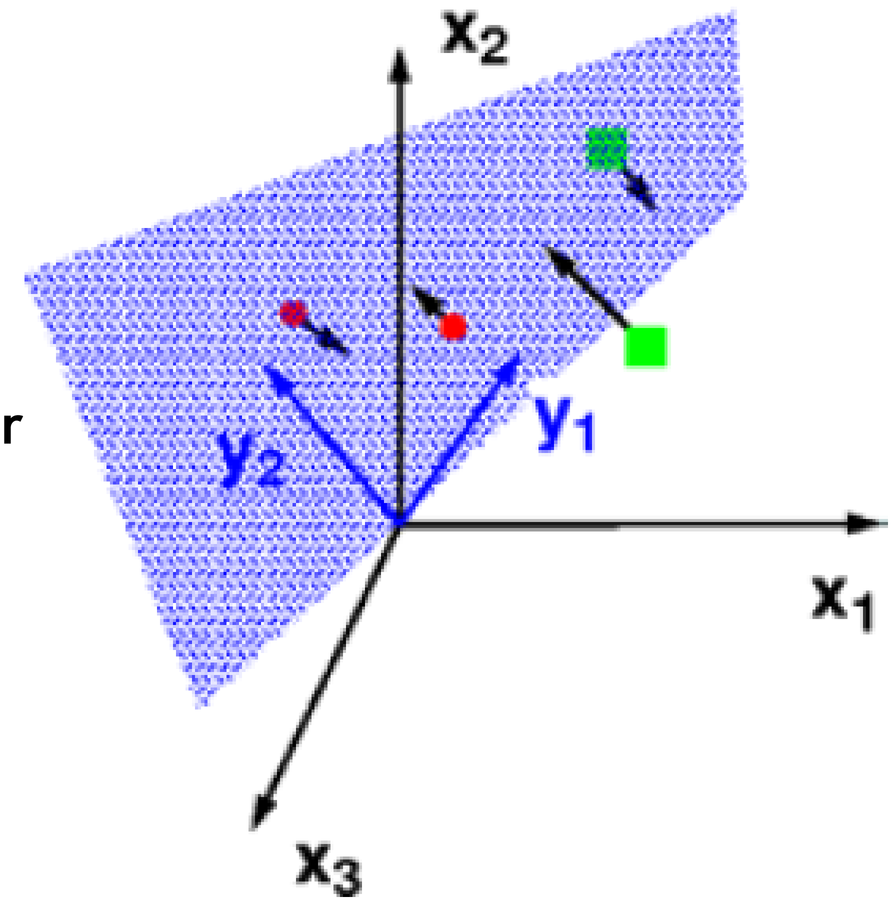


Get a compact representation by keeping only the first k eigenvectors!



Object Detection by Distance TO Eigenspace

- Is an image window ω likely to contain a learned object?
  - Project window to subspace and reconstruct as earlier.
  - Compute the distance between ω and the reconstruction (reprojection error).
  - Local minima of distance over all image locations ⇒ object locations



Eigenfaces: Key Idea

- Assume that most face images lie on a low-dimensional subspace determined by the first k directions of maximum variance (where $k < d$).
- Use PCA to determine the vectors $u_1, \dots u_k$ that span that subspace:
  $x \approx \mu + w_1 u_1 + w_2 u_2 + \dots + w_k u_k$
- Represent each face using its "face space" coordinates $(w_1, \dots w_k)$
- Perform nearest-neighbor recognition in "face space"

M. Turk and A. Pentland, Face Recognition using Eigenfaces, CVPR 1991

Eigenfaces Example

- **Training images** $x_1, \ldots, x_N$

Slide credit: Svetlana Lazebnik

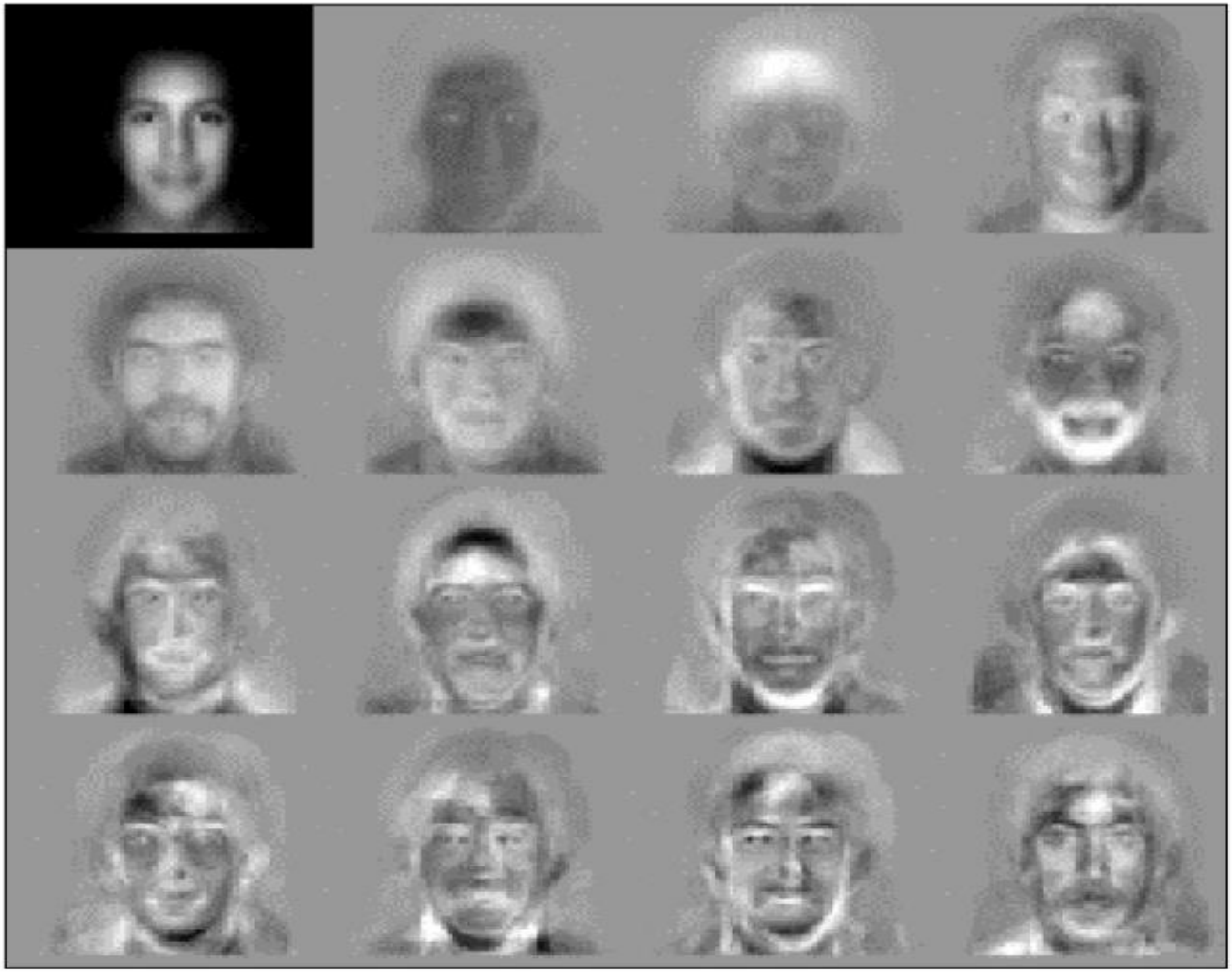
Eigenfaces Example

Top eigenvectors: $u_1, \ldots u_k$

Mean: μ

Slide credit: Svetlana Lazebnik

Eigenface Example 2 (Better Alignment)



Slide credit: Peter Belhumeur

Eigenfaces Example

- **Face x in “face space” coordinates:**

$$\mathbf{x} \rightarrow [\mathbf{u}_1^{\mathrm{T}}(\mathbf{x} - \mu), \ldots, \mathbf{u}_k^{\mathrm{T}}(\mathbf{x} - \mu)]$$
$$= \quad w_1, \ldots, w_k$$

Eigenfaces Example

- Face x in "face space" coordinates:

$$\mathbf{x} \rightarrow [\mathbf{u}_1^{\mathrm{T}}(\mathbf{x}-\mu), \ldots, \mathbf{u}_k^{\mathrm{T}}(\mathbf{x}-\mu)]$$
$$= \quad w_1, \ldots, w_k$$

- Reconstruction:

 =  + 

$$\mathbf{x} = \mu + w_1\mathbf{u}_1 + w_2\mathbf{u}_2 + w_3\mathbf{u}_3 + w_4\mathbf{u}_4 + \ldots$$

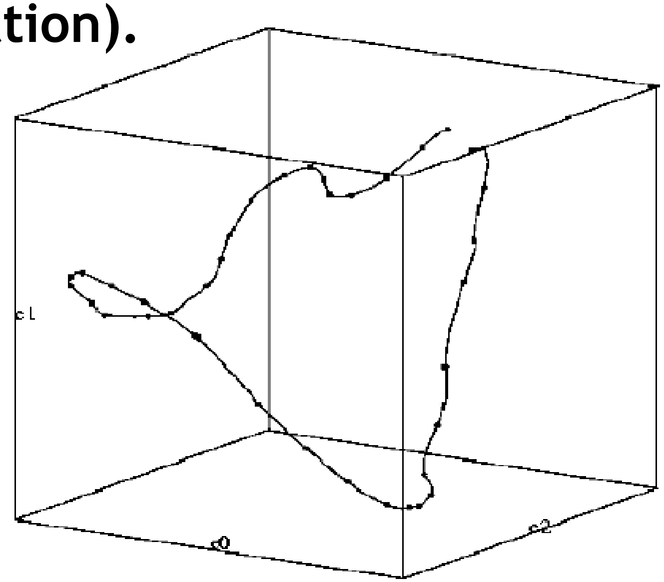
Recognition with Eigenspaces

- Process labeled training images:
  - Find mean μ and covariance matrix Σ
  - Find k principal components (eigenvectors of Σ) $u_1,\ldots u_k$
  - Project each training image x_i onto subspace spanned by principal components:
    $(w_{i1},\ldots,w_{ik}) = (u_1^T(x_i - \mu), \ldots, u_k^T(x_i - \mu))$

- Given novel image x:
  - Project onto subspace:
    $(w_1,\ldots,w_k) = (u_1^T(x - \mu), \ldots, u_k^T(x - \mu))$
  - Optional: check reconstruction error $x - \hat{x}$ to determine whether image is really a face
  - Classify as closest training face in k-dimensional subspace

Obj. Identification by Distance IN Eigenspace

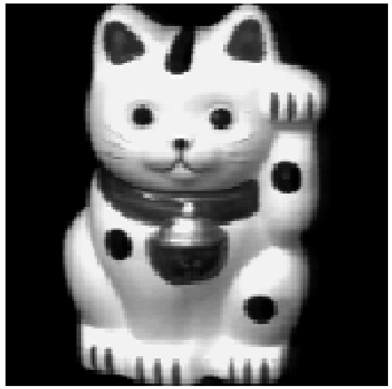
- Objects are represented as coordinates in an n-dim. eigenspace.
- Example:
  - 3D space with points representing individual objects or a manifold representing parametric eigenspace (*e.g.*, orientation, pose, illumination).

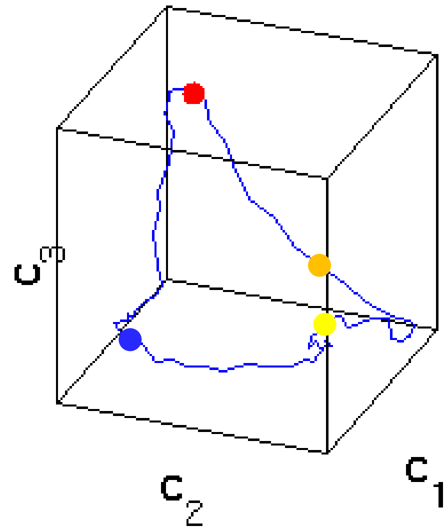


- Estimate parameters by finding the NN in the eigenspace

Slide credit: Ales Leonardis

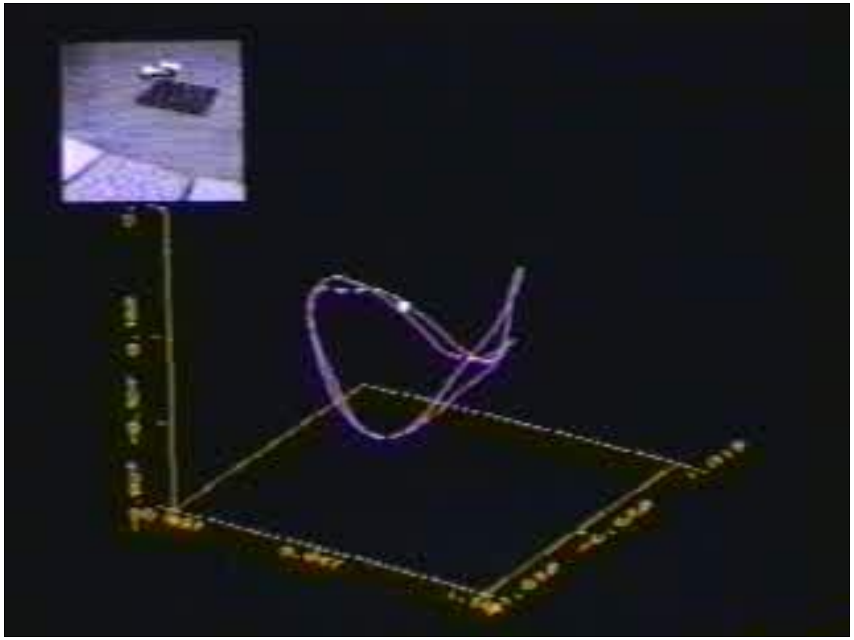
Parametric Eigenspace





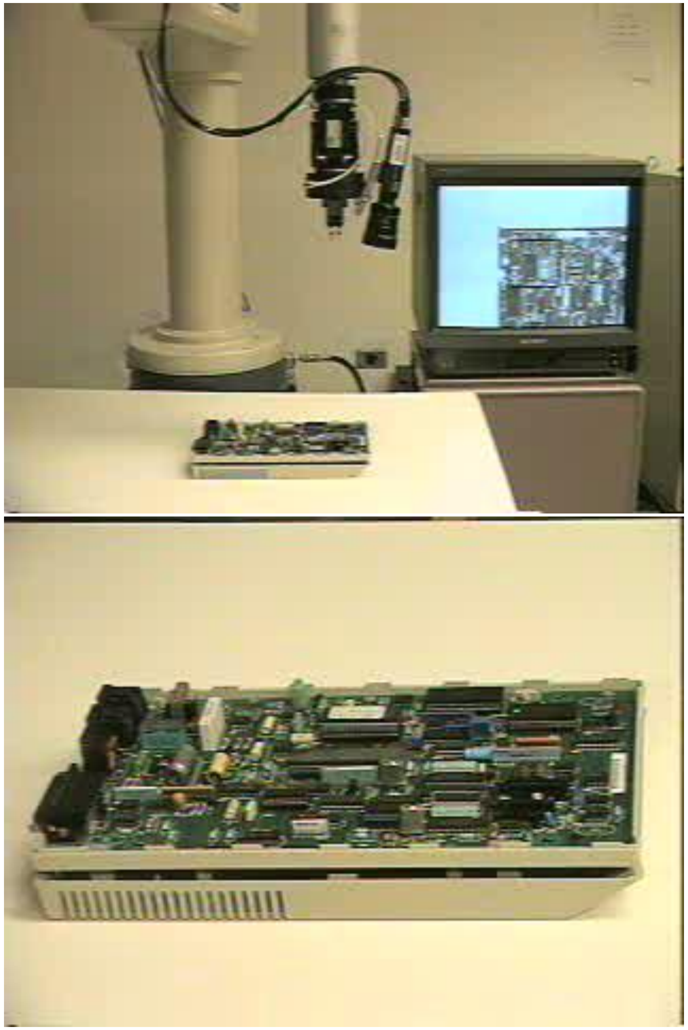
- **Object identification / pose estimation**
  - **Find nearest neighbor in eigenspace [Murase & Nayar, IJCV'95]**

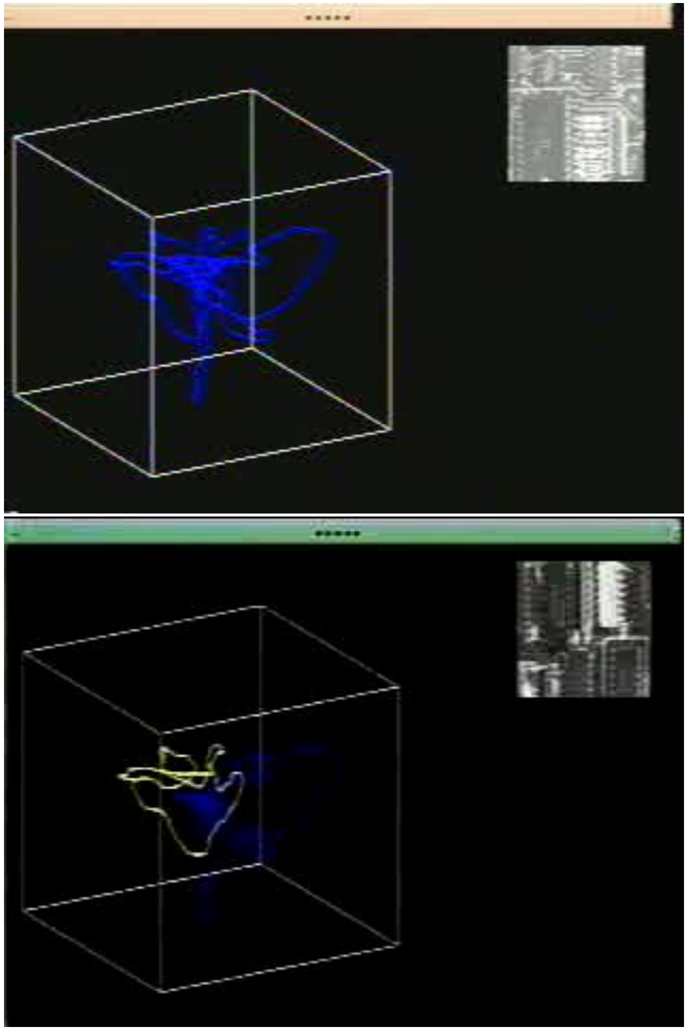
Applications: Recognition, Pose Estimation



H. Murase and S. Nayar, Visual learning and recognition of 3-d objects from appearance, IJCV 1995

Applications: Visual Inspection





S. K. Nayar, S. A. Nene, and H. Murase, Subspace Methods for Robot Vision, IEEE Transactions on Robotics and Automation, 1996.

Important Footnote

- **Don't really implement PCA this way!**
  - Why?

1. **How big is Σ?**
   - $n \times n$, where n is the number of pixels in an image!
   - However, we only have m training examples, typically $m << n$.

   $\Rightarrow$ Σ will at most have rank m!

2. **You only need the first k eigenvectors**

Slide credit: Peter Belhumeur

Singular Value Decomposition (SVD)

- Any m×n matrix A may be factored such that

$$A = U\Sigma V^T$$

$$[m \times n] = [m \times m][m \times n][n \times n]$$

- U: $m \times m$, orthogonal matrix
  - Columns of U are the eigenvectors of AA^T
- V: $n \times n$, orthogonal matrix
  - Columns are the eigenvectors of A^TA
- Σ: $m \times n$, diagonal with non-negative entries (σ_1, σ_2,…, σ_s) with $s = \min(m,n)$ are called the singular values.
  - Singular values are the square roots of the eigenvalues of both AA^T and A^TA. *Columns of U are corresponding eigenvectors!*
  - Result of SVD algorithm: $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_s$

Slide credit: Peter Belhumeur

SVD Properties

- Matlab: `[u s v] = svd(A)`
  - where $A = u*s*v'$
- `r = rank(A)`
  - Number of non-zero singular values
- U, V give us orthonormal bases for the subspaces of A
  - first r columns of U: *column space* of A
  - last m-r columns of U: *left nullspace* of A
  - first r columns of V: *row space* of A
  - last n-r columns of V: *nullspace* of A
- For $d \leq r$, the first d columns of U provide the best d-dimensional basis for columns of A in least-squares sense

Performing PCA with SVD

- Singular values of A are the square roots of eigenvalues of both AA^T and A^TA.
  - Columns of U are the corresponding eigenvectors.

- And $\sum_{i=1}^{n} a_i a_i^T = [a_1 \quad \ldots \quad a_n][a_1 \quad \ldots \quad a_n]^T = AA^T$

- Covariance matrix

$$\Sigma = \frac{1}{n} \sum_{i=1}^{n} (\vec{x}_i - \vec{\mu})(\vec{x}_i - \vec{\mu})^T$$

- So, ignoring the factor $1/n$, subtract mean image μ from each input image, create data matrix $A = (\vec{x}_i - \vec{\mu})$, and perform (thin) SVD on the data matrix.

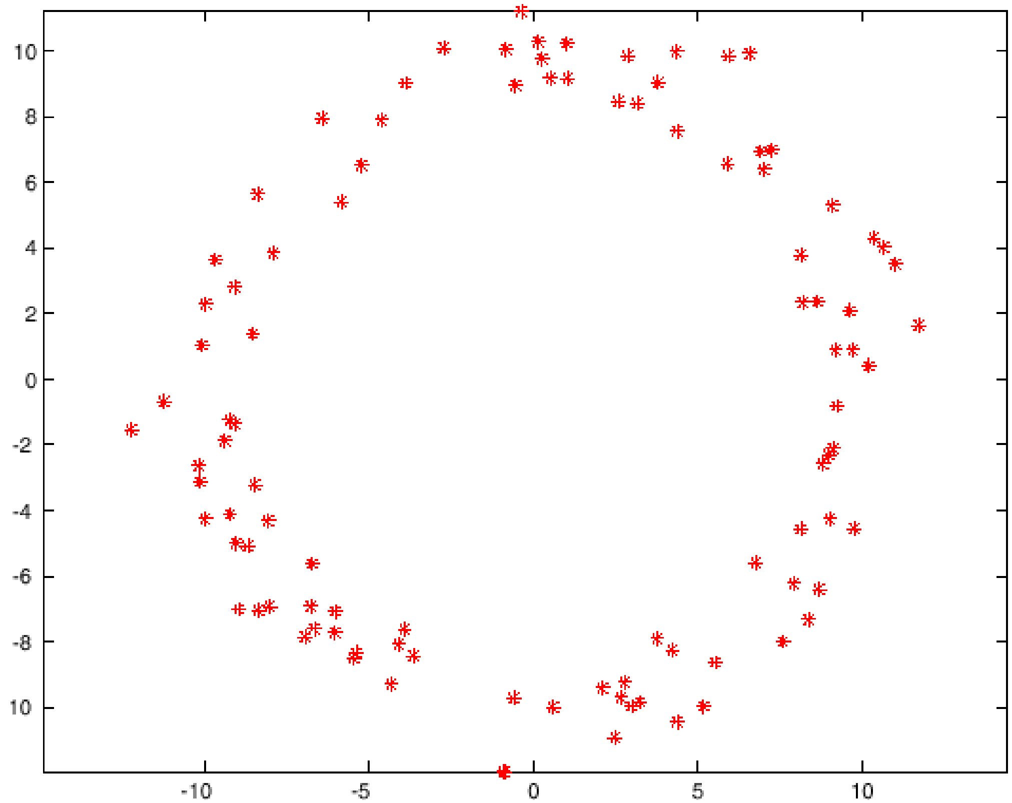
Limitations

- Global appearance method: not robust to misalignment, background variation

- Easy fix (with considerable manual overhead)
  - Need to align the training examples

Limitations

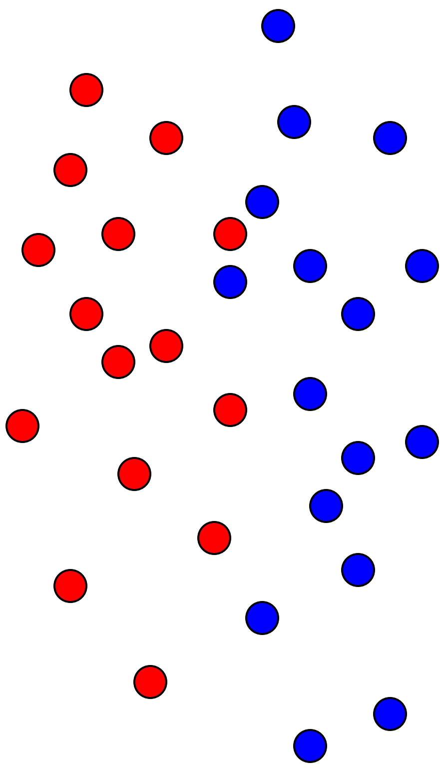
- **PCA assumes that the data has a Gaussian distribution (mean μ, covariance matrix Σ)**



  - **The shape of this dataset is not well described by its principal components**

Limitations

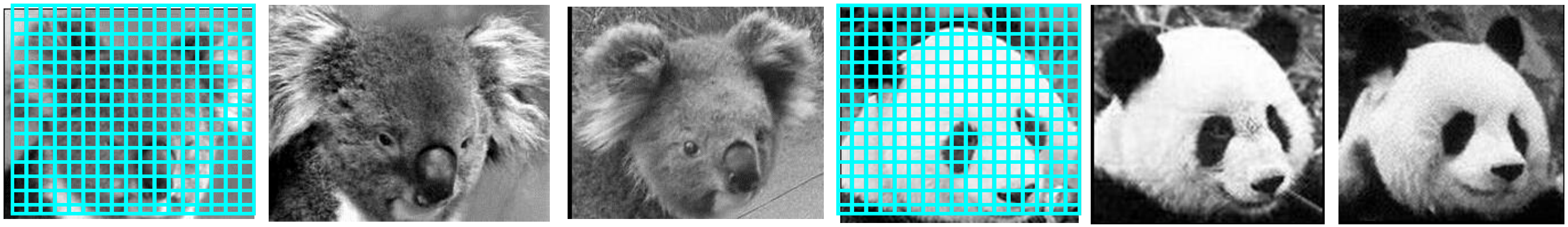
- **The direction of maximum variance is not always good for classification**



Topics of This Lecture

- Subspace Methods for Recognition
  - Motivation
- Principal Component Analysis (PCA)
  - Derivation
  - Object recognition with PCA
  - Eigenimages/Eigenfaces
  - Limitations
- **Discussion: Global representations for recognition**
  - **Vectors of pixel intensities**
  - **Histograms**
  - **Localized Histograms**
- Application: Image completion

Feature Extraction: Global Appearance



- **Simple holistic descriptions of image content**
  - **Vector of pixel intensities**

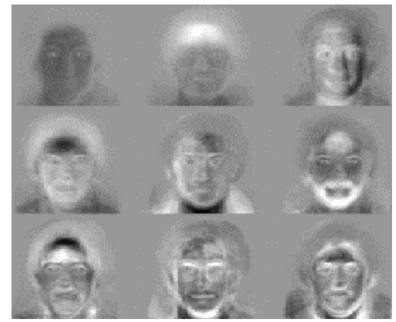
Slide adapted from Kristen Grauman

Eigenfaces: Global Appearance Description

- This can also be applied in a sliding-window framework…

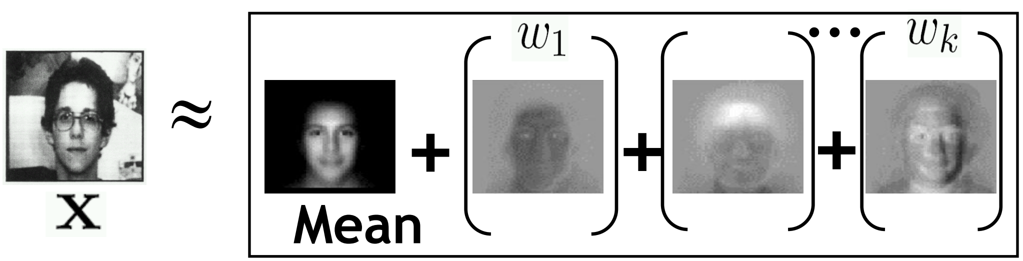
Training images

Mean



Eigenvectors computed from covariance matrix

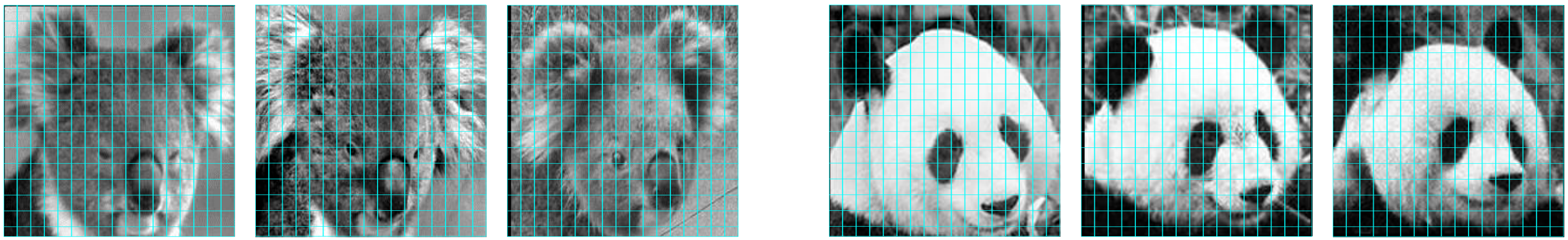
Generate low-dimensional representation of appearance with a linear subspace.



Project new images to "face space".

Recognition via nearest neighbors in face space

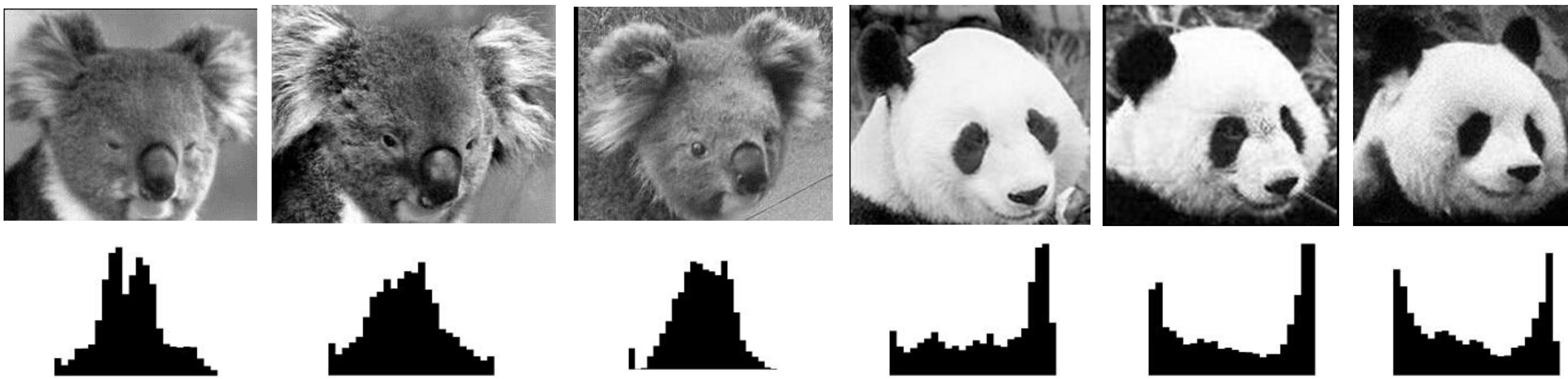
Feature Extraction: Global Appearance



- **Simple holistic descriptions of image content**
  - **Vector of pixel intensities**

  ⇒ Pixel based representations sensitive to small shifts!

Feature Extraction: Global Appearance



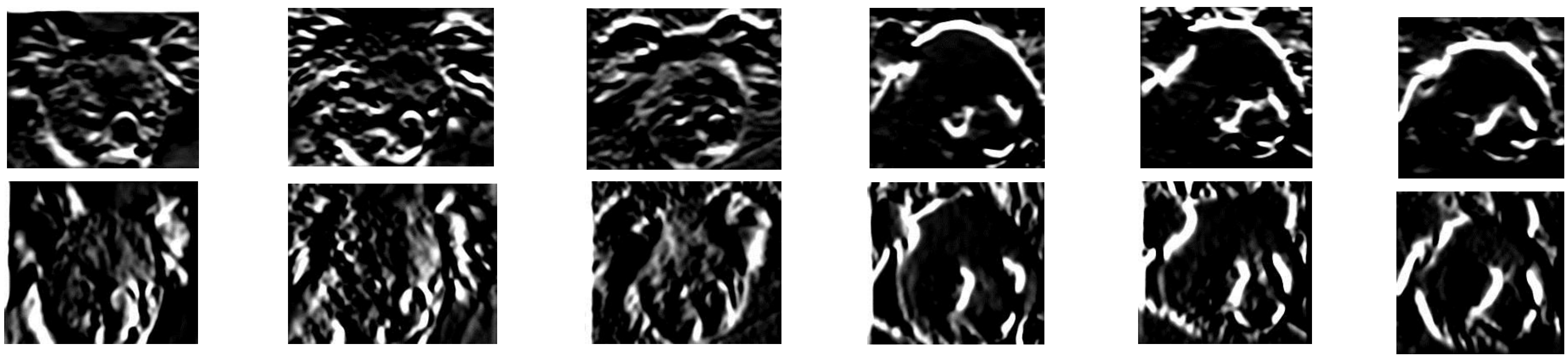
- Simple holistic descriptions of image content
  - Vector of pixel intensities
  - Grayscale / color histograms

  ⇒ Color or grayscale-based appearance description can be sensitive to illumination and intra-class appearance variation!

Cartoon example: an albino koala

Gradient-based Representations

- Better: Edges, contours, and (oriented) intensity gradients



Matching Edge Templates

- Example: Chamfer matching

Input image | Edges detected | Distance transform | Template shape | Best match

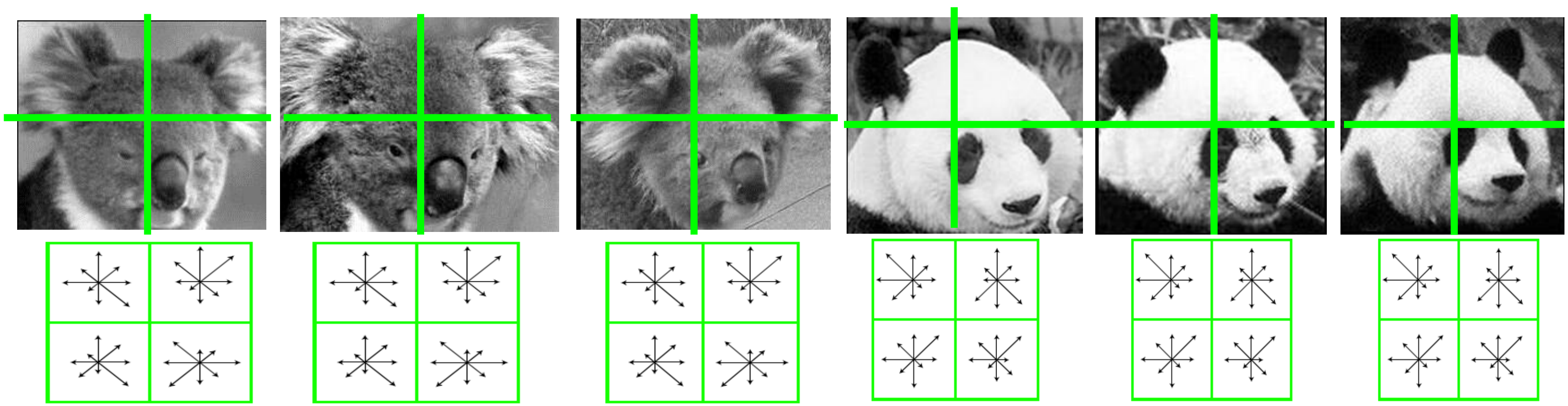
At each window position, compute average min distance between points on template (T) and input (I).

$$D_{chamfer}(T, I) \equiv \frac{1}{|T|} \sum_{t \in T} d_I(t)$$

[Gavrila & Philomin, ICCV 1999]

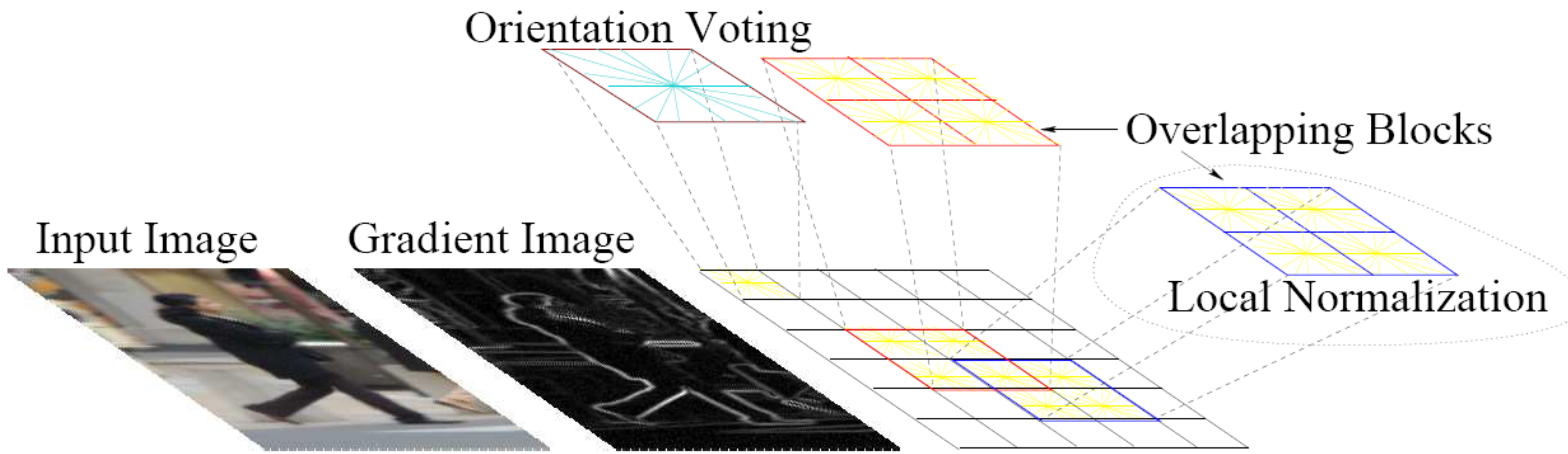
Gradient-based Representations

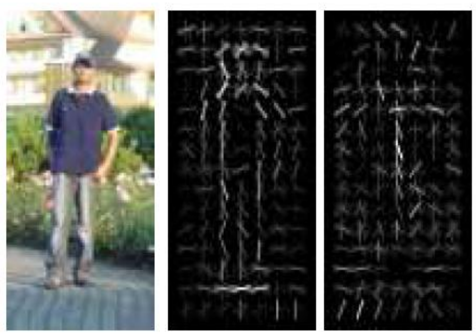
- Improved discriminance: localized gradients



- Summarize local distribution of gradients with histogram
  - Locally orderless: offers invariance to small shifts and rotations
  - Contrast-normalization: try to correct for variable illumination

Gradient-based Representations: Histograms of Oriented Gradients (HOG)





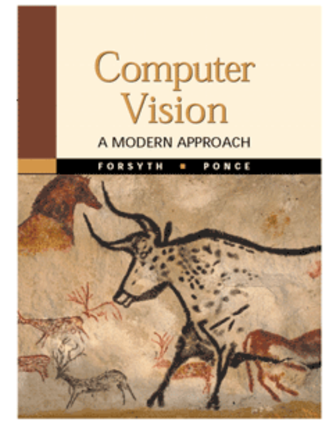
Map each grid cell in the input window to a histogram counting the gradients per orientation.

Code available:
http://pascal.inrialpes.fr/soft/olt/

[Dalal & Triggs, CVPR 2005]

References and Further Reading

- Background information on PCA can be found in Chapter 22.3 of
  - D. Forsyth, J. Ponce,
    Computer Vision – A Modern Approach.
    Prentice Hall, 2003



- Important Papers (available on webpage)
  - M. Turk, A. Pentland
    Eigenfaces for Recognition
    J. Cognitive Neuroscience, Vol. 3(1), 1991.
  - P.N. Belhumeur, J.P. Hespanha, D.J. Kriegman
    Eigenfaces vs. Fisherfaces: Recognition Using Class Specific Linear Projection, IEEE Trans. PAMI, Vol. 19(7), 1997.