# Computer Vision – Lecture 7

## Segmentation as Energy Minimization

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# Announcements

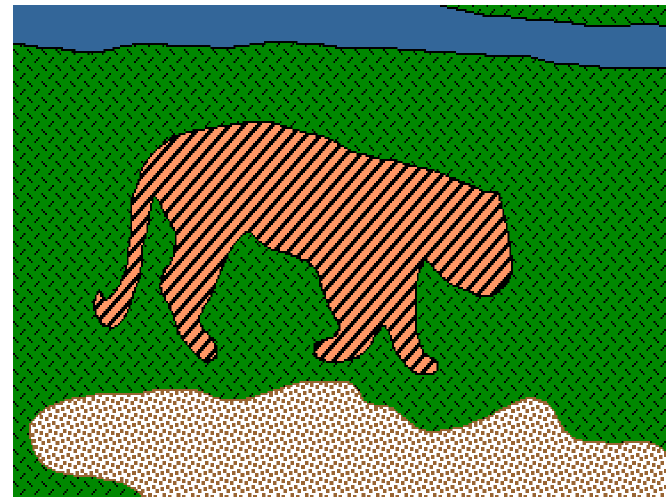
- **Please don’t forget to register for the exam!**
  - **On the Campus system**

# Course Outline

- Image Processing Basics
- Segmentation
  - Segmentation and Grouping
  - Segmentation as Energy Minimization
- Recognition
  - Global Representations
  - Subspace representations
- Local Features & Matching
- Object Categorization
- 3D Reconstruction
- Motion and Tracking

# Recap: Image Segmentation

- Goal: identify groups of pixels that go together



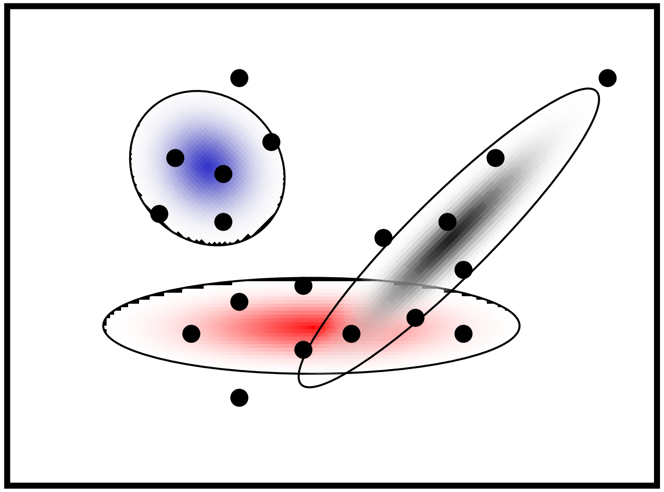
# Recap: K-Means Clustering

- **Basic idea: randomly initialize the *k* cluster centers, and iterate between the two steps we just saw.**
  1. Randomly initialize the cluster centers, $c_1, \ldots, c_K$
  2. Given cluster centers, determine points in each cluster
     - For each point p, find the closest $c_i$. Put p into cluster i
  3. Given points in each cluster, solve for $c_i$
     - Set $c_i$ to be the mean of points in cluster i
  4. If $c_i$ have changed, repeat Step 2

- **Properties**
  - Will always converge to *some* solution
  - Can be a "local minimum"
    - Does not always find the global minimum of objective function:

$$\sum_{\text{clusters } i} \; \sum_{\text{points p in cluster } i} ||p - c_i||^2$$

# Recap: Expectation Maximization (EM)



- **Goal**
  - Find blob parameters $\theta$ that maximize the likelihood function:

$$p(data|\theta) = \prod_{n=1}^{N} p(\mathbf{x}_n|\theta)$$

- **Approach:**
  1. E-step: given current guess of blobs, compute ownership of each point
  2. M-step: given ownership probabilities, update blobs to maximize likelihood function
  3. Repeat until convergence

see lecture Machine Learning!

# Recap: EM Algorithm

- **Expectation-Maximization (EM) Algorithm**
  - E-Step: softly assign samples to mixture components

$$\gamma_j(\mathbf{x}_n) \leftarrow \frac{\pi_j \mathcal{N}(\mathbf{x}_n|\boldsymbol{\mu}_j, \boldsymbol{\Sigma}_j)}{\sum_{k=1}^{K} \pi_k \mathcal{N}(\mathbf{x}_n|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)} \qquad \forall j = 1,\ldots,K, \quad n = 1,\ldots,N$$

  - M-Step: re-estimate the parameters (separately for each mixture component) based on the soft assignments

$$\hat{N}_j \leftarrow \sum_{n=1}^{N} \gamma_j(\mathbf{x}_n)$$ = soft number of samples labeled $j$

$$\hat{\pi}_j^{\text{new}} \leftarrow \frac{\hat{N}_j}{N}$$

$$\hat{\boldsymbol{\mu}}_j^{\text{new}} \leftarrow \frac{1}{\hat{N}_j} \sum_{n=1}^{N} \gamma_j(\mathbf{x}_n)\mathbf{x}_n$$

$$\hat{\boldsymbol{\Sigma}}_j^{\text{new}} \leftarrow \frac{1}{\hat{N}_j} \sum_{n=1}^{N} \gamma_j(\mathbf{x}_n)(\mathbf{x}_n - \hat{\boldsymbol{\mu}}_j^{\text{new}})(\mathbf{x}_n - \hat{\boldsymbol{\mu}}_j^{\text{new}})^{\mathrm{T}}$$

Slide adapted from Bernt Schiele

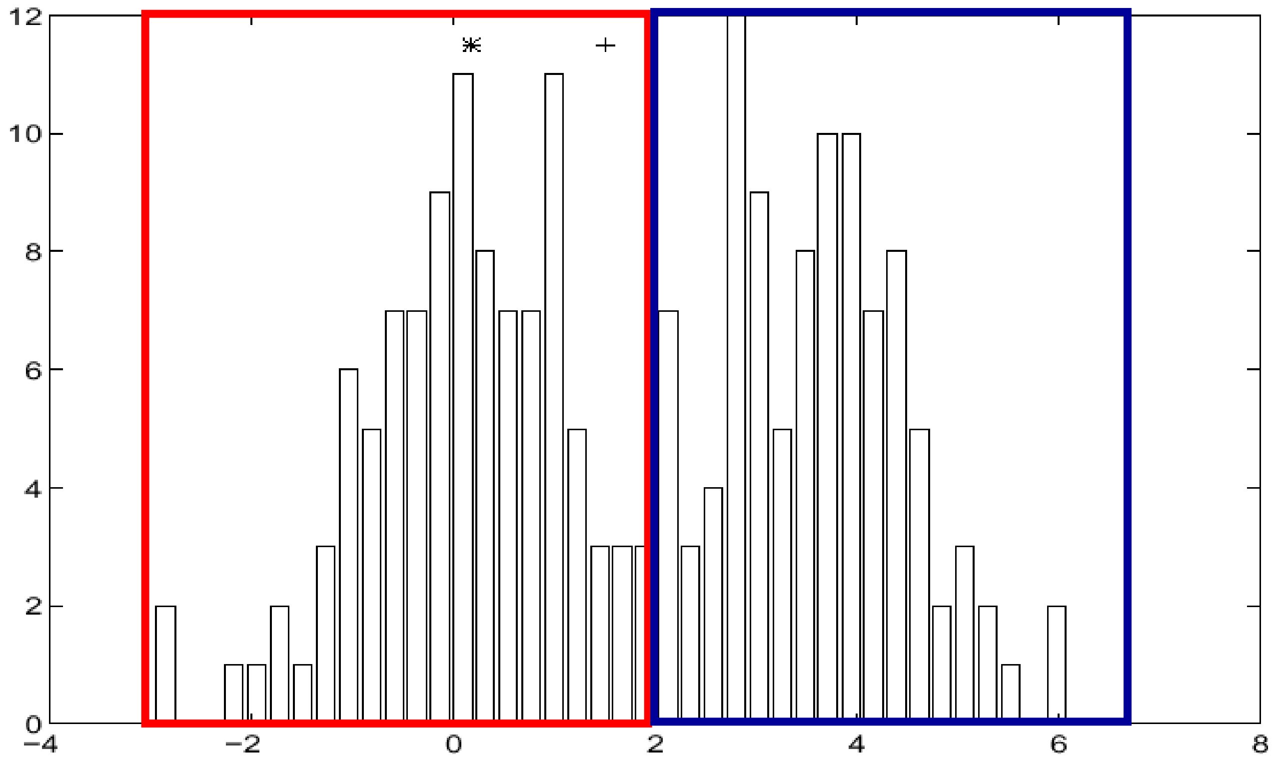
# MoG Color Models for Image Segmentation

(a) input image (b) user input (c) inferred segmentation

- **User assisted image segmentation**
  - User marks two regions for foreground and background.
  - Learn a MoG model for the color values in each region.
  - Use those models to classify all other pixels.

  $\Rightarrow$ Simple segmentation procedure
  (building block for more complex applications)

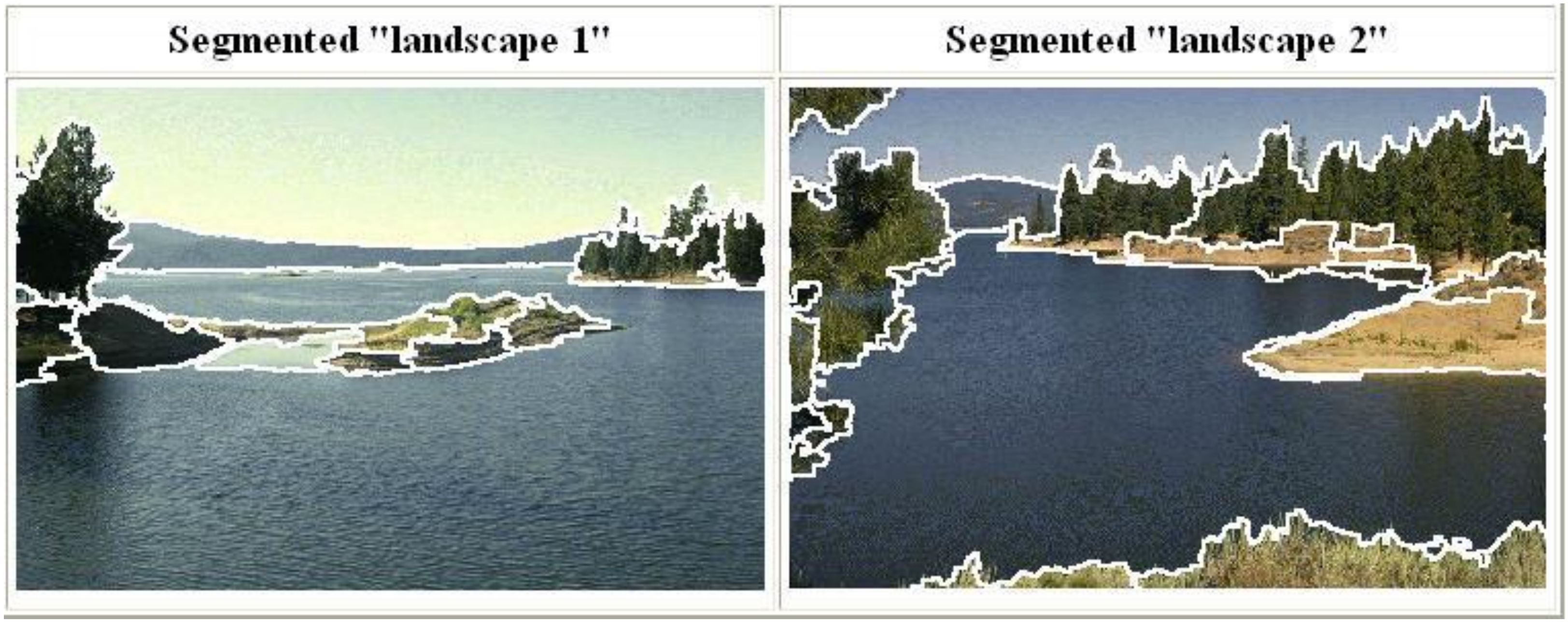
# Finding Modes in a Histogram




- How many modes are there?
  - *Mode* = local maximum of the density of a given distribution
  - Easy to see, hard to compute

# Mean-Shift Segmentation

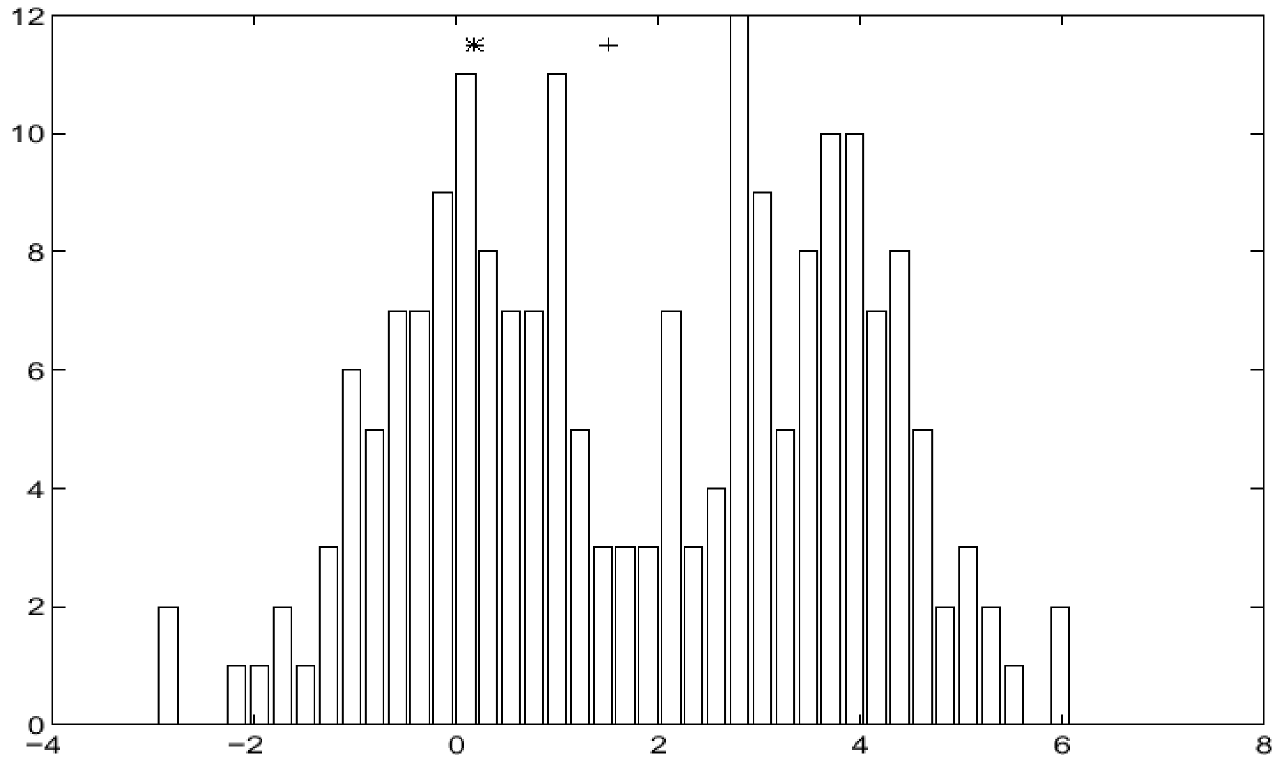
- An advanced and versatile technique for clustering-based segmentation




http://www.caip.rutgers.edu/~comanici/MSPAMI/msPamiResults.html

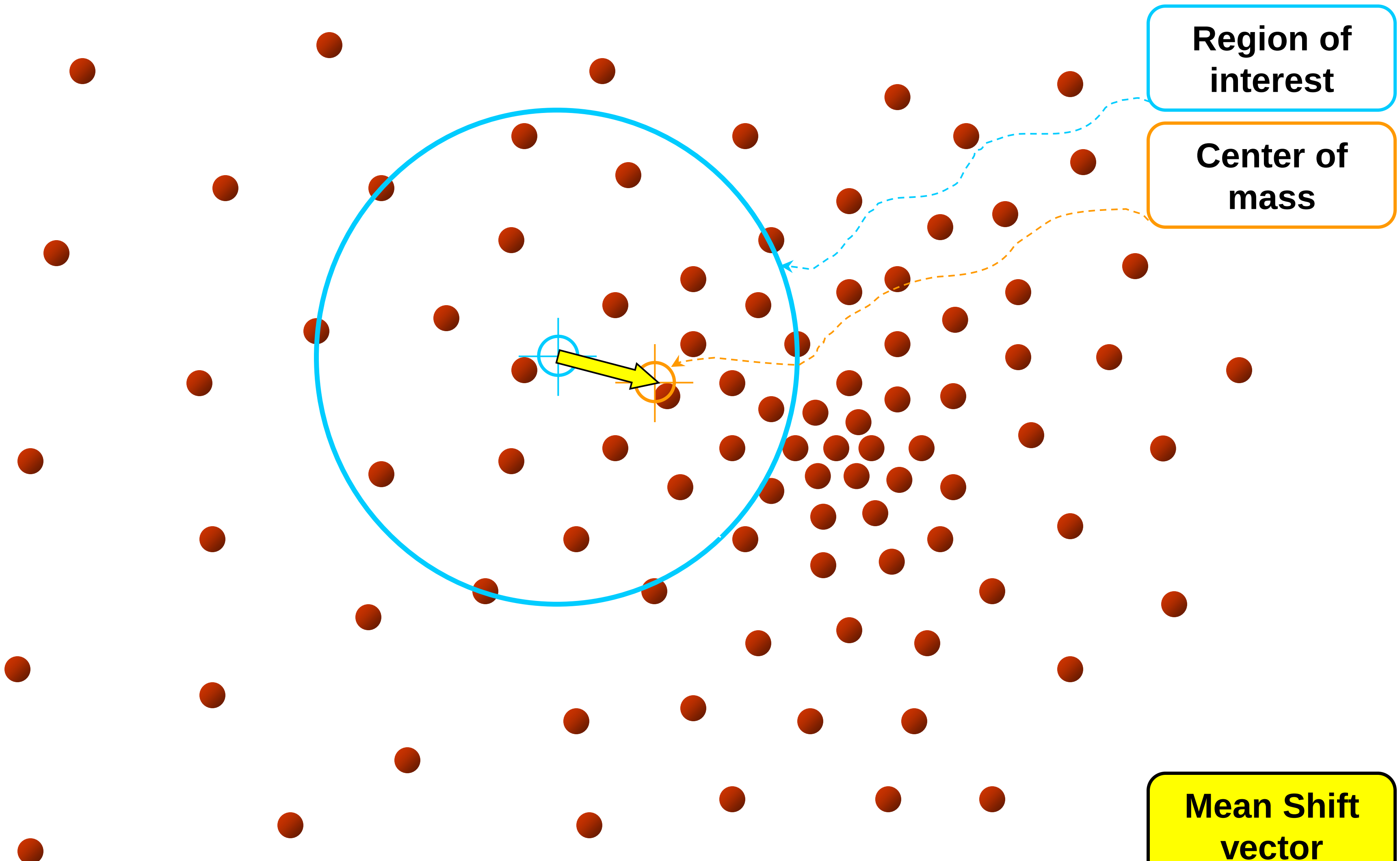
D. Comaniciu and P. Meer, Mean Shift: A Robust Approach toward Feature Space Analysis, PAMI 2002.

# Mean-Shift Algorithm




- **Iterative Mode Search**
  1. Initialize random seed, and window W
  2. Calculate center of gravity (the "mean") of W: $\sum_{x \in W} x H(x)$
  3. Shift the search window to the mean
  4. Repeat Step 2 until convergence

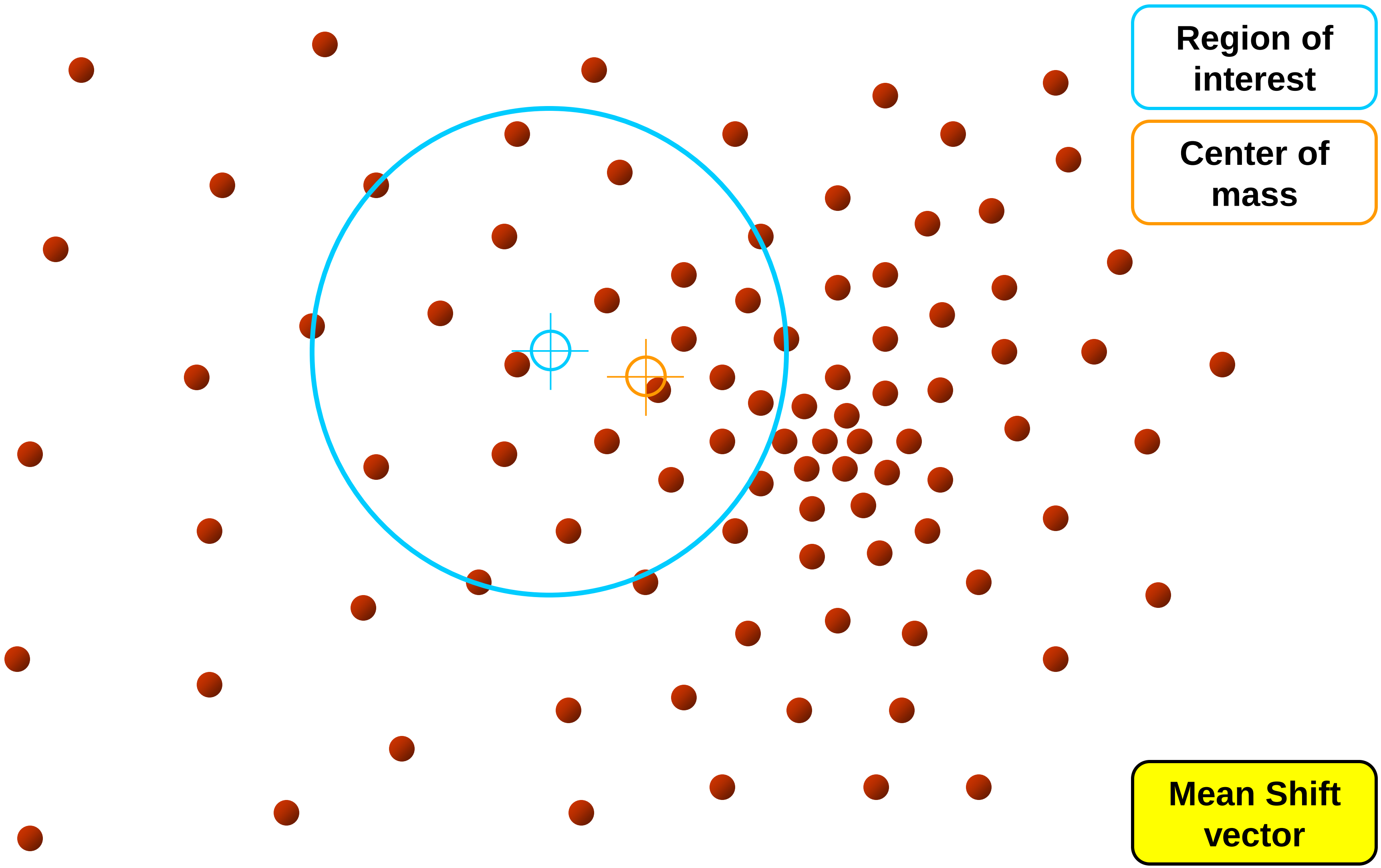
# Mean-Shift






Slide by Y. Ukrainitz & B. Sarel

# Mean-Shift



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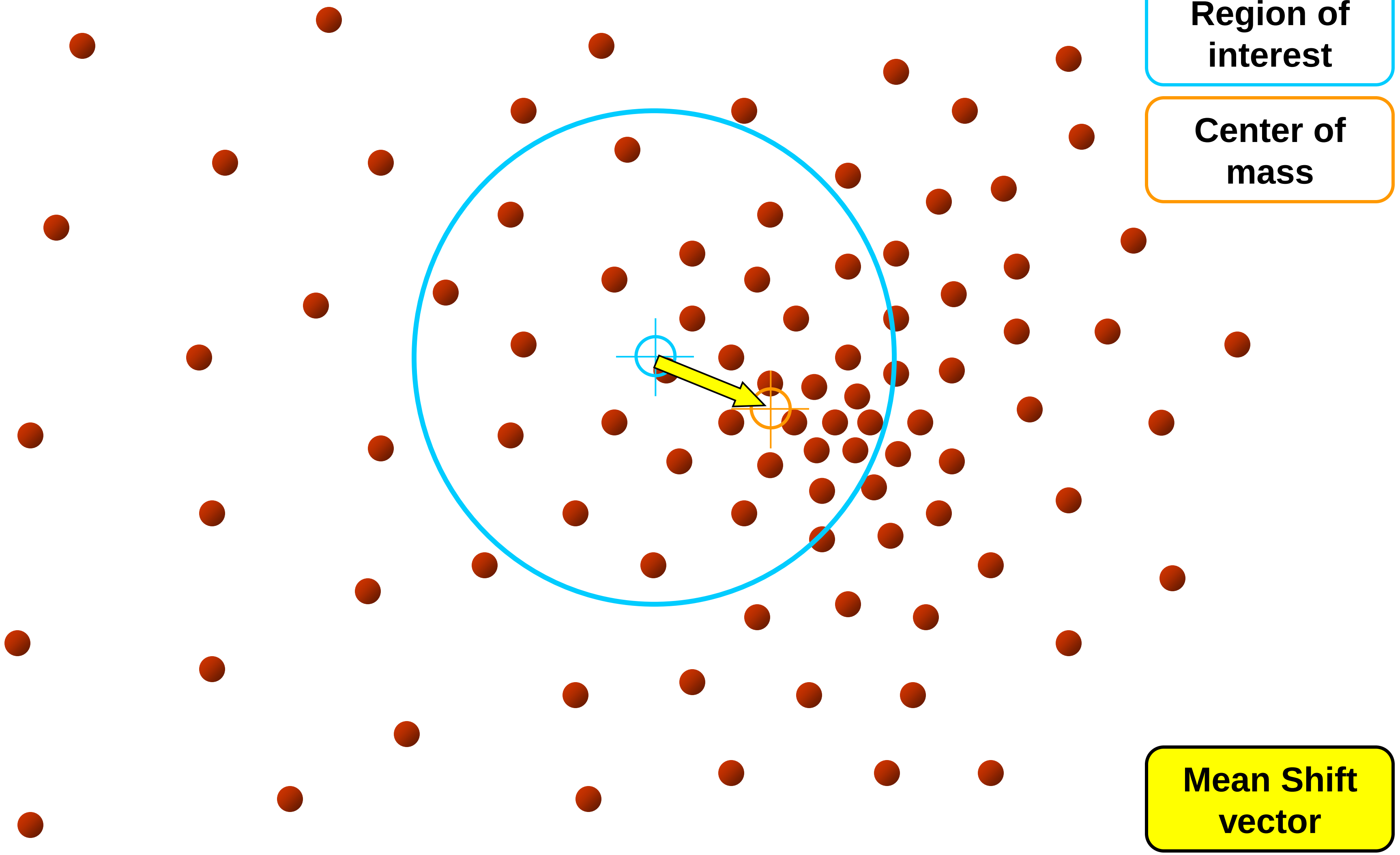
# Mean-Shift



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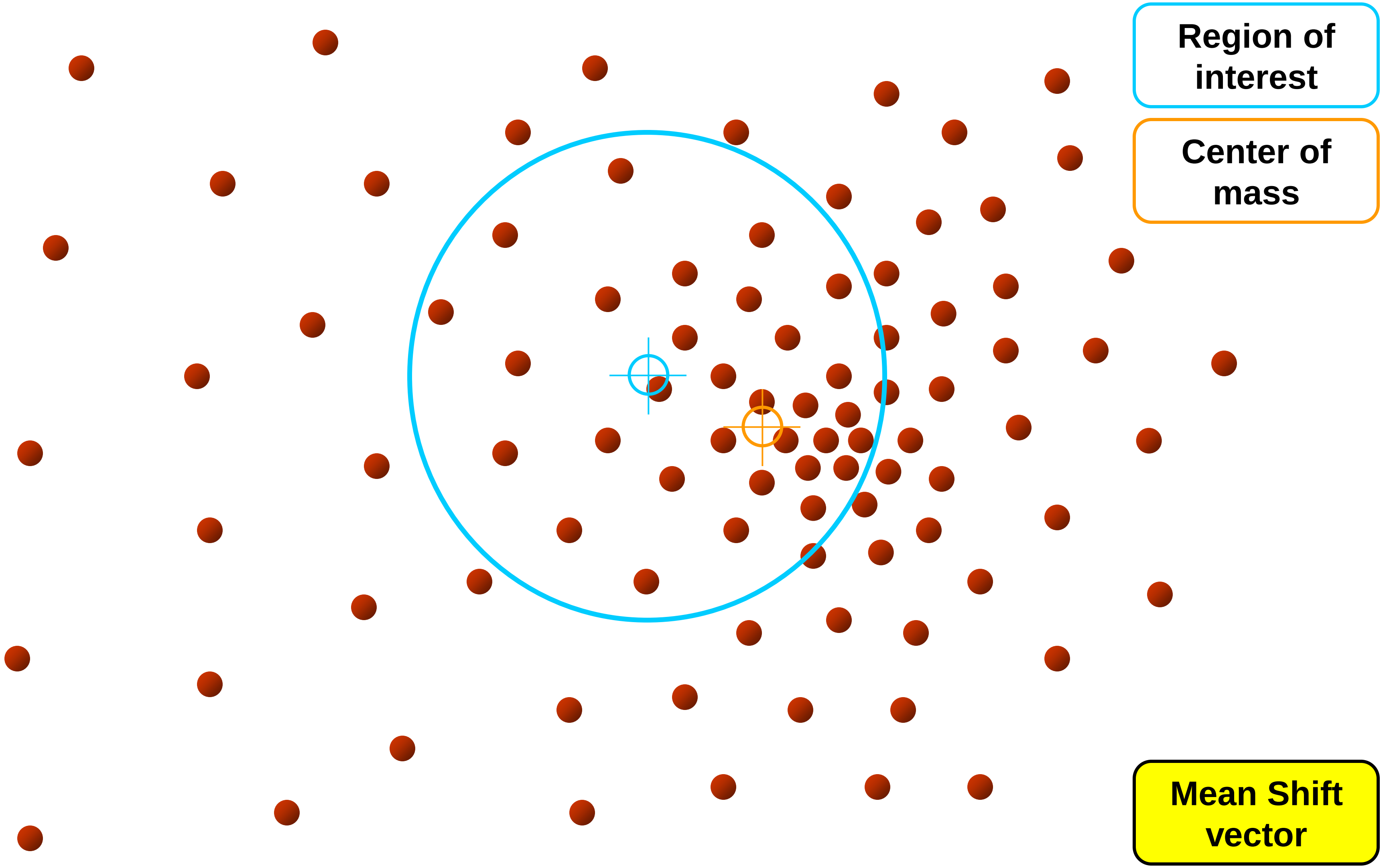
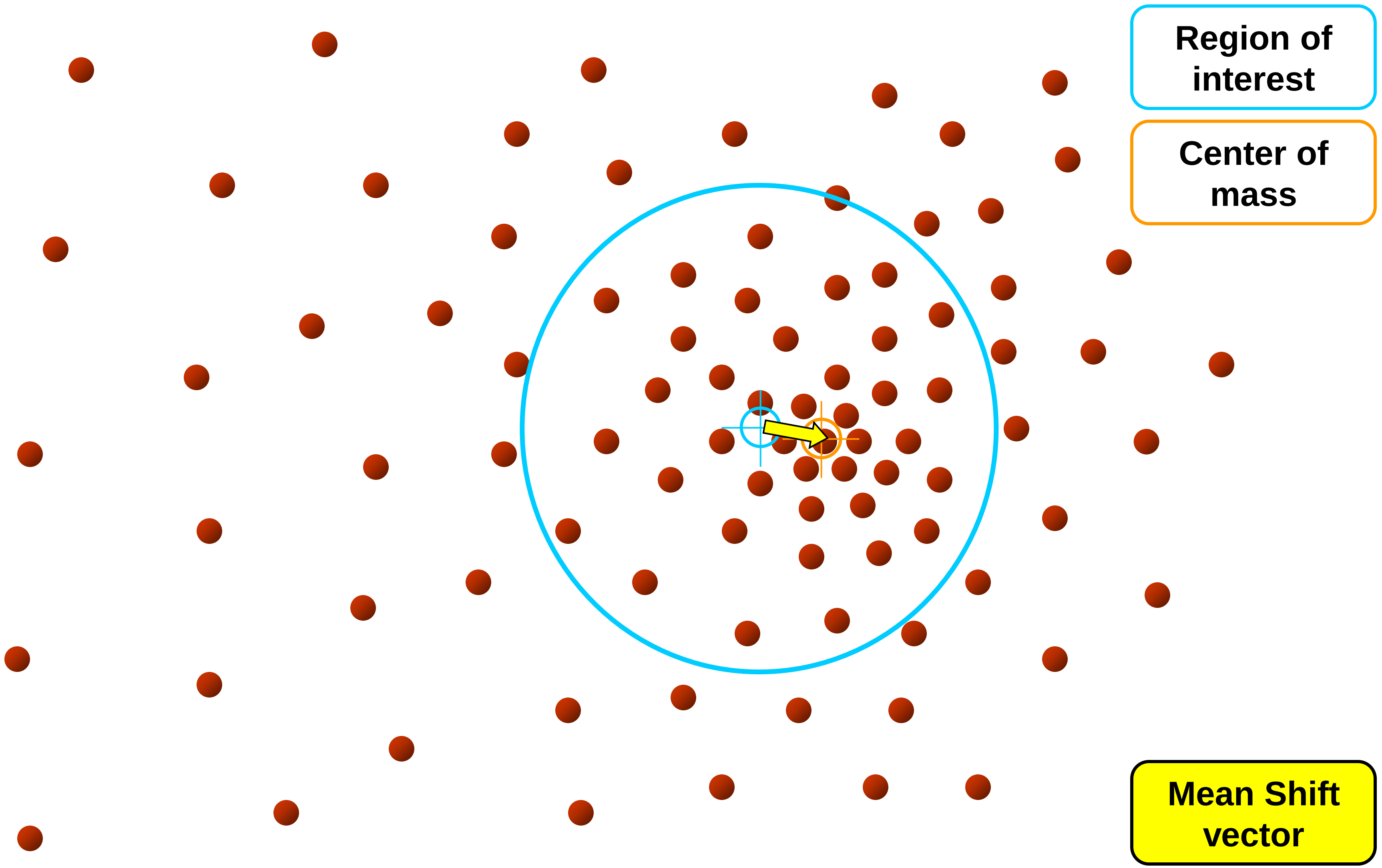
# Mean-Shift



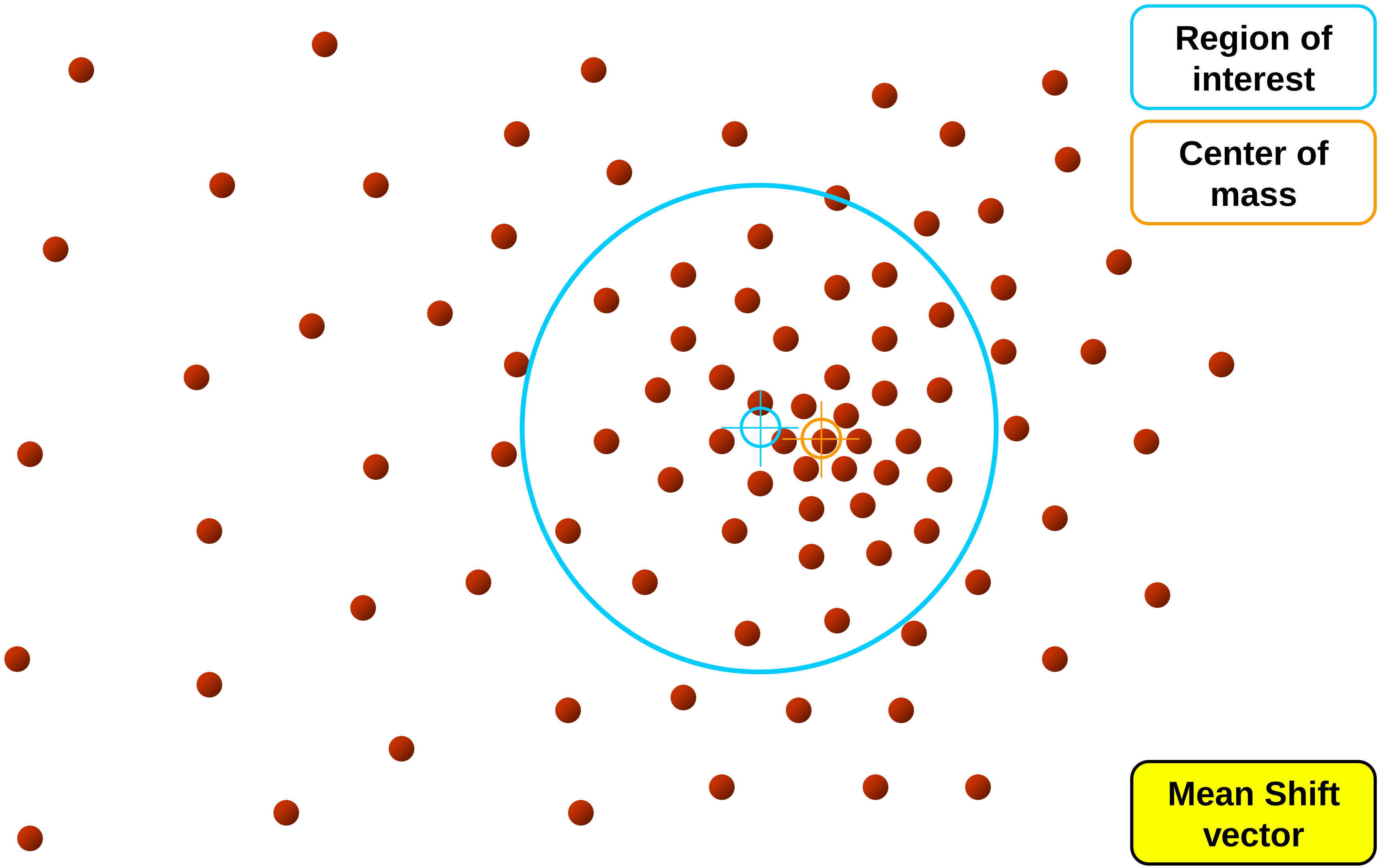



Slide by Y. Ukrainitz & B. Sarel

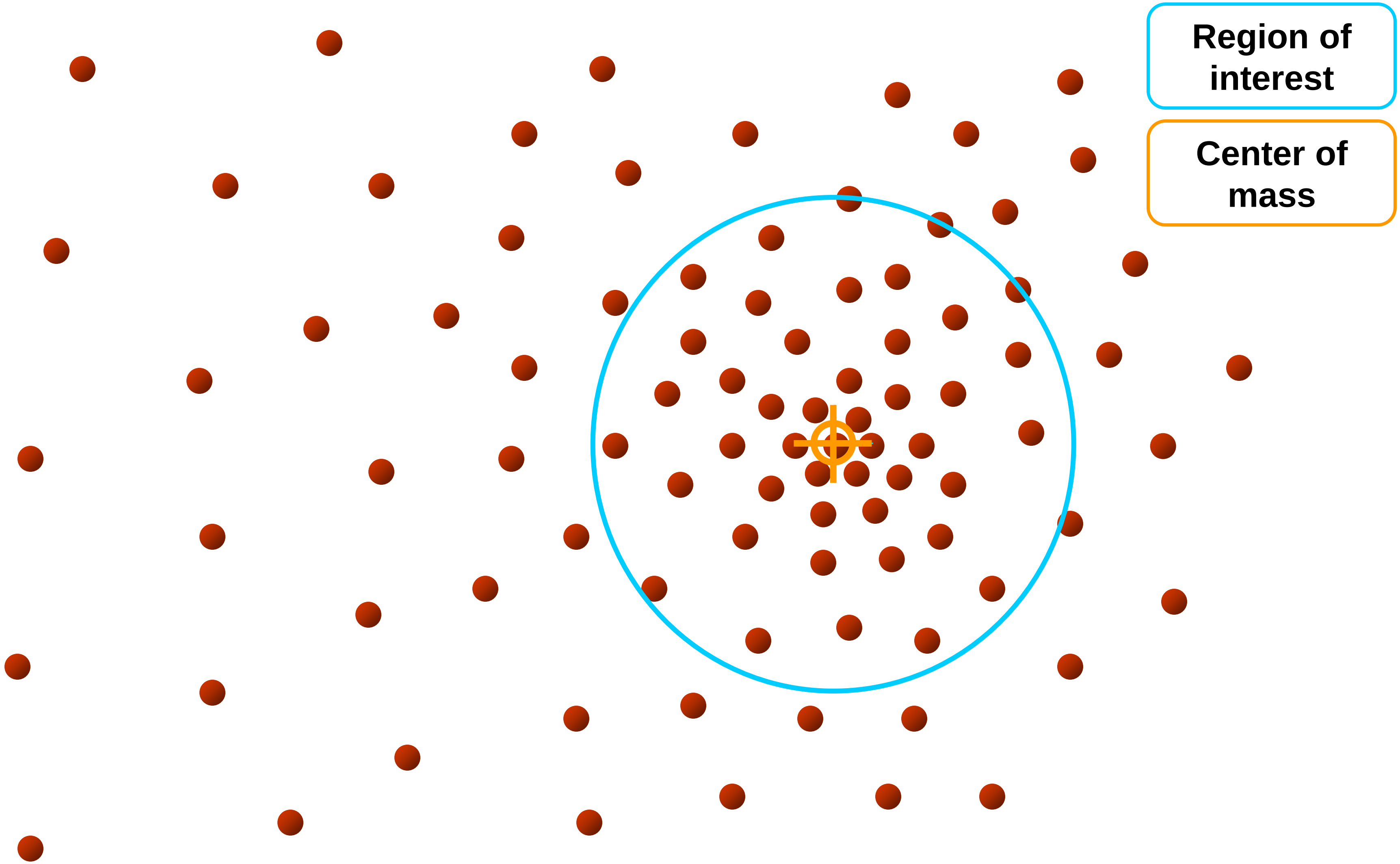
# Mean-Shift



# Mean-Shift



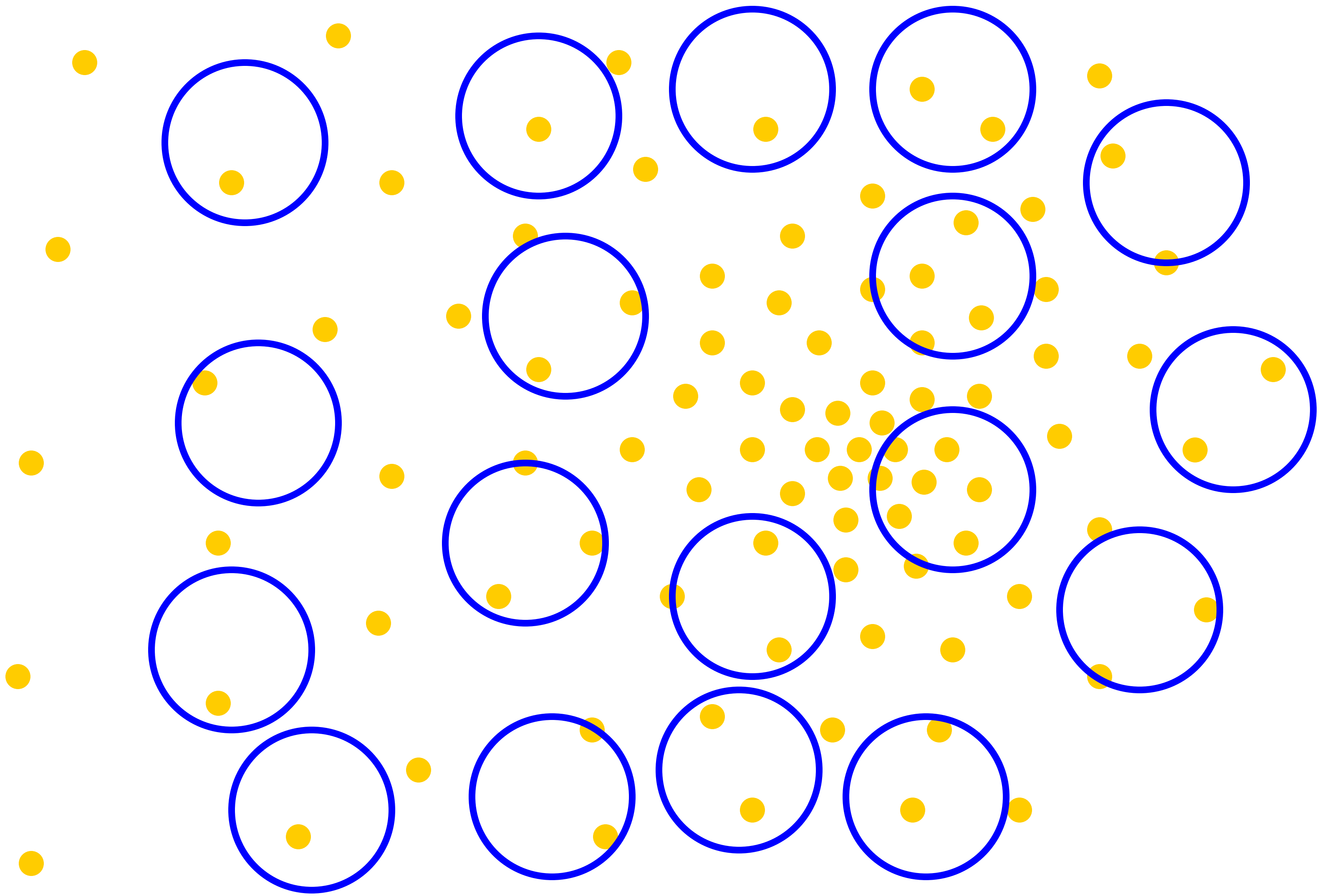
# Mean-Shift






Slide by Y. Ukrainitz & B. Sarel

# Real Modality Analysis

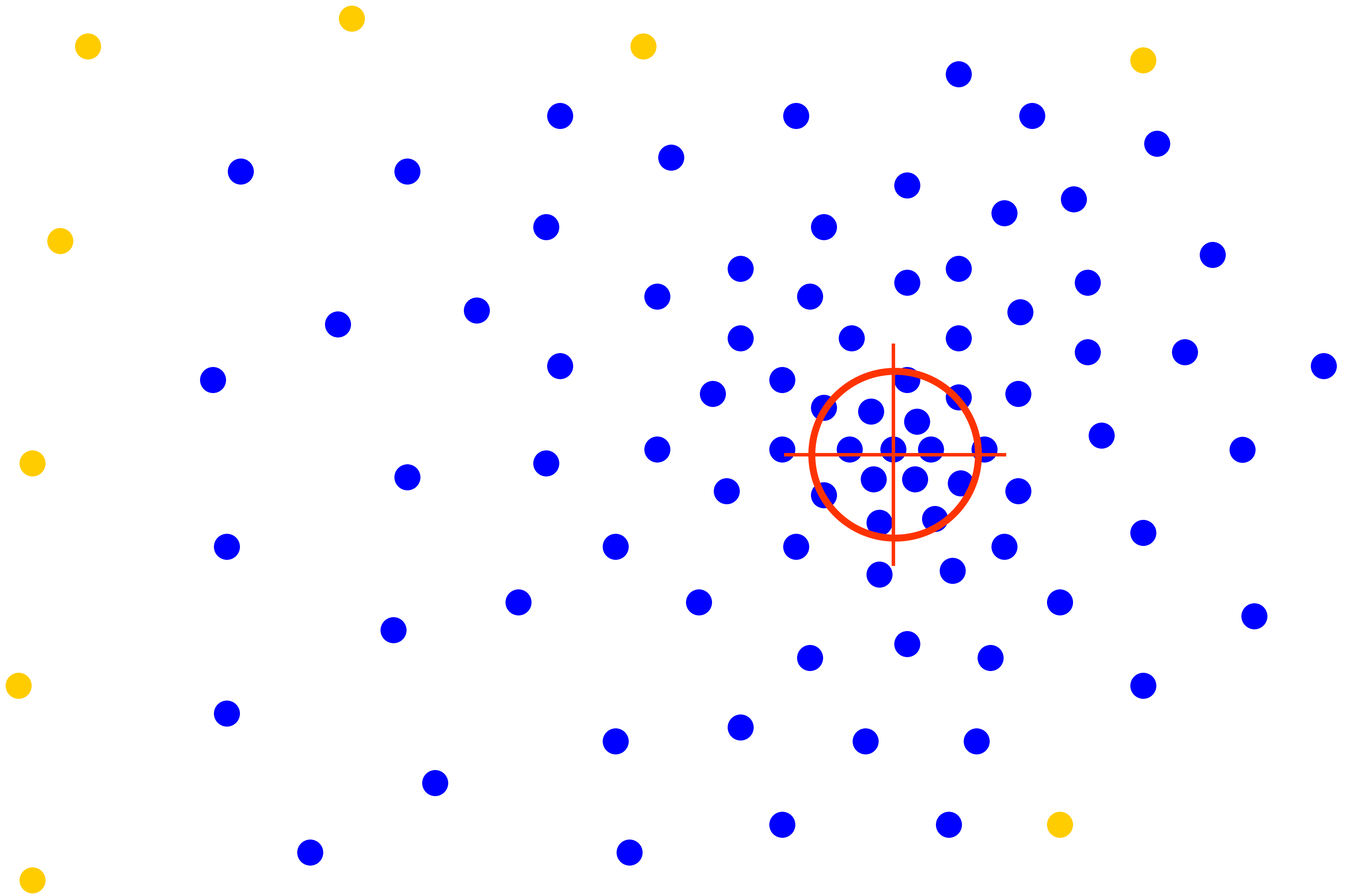


Tessellate the space with windows

Run the procedure in parallel

Slide by Y. Ukrainitz & B. Sarel

# Real Modality Analysis



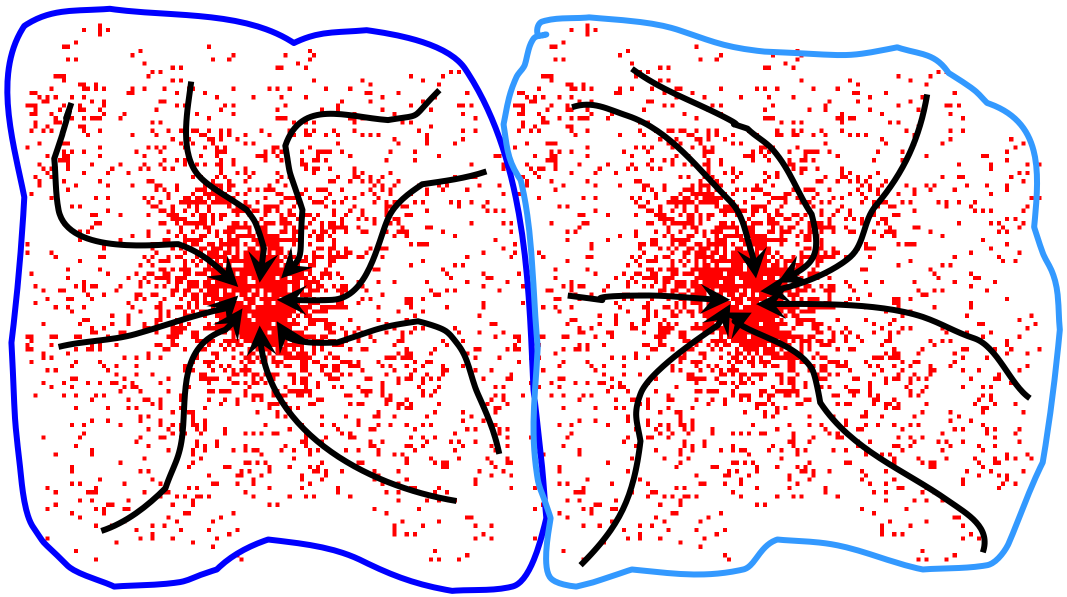
The blue data points were traversed by the windows towards the mode.



Slide by Y. Ukrainitz & B. Sarel

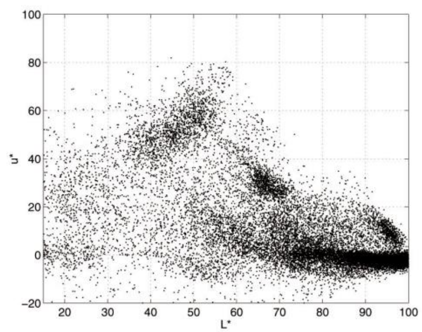
# Mean-Shift Clustering

- Cluster: all data points in the attraction basin of a mode
- Attraction basin: the region for which all trajectories lead to the same mode

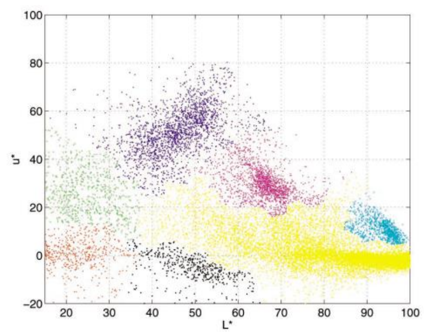


# Mean-Shift Clustering/Segmentation

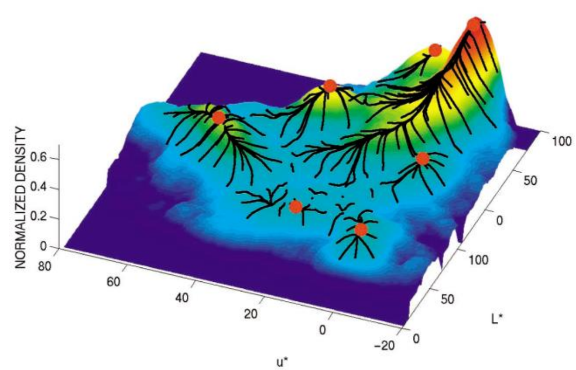
- Find features (color, gradients, texture, etc)
- Initialize windows at individual pixel locations
- Perform mean shift for each window until convergence
- Merge windows that end up near the same “peak” or mode



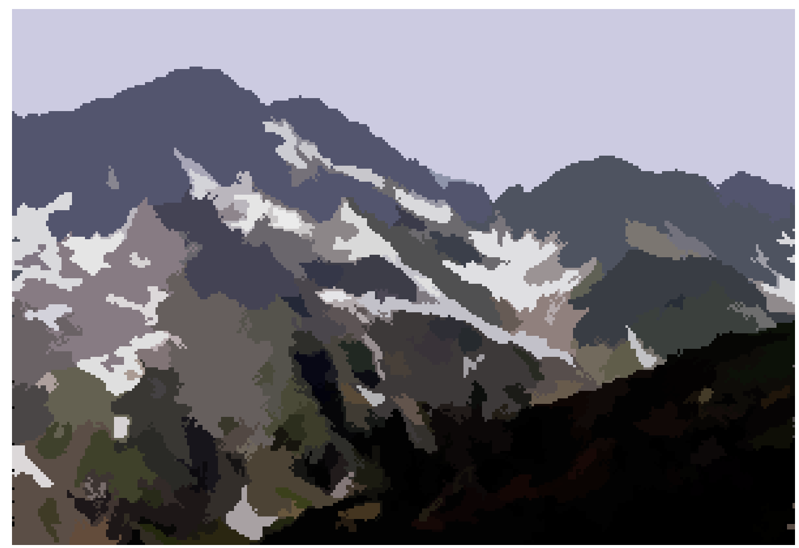

(a)




(b)



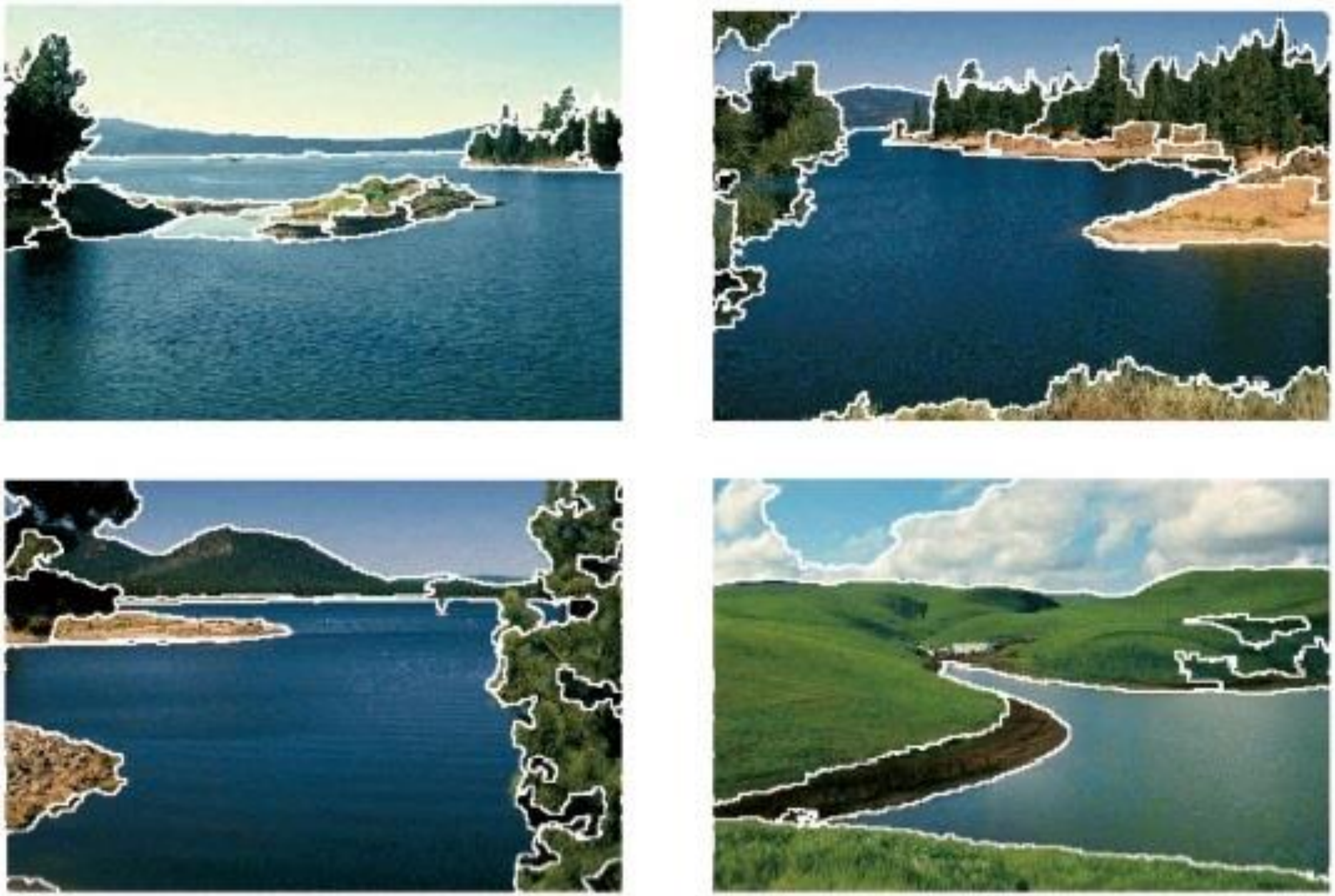
# Mean-Shift Segmentation Results



http://www.caip.rutgers.edu/~comanici/MSPAMI/msPamiResults.html

Slide credit: Svetlana Lazebnik

# More Results



Slide credit: Svetlana Lazebnik

# More Results

Slide credit: Svetlana Lazebnik

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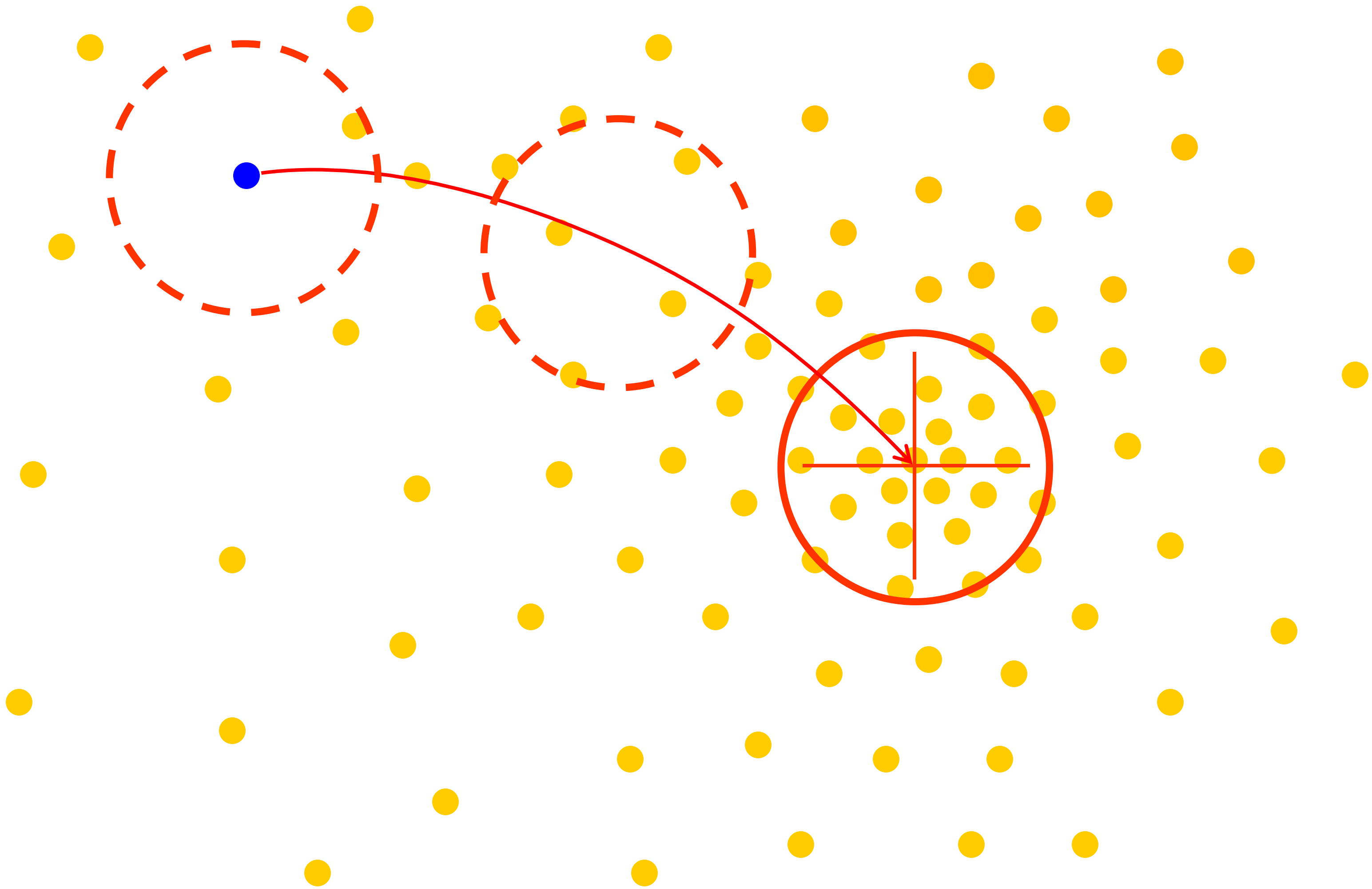
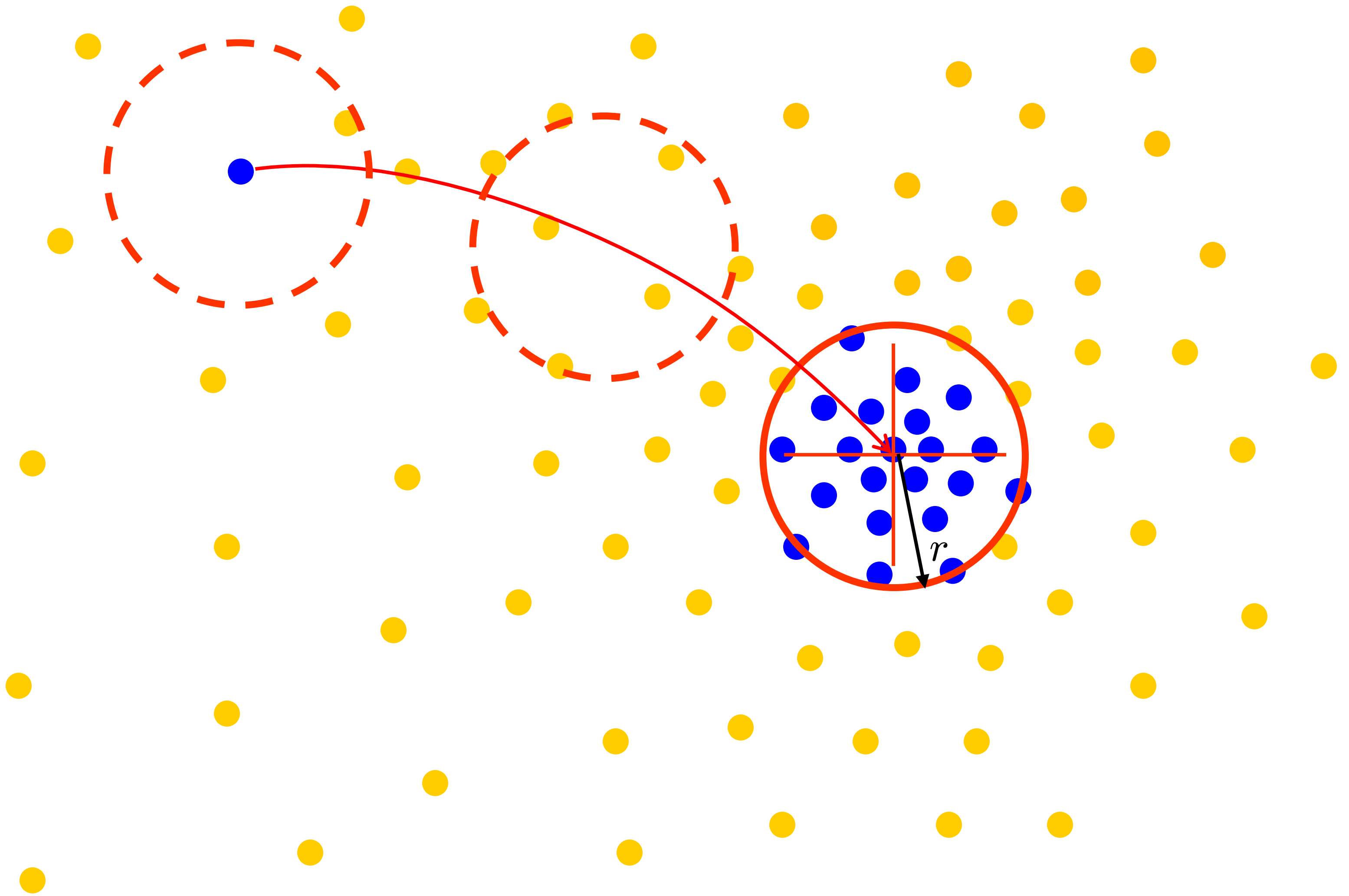
# Problem: Computational Complexity



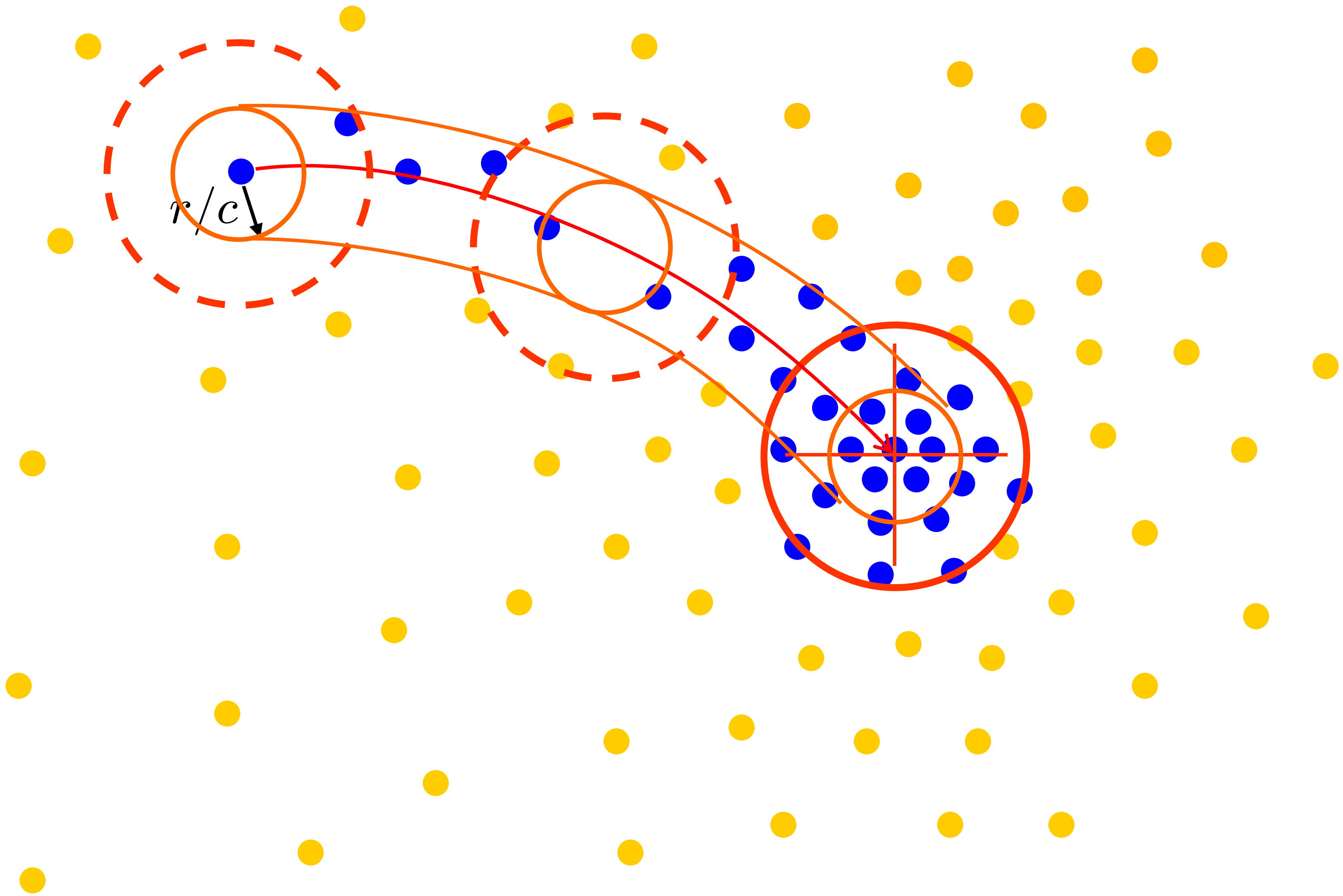
- Need to shift many windows…
- Many computations will be redundant.

# Speedups: Basin of Attraction




1. **Assign all points within radius r of end point to the mode.**

# Speedups




**2. Assign all points within radius r/c of the search path to the mode.**

# Summary Mean-Shift

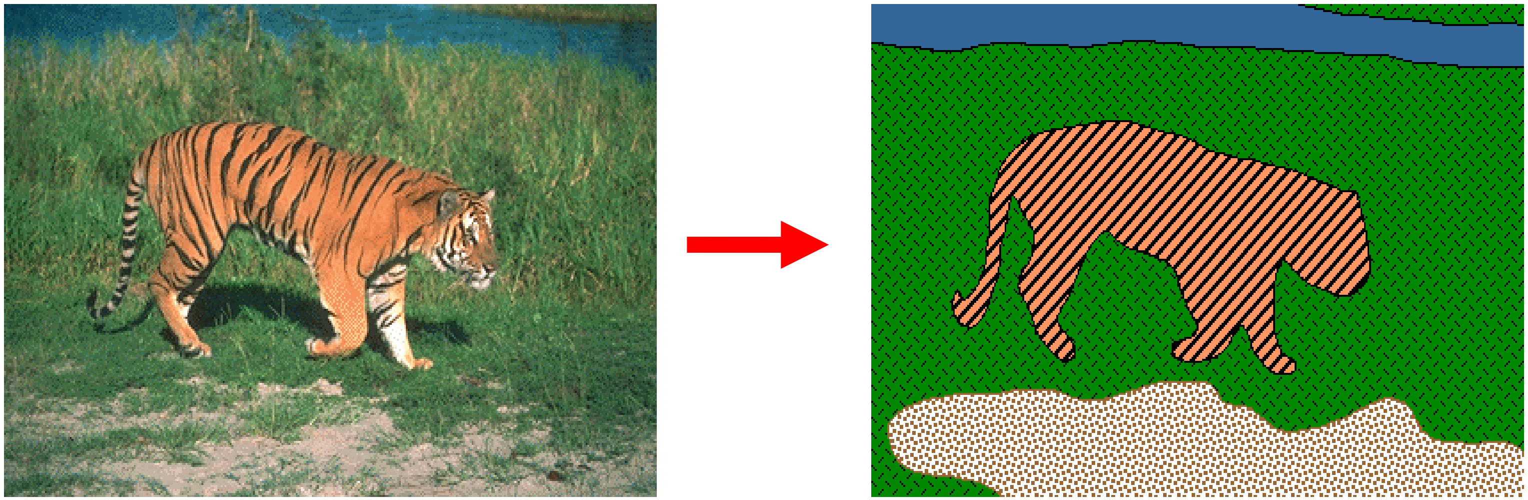
- Pros
  - General, application-independent tool
  - Model-free, does not assume any prior shape (spherical, elliptical, etc.) on data clusters
  - Just a single parameter (window size h)
    - h has a physical meaning (unlike k-means)
  - Finds variable number of modes
  - Robust to outliers

- Cons
  - Output depends on window size
  - Window size (bandwidth) selection is not trivial
  - Computationally (relatively) expensive (~2s/image)
  - Does not scale well with dimension of feature space

Slide credit: Svetlana Lazebnik

# Back to the Image Segmentation Problem…

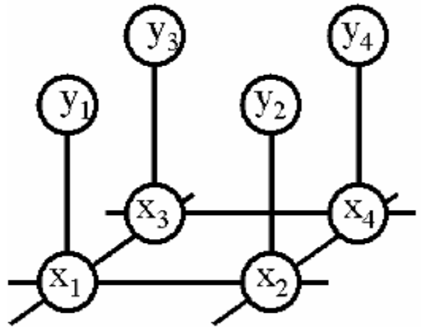
- Goal: identify groups of pixels that go together



- Up to now, we have focused on ways to group pixels into image segments based on their appearance…
  - Segmentation as clustering.
- We also want to enforce region constraints.
  - Spatial consistency
  - Smooth borders

# Topics of This Lecture

- Segmentation as Energy Minimization
  - Markov Random Fields
  - Energy formulation

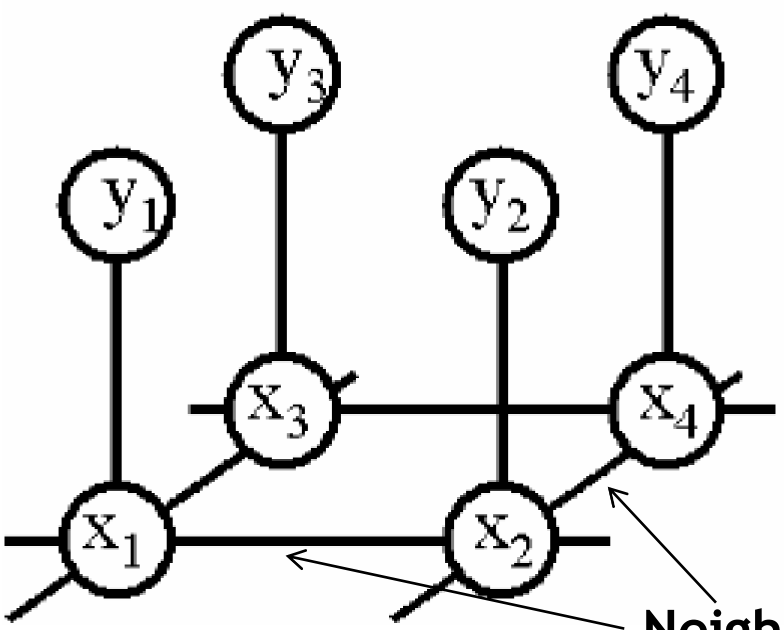



- Graph cuts for image segmentation
  - Basic idea
  - s-t Mincut algorithm
  - Extension to non-binary case

- Applications
  - Interactive segmentation

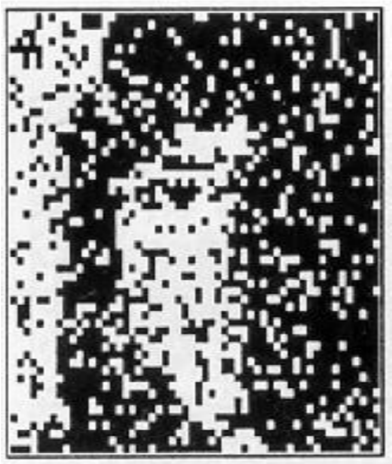
# Markov Random Fields

- Allow rich probabilistic models for images
- But built in a local, modular way
  - Learn local effects, get global effects out



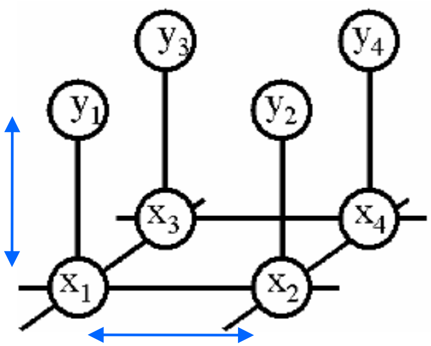
# MRF Nodes as Pixels

Original image



Degraded image

Reconstruction from MRF modeling pixel neighborhood statistics




$\Phi(x_i, y_i)$

$\Psi(x_i, x_j)$

# Network Joint Probability

$$p(\mathbf{x}, \mathbf{y}) = \prod_i \Phi(x_i, y_i) \prod_{i,j} \Psi(x_i, x_j)$$

Scene

Image

Image-scene compatibility function

Local observations

Scene-scene compatibility function

Neighboring scene nodes

# Energy Formulation

- **Joint probability**

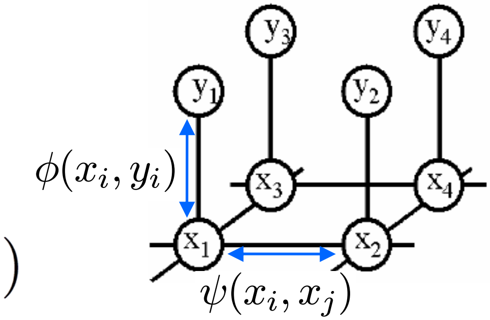
$$p(\mathbf{x},\mathbf{y}) = \prod_i \Phi(x_i, y_i) \prod_{i,j} \Psi(x_i, x_j)$$

- **Maximizing the joint probability is the same as minimizing the negative log**

$$\begin{aligned} -\log p(\mathbf{x},\mathbf{y}) &= -\sum_i \log \Phi(x_i, y_i) - \sum_{i,j} \log \Psi(x_i, x_j) \\ E(\mathbf{x},\mathbf{y}) &= \sum_i \phi(x_i, y_i) + \sum_{i,j} \psi(x_i, x_j) \end{aligned}$$

- **This is similar to free-energy problems in statistical mechanics (spin glass theory). We therefore draw the analogy and call $E$ an *energy function*.**
- **$\phi$ and $\psi$ are called *potentials*.**

# Energy Formulation

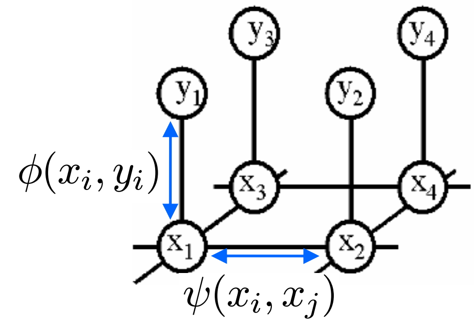



- Energy function

$$E(\mathbf{x}, \mathbf{y}) = \underbrace{\sum_i \phi(x_i, y_i)}_{\text{Single-node potentials}} + \underbrace{\sum_{i,j} \psi(x_i, x_j)}_{\text{Pairwise potentials}}$$

- Single-node potentials $\phi$ ("unary potentials")
  - Encode local information about the given pixel/patch
  - How likely is a pixel/patch to belong to a certain class (e.g. foreground/background)?
- Pairwise potentials $\psi$
  - Encode neighborhood information
  - How different is a pixel/patch's label from that of its neighbor? (e.g. based on intensity/color/texture difference, edges)

# Energy Minimization



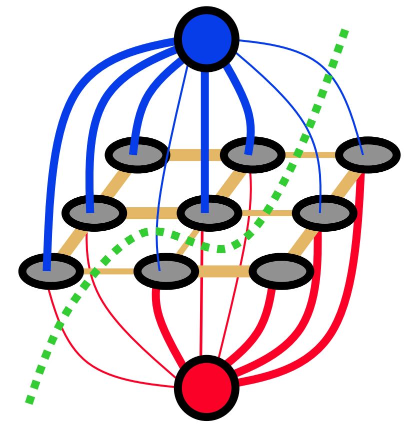

- **Goal:**
  - Infer the optimal labeling of the MRF.
- **Many inference algorithms are available, e.g.**
  - Gibbs sampling, simulated annealing
  - Iterated conditional modes (ICM)
  - Variational methods
  - Belief propagation
  - Graph cuts

see lecture Machine Learning!

- **Recently, Graph Cuts have become a popular tool**
  - Only suitable for a certain class of energy functions
  - But the solution can be obtained very fast for typical vision problems (~1MPixel/sec).

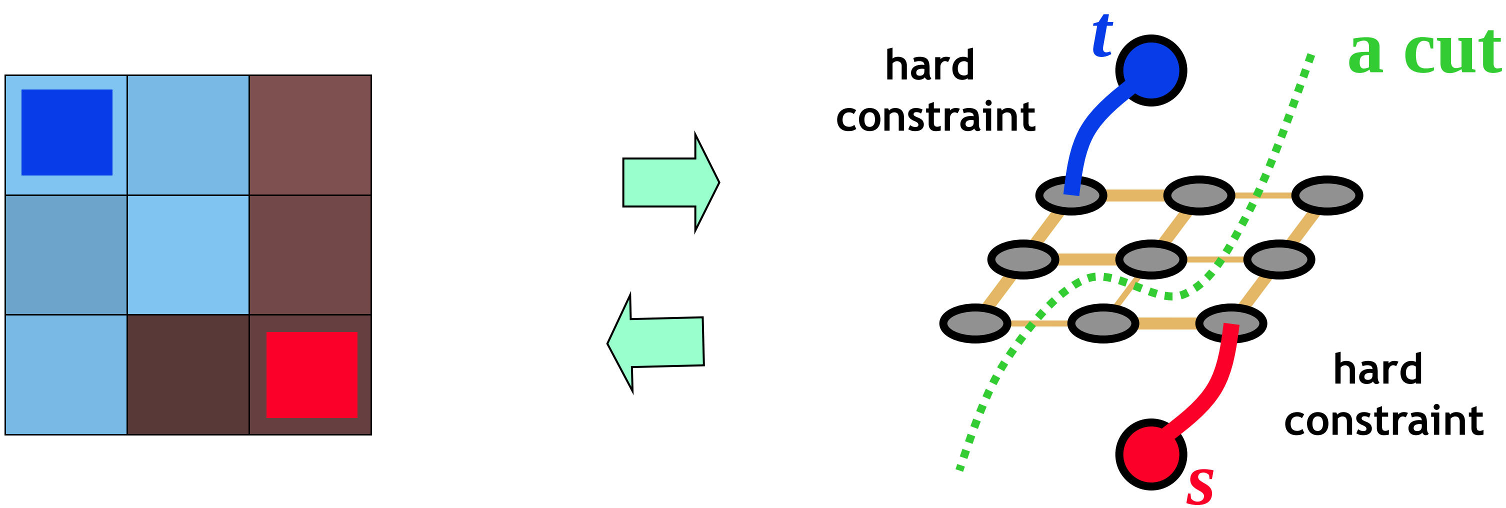
# Topics of This Lecture

- Segmentation as Energy Minimization
  - Markov Random Fields
  - Energy formulation
- Graph cuts for image segmentation
  - Basic idea
  - s-t Mincut algorithm
  - Extension to non-binary case
- Applications
  - Interactive segmentation



# Graph Cuts for Optimal Boundary Detection

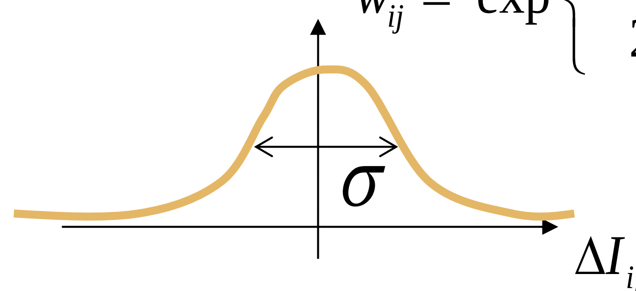
- **Idea: convert MRF into source-sink graph**




**Minimum cost cut can be computed in polynomial time**

**(max-flow/min-cut algorithms)**

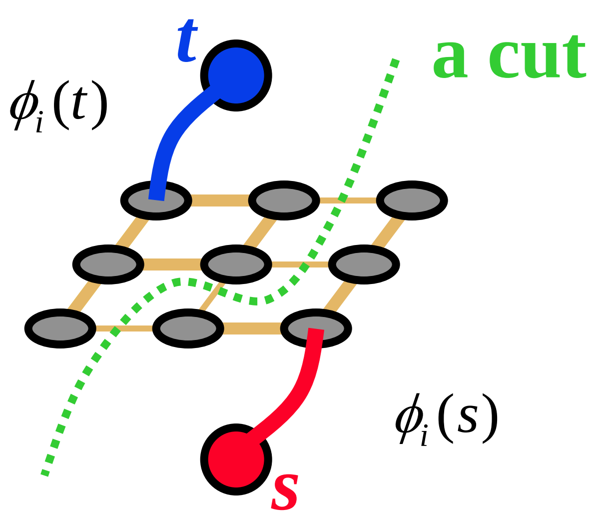
$$w_{ij} = \exp\left\{ -\frac{\Delta I_{ij}}{2\sigma^2} \right\}$$



# Simple Example of Energy

$$E(\mathbf{x},\mathbf{y}) = \sum_i \phi_i(x_i) + \sum_{i,j} w_{ij} \cdot \delta(x_i \neq x_j)$$

**Unary terms** **Pairwise terms**

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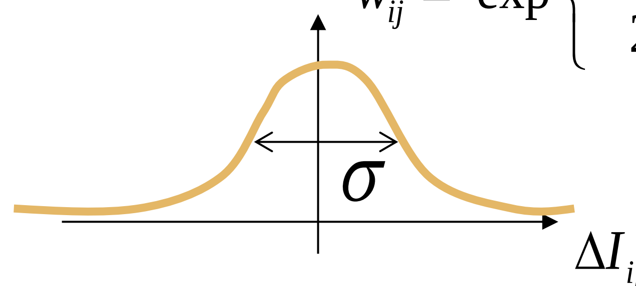
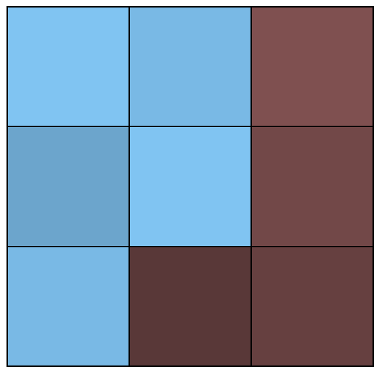
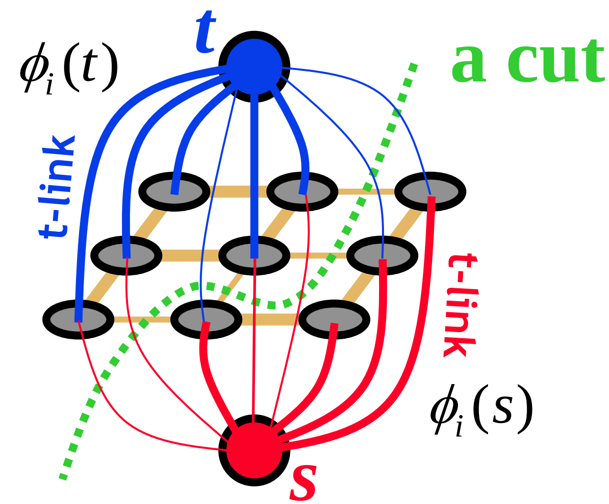



$$x \in \{s, t\}$$

**(binary object segmentation)**

# Adding Regional Properties






**Regional bias example**

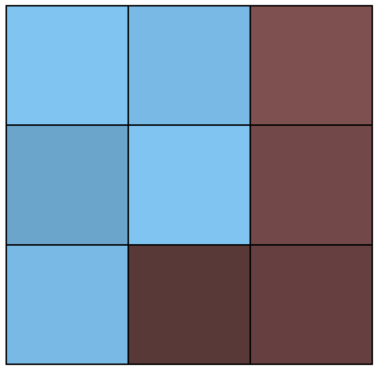
**Suppose $I^s$ and $I^t$ are given "expected" intensities of object and background**

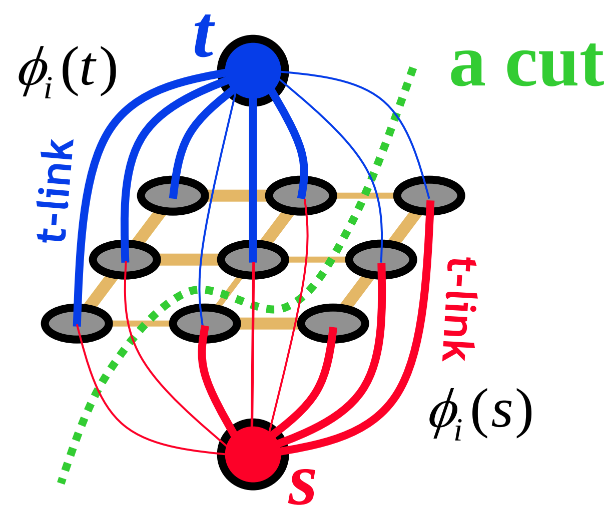
$$\phi_i(s) \propto \exp\left(-\| I_i - I^s \|^2 / 2\sigma^2\right)$$

$$\phi_i(t) \propto \exp\left(-\| I_i - I^t \|^2 / 2\sigma^2\right)$$

**NOTE: hard constrains are not required, in general.**

# Adding Regional Properties






"expected" intensities of **object** and **background** $I^s$ and $I^t$ can be re-estimated

$$\phi_i(s) \propto \exp\left(-\| I_i - I^s \|^2 / 2\sigma^2\right)$$

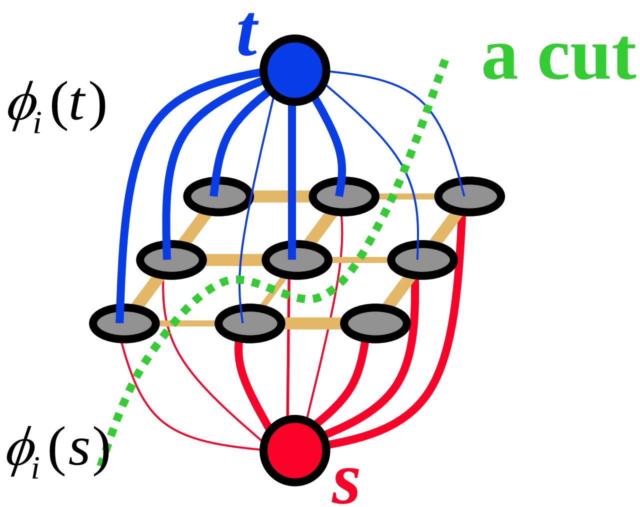
$$\phi_i(t) \propto \exp\left(-\| I_i - I^t \|^2 / 2\sigma^2\right)$$

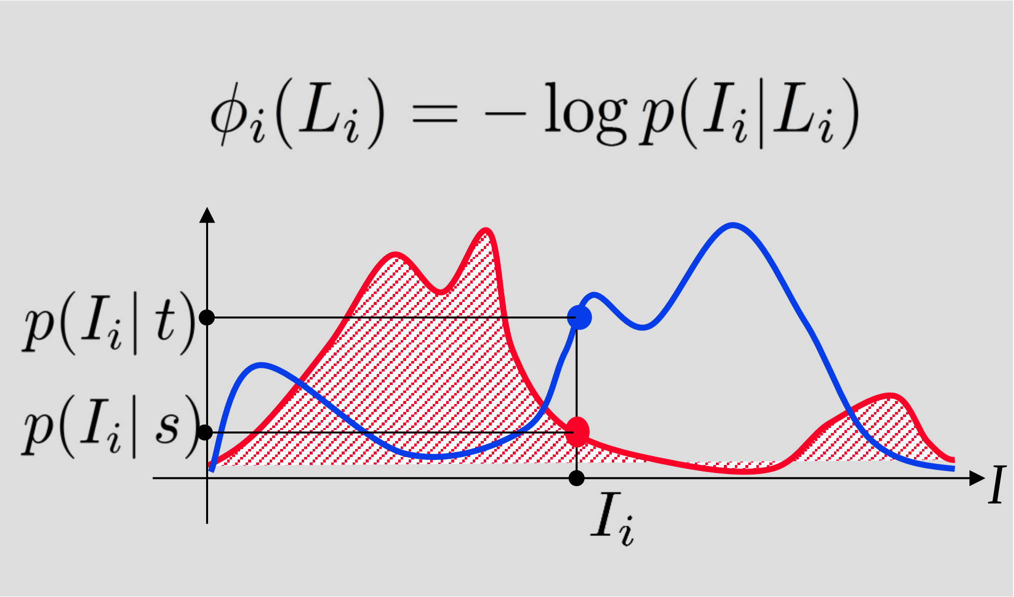
**EM-style optimization**


[Boykov & Jolly, ICCV'01]

# Adding Regional Properties

- More generally, regional bias can be based on any intensity models of object and background




given object and background intensity histograms

# How to Set the Potentials? Some Examples

- **Color potentials**
  - **e.g., modeled with a Mixture of Gaussians**

$$\phi(x_i, y_i; \theta_\phi) = \log \sum_k \theta_\phi(x_i, k) p(k|x_i) \mathcal{N}(y_i; \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$$

- **Edge potentials**
  - **E.g., a "contrast sensitive Potts model"**

$$\psi(x_i, x_j, g_{ij}(\mathbf{y}); \theta_\psi) = -\theta_\psi g_{ij}(\mathbf{y}) \delta(x_i \neq x_j)$$

**where**

$$g_{ij}(\mathbf{y}) = e^{-\beta \|y_i - y_j\|^2} \qquad \beta = \frac{1}{2}\left(\text{avg}\left(\|y_i - y_j\|^2\right)\right)^{-1}$$

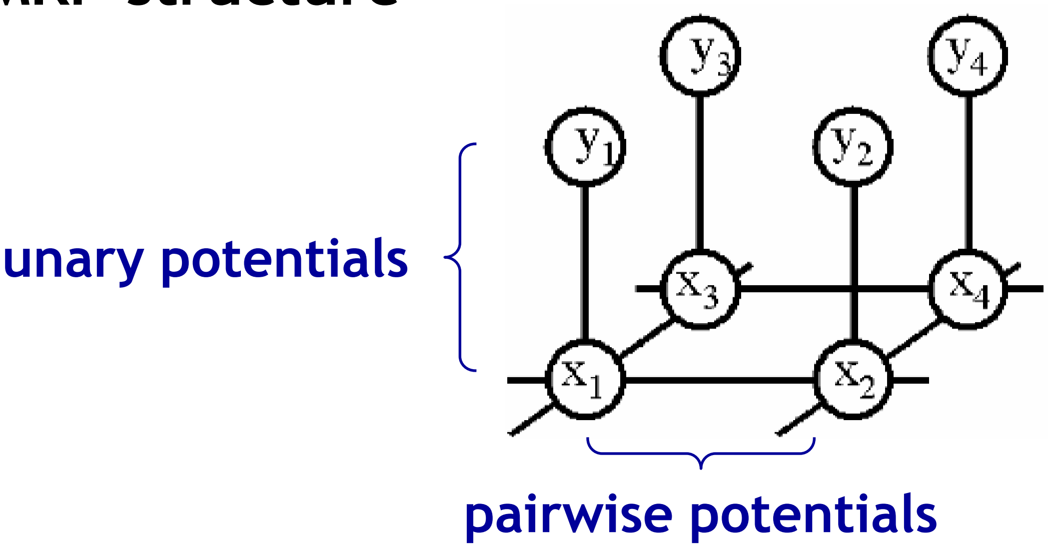
- **Parameters $\theta_\phi$, $\theta_\psi$ need to be learned, too!**



[Shotton & Winn, ECCV'06]

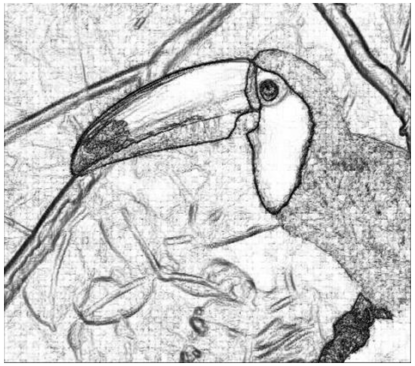
# Example: MRF for Image Segmentation

- **MRF structure**




Data (D)

Unary likelihood



Pair-wise Terms

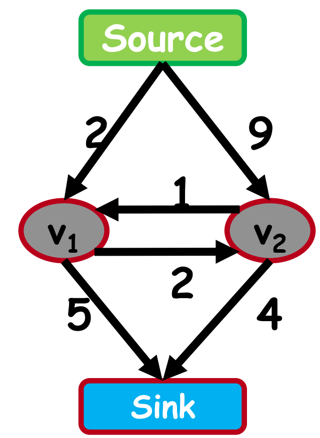
MAP Solution



Slide adapted from Phil Torr

# Topics of This Lecture

- Segmentation as Energy Minimization
  - Markov Random Fields
  - Energy formulation
- Graph cuts for image segmentation
  - Basic idea
  - s-t Mincut algorithm
  - Extension to non-binary case
- Applications
  - Interactive segmentation



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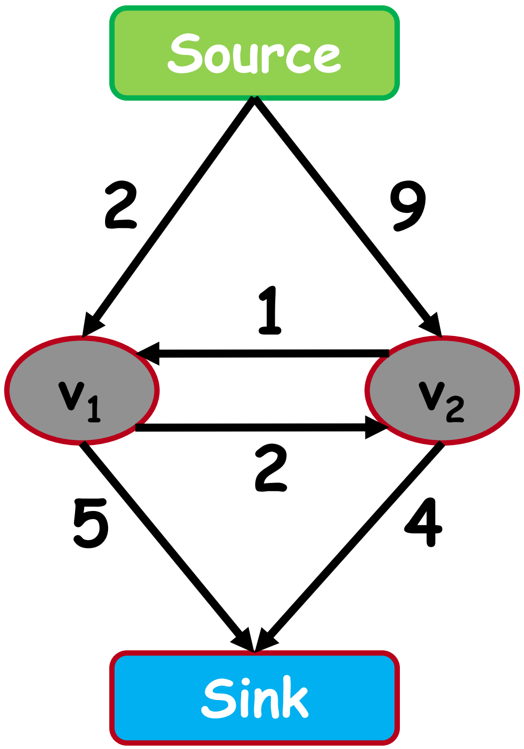
# How Does it Work? The s-t-Mincut Problem



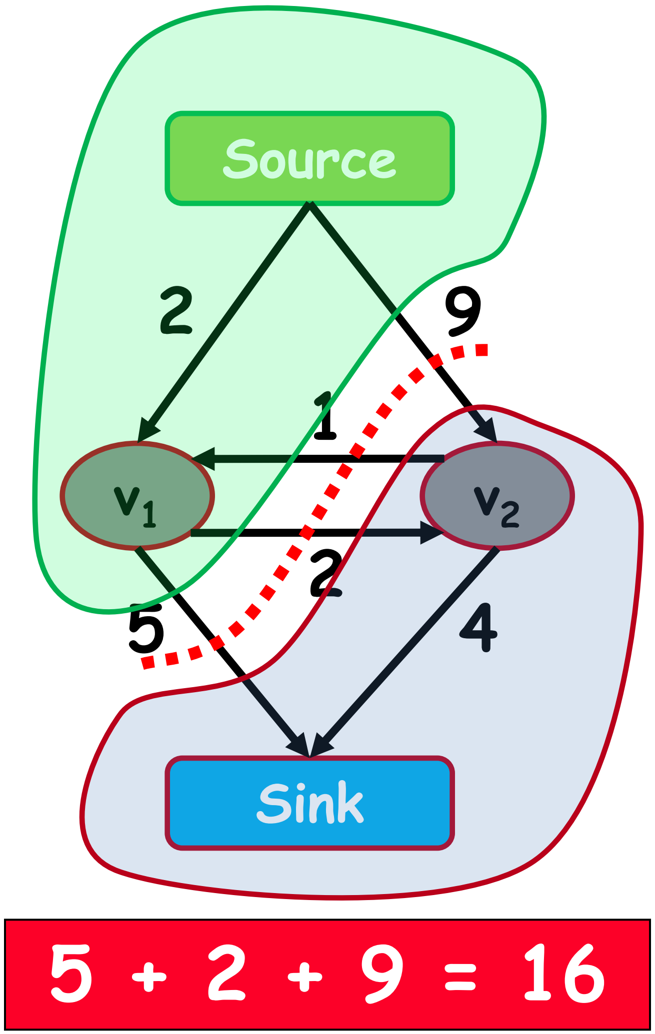

**Graph (V, E, C)**

Vertices $V = \{v_1, v_2 \dots v_n\}$

Edges $E = \{(v_1, v_2) \dots\}$

Costs $C = \{c_{(1, 2)} \dots\}$

# The s-t-Mincut Problem




## What is an st-cut?

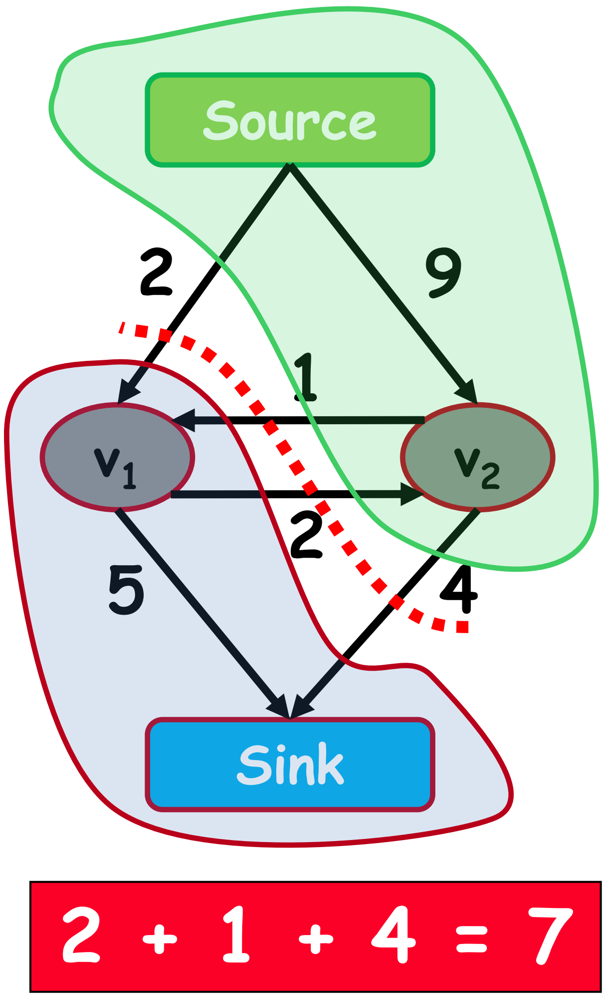
An st-cut (S,T) divides the nodes between source and sink.

## What is the cost of a st-cut?

Sum of cost of all edges going from S to T

Slide credit: Pushmeet Kohli

# The s-t-Mincut Problem




**What is an st-cut?**

An st-cut (S,T) divides the nodes between source and sink.

**What is the cost of a st-cut?**

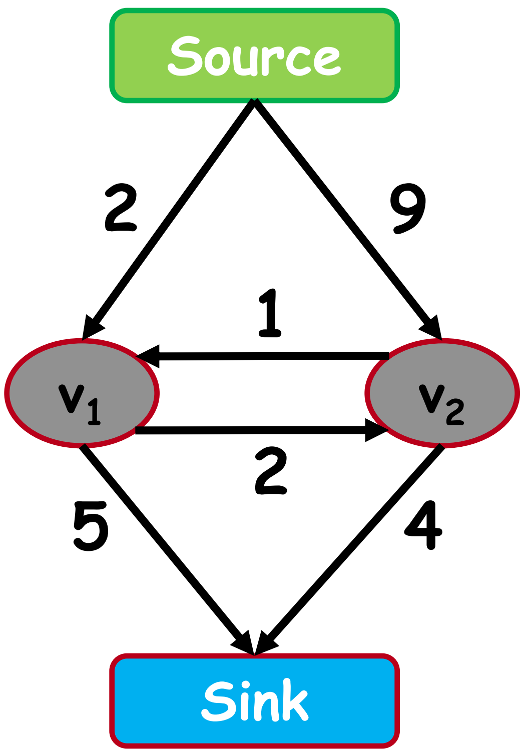
Sum of cost of all edges going from S to T

**What is the st-mincut?**

st-cut with the minimum cost

Slide credit: Pushmeet Kohli

# How to Compute the s-t-Mincut?




Solve the dual maximum flow problem

Compute the maximum flow between Source and Sink

**Constraints**

Edges: Flow < Capacity

Nodes: Flow in = Flow out

**Min-cut/Max-flow Theorem**

In every network, the maximum flow equals the cost of the st-mincut

# History of Maxflow Algorithms

## Augmenting Path and Push-Relabel

| year | discoverer(s) | bound |
|---|---|---|
| 1951 | Dantzig | $O(n^2mU)$ |
| 1955 | Ford & Fulkerson | $O(m^2U)$ |
| 1970 | Dinitz | $O(n^2m)$ |
| 1972 | Edmonds & Karp | $O(m^2 \log U)$ |
| 1973 | Dinitz | $O(nm \log U)$ |
| 1974 | Karzanov | $O(n^3)$ |
| 1977 | Cherkassky | $O(n^2m^{1/2})$ |
| 1980 | Galil & Naamad | $O(nm \log^2 n)$ |
| 1983 | Sleator & Tarjan | $O(nm \log n)$ |
| 1986 | Goldberg & Tarjan | $O(nm \log(n^2/m))$ |
| 1987 | Ahuja & Orlin | $O(nm + n^2 \log U)$ |
| 1987 | Ahuja et al. | $O(nm \log(n\sqrt{\log U}/m))$ |
| 1989 | Cheriyan & Hagerup | $E(nm + n^2 \log^2 n)$ |
| 1990 | Cheriyan et al. | $O(n^3/\log n)$ |
| 1990 | Alon | $O(nm + n^{8/3} \log n)$ |
| 1992 | King et al. | $O(nm + n^{2+\epsilon})$ |
| 1993 | Phillips & Westbrook | $O(nm(\log_{m/n} n + \log^{2+\epsilon} n))$ |
| 1994 | King et al. | $O(nm \log_{m/(n \log n)} n)$ |
| 1997 | Goldberg & Rao | $O(m^{3/2} \log(n^2/m) \log U)$<br>$O(n^{2/3}m \log(n^2/m) \log U)$ |

$n$: #nodes

$m$: #edges

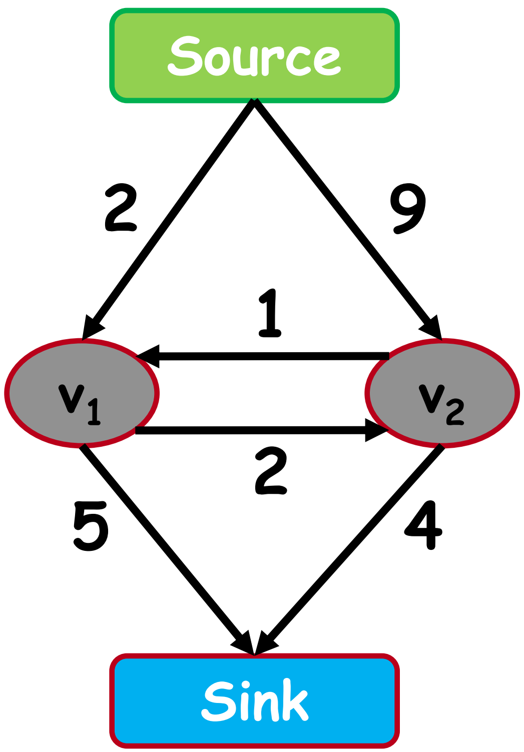
$U$: maximum edge weight

**Algorithms assume non-negative edge weights**

Slide credit: Andrew Goldberg

# Maxflow Algorithms

Flow = 0

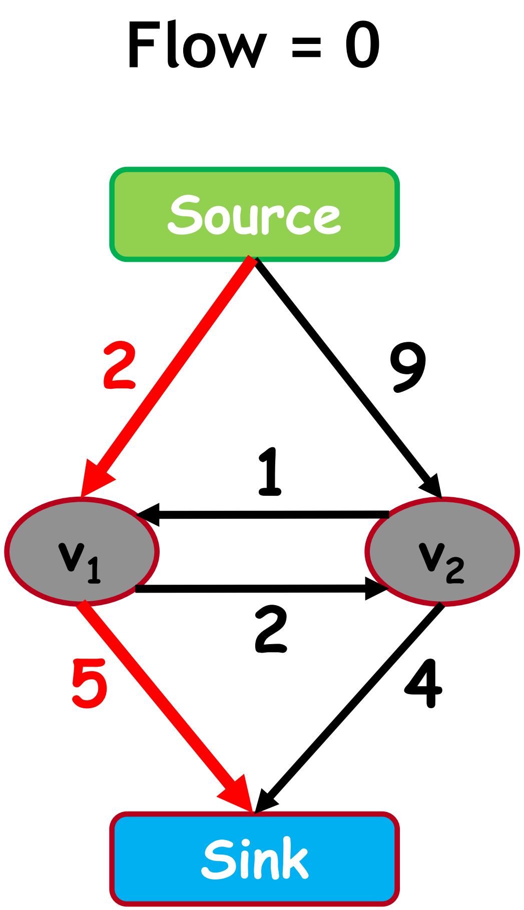



Augmenting Path Based Algorithms

1. Find path from source to sink with positive capacity
2. Push maximum possible flow through this path
3. Repeat until no path can be found

**Algorithms assume non-negative capacity**

# Maxflow Algorithms




Augmenting Path Based Algorithms

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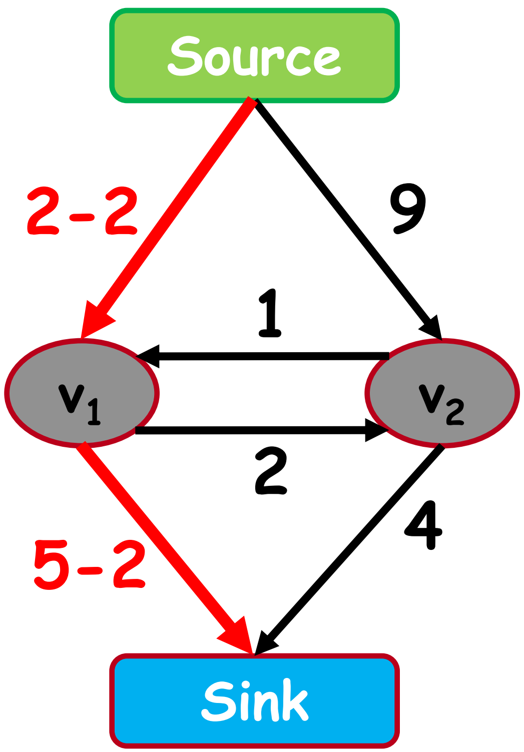
**Algorithms assume non-negative capacity**

# Maxflow Algorithms

Flow = 0 + 2




Augmenting Path Based Algorithms

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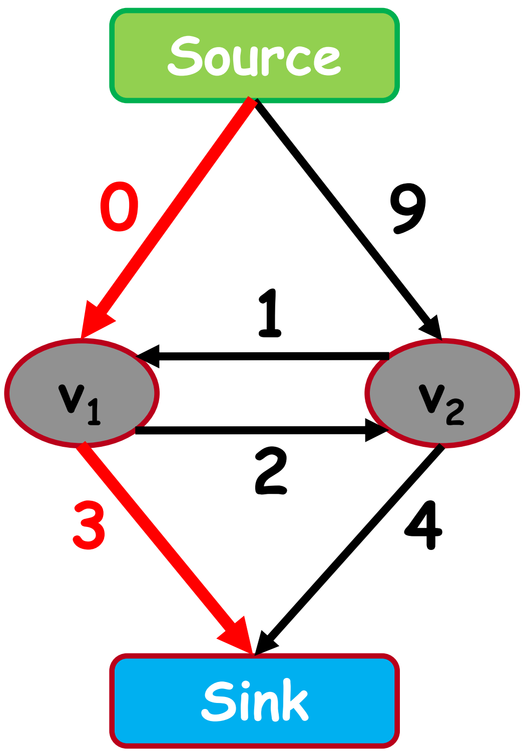
# Maxflow Algorithms

Flow = 2

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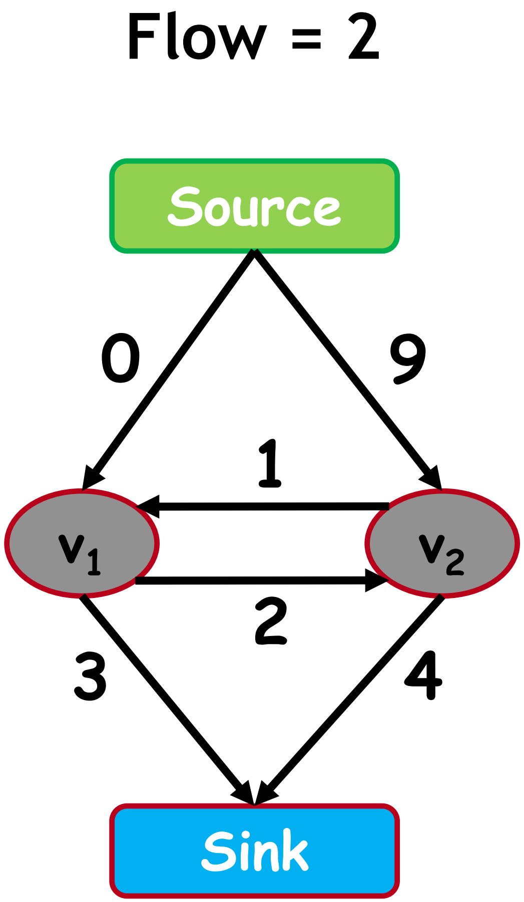


Augmenting Path Based Algorithms

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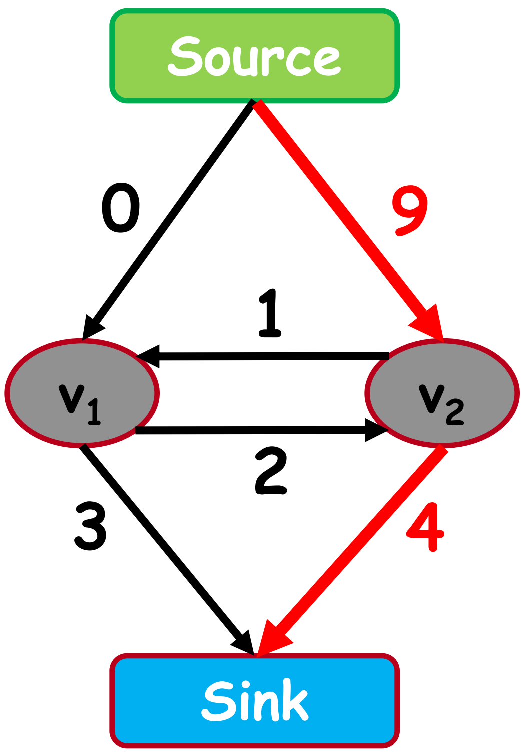
# Maxflow Algorithms

Flow = 2




## Augmenting Path Based Algorithms

1. Find path from source to sink with positive capacity
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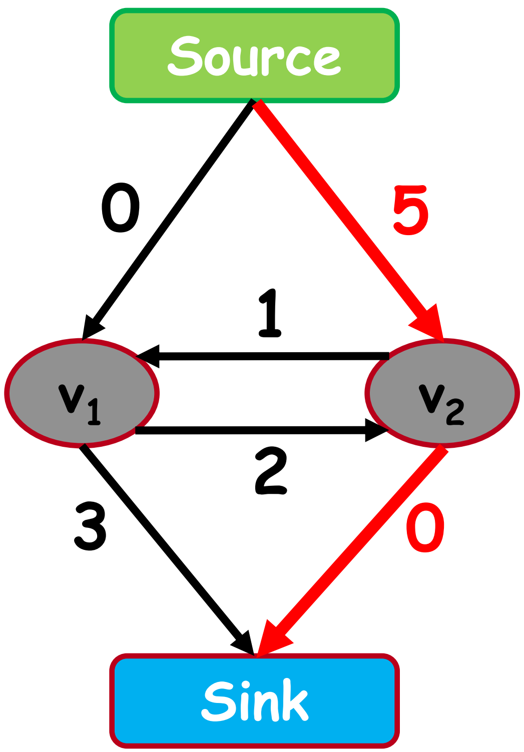
**Algorithms assume non-negative capacity**

# Maxflow Algorithms

Flow = 2 + 4




Augmenting Path Based Algorithms

1. Find path from source to sink with positive capacity
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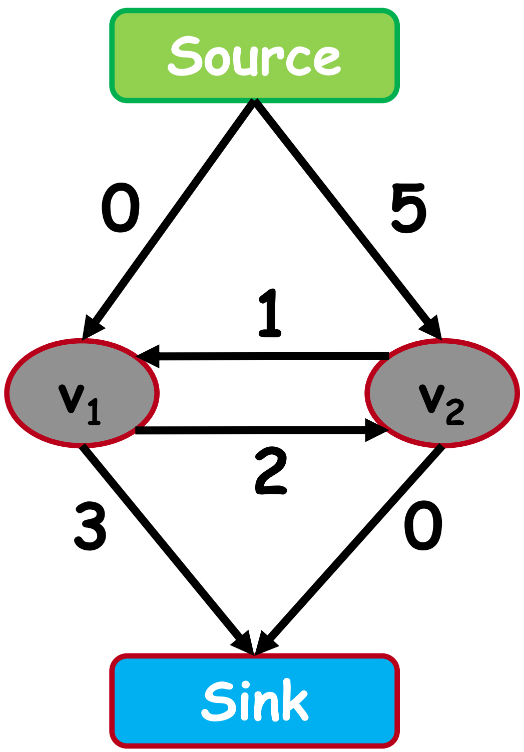
**Algorithms assume non-negative capacity**

Slide credit: Pushmeet Kohli

# Maxflow Algorithms

Flow = 6




Augmenting Path Based Algorithms

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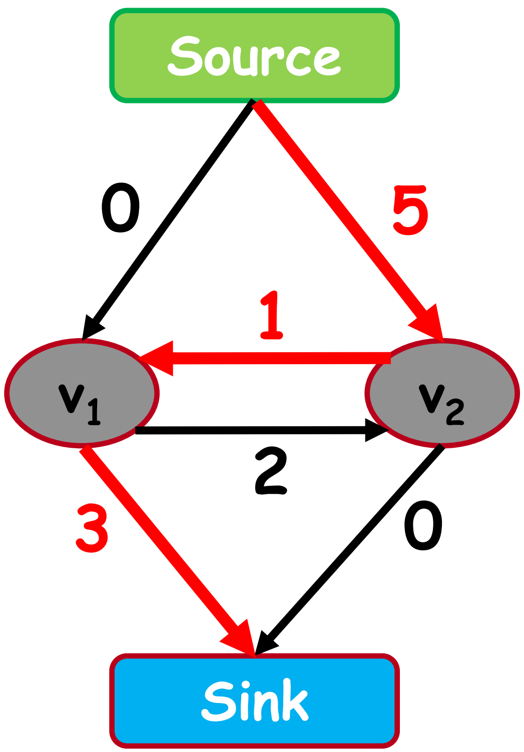
Algorithms assume non-negative capacity

# Maxflow Algorithms

Flow = 6




Augmenting Path Based Algorithms

1. Find path from source to sink with positive capacity
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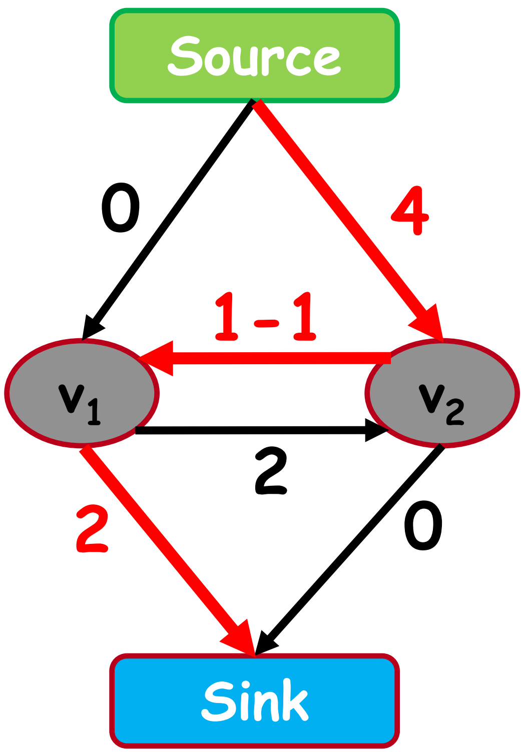
Algorithms assume non-negative capacity

# Maxflow Algorithms

Flow = 6 + 1




## Augmenting Path Based Algorithms

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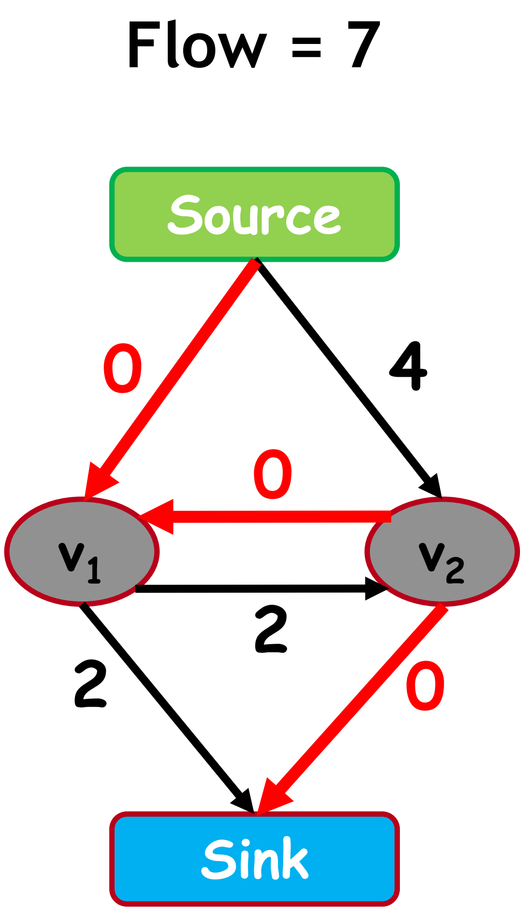
# Maxflow Algorithms




Augmenting Path Based Algorithms

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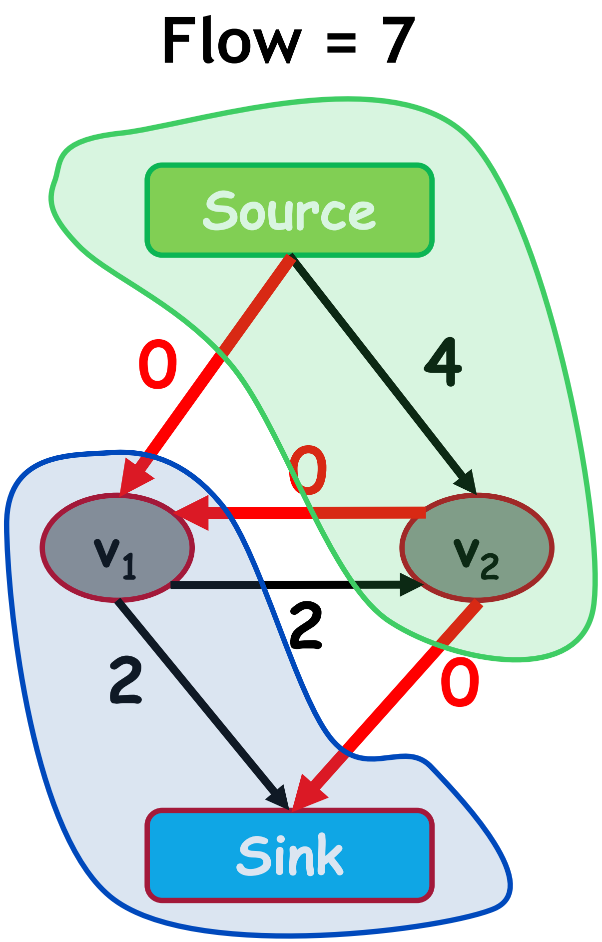
# Maxflow Algorithms




## Augmenting Path Based Algorithms

1. Find path from source to sink with positive capacity
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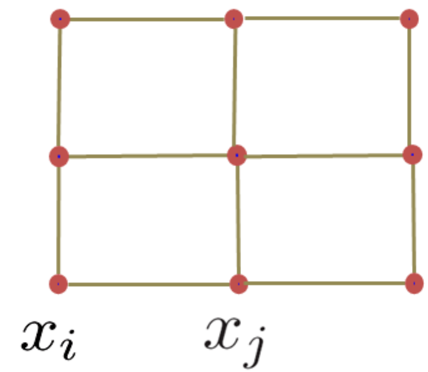
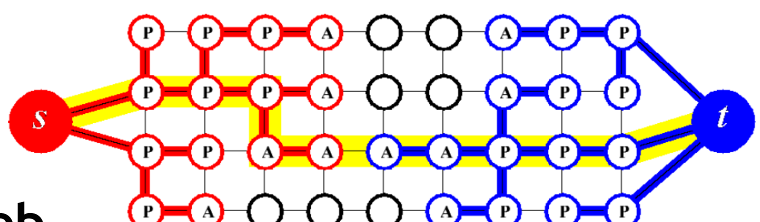
**Algorithms assume non-negative capacity**

# Applications: Maxflow in Computer Vision

- Specialized algorithms for vision problems
  - Grid graphs
  - Low connectivity (m ~ O(n))




- Dual search tree augmenting path algorithm
  [Boykov and Kolmogorov PAMI 2004]
  - Finds approximate shortest augmenting paths efficiently.
  - High worst-case time complexity.
  - Empirically outperforms other algorithms on vision problems.
  - Efficient code available on the web
    http://www.cs.ucl.ac.uk/staff/V.Kolmogorov/software.html




Slide credit: Pushmeet Kohli

# When Can s-t Graph Cuts Be Applied?

Regional term — Boundary term

$$E(L) = \sum_{p} E_p(L_p) + \sum_{pq \in N} E(L_p, L_q)$$

t-links — n-links

$$L_p \in \{s, t\}$$

- s-t graph cuts can only globally minimize binary energies that are submodular. [Boros & Hummer, 2002, Kolmogorov & Zabih, 2004]

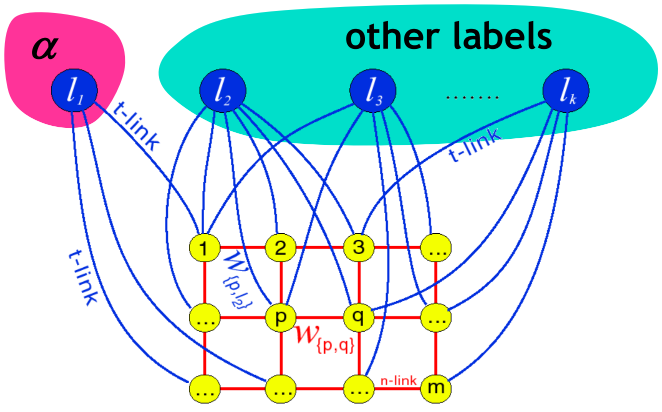
***E(L)*** **can be minimized by *s-t* graph cuts** $\Leftrightarrow$ $E(s,s) + E(t,t) \leq E(s,t) + E(t,s)$

Submodularity ("convexity")

- Non-submodular cases can still be addressed with some optimality guarantees.
  - Current research topic

# Topics of This Lecture

- Segmentation as Energy Minimization
  - Markov Random Fields
  - Energy formulation
- **Graph cuts for image segmentation**
  - **Basic idea**
  - **s-t Mincut algorithm**
  - **Extension to non-binary case**
- Applications
  - Interactive segmentation

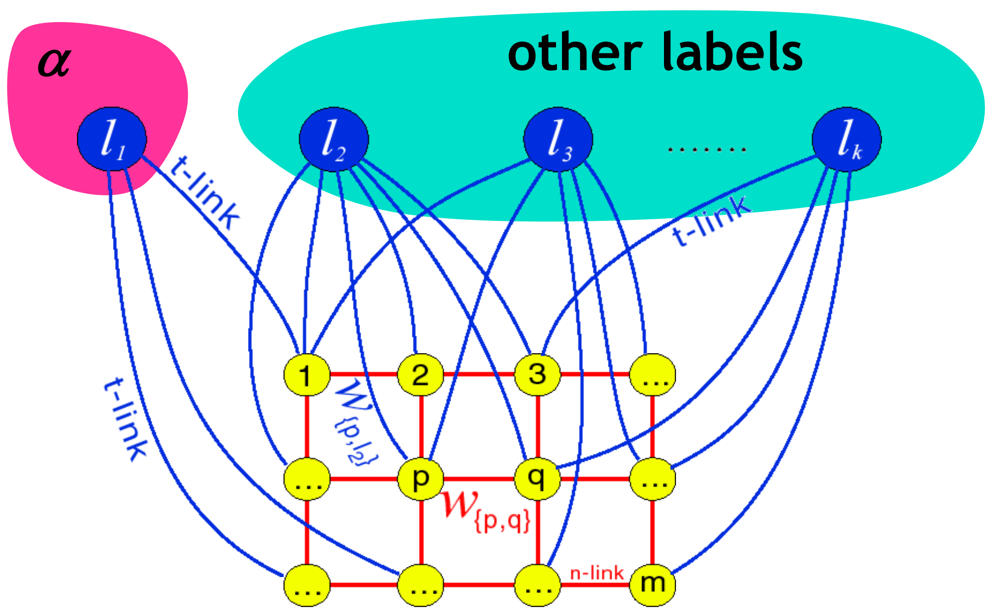


# Dealing with Non-Binary Cases

- Limitation to binary energies is often a nuisance.
  - ⇒ E.g. binary segmentation only…
- We would like to solve also multi-label problems.
  - The bad news: Problem is NP-hard with 3 or more labels!
- There exist some approximation algorithms which extend graph cuts to the multi-label case:
  - $\alpha$-Expansion
  - $\alpha\beta$-Swap
- They are no longer guaranteed to return the globally optimal result.
  - But $\alpha$-Expansion has a guaranteed approximation quality (2-approx) and converges in a few iterations.

# α-Expansion Move

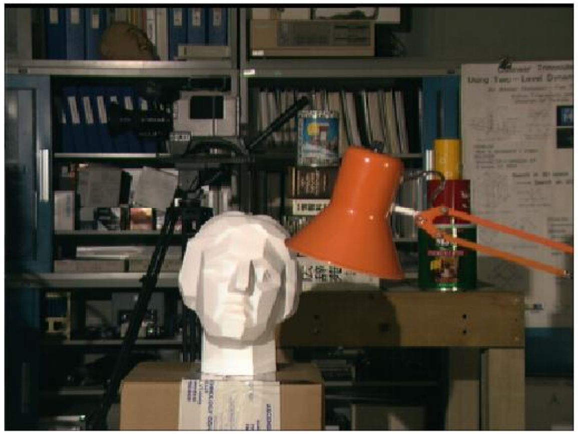
- **Basic idea:**
  - Break multi-way cut computation into a sequence of binary s-t cuts.

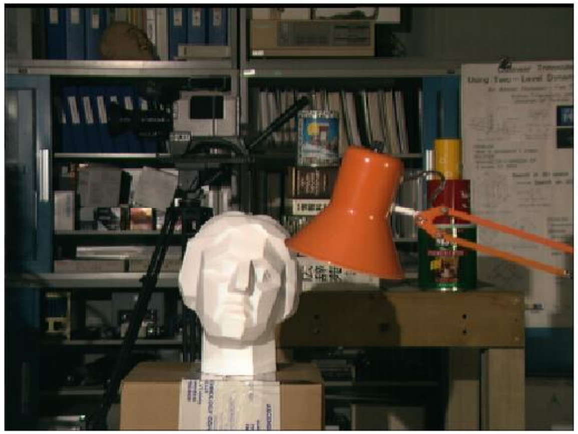


# $\alpha$-Expansion Algorithm

1. Start with any initial solution
2. For each label "$\alpha$" in any (e.g. random) order:
   1. Compute optimal $\alpha$-expansion move (s-t graph cuts).
   2. Decline the move if there is no energy decrease.
3. Stop when no expansion move would decrease energy.

# Example: Stereo Vision





Original pair of "stereo" images


Depth map

# α-Expansion Moves

- In each α-expansion a given label "α" grabs space from other labels

initial solution

-expansion

-expansion

-expansion

-expansion

-expansion

-expansion

-expansion

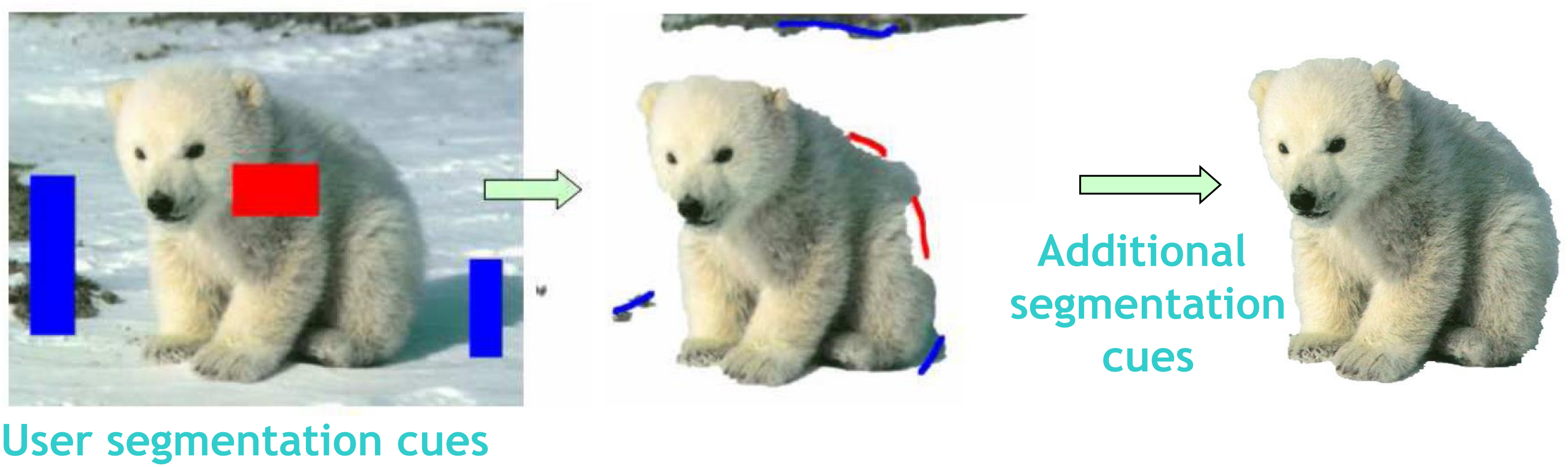
For each move, we choose the expansion that gives the largest decrease in the energy: ⇒ binary optimization problem

# Topics of This Lecture

- Segmentation as Energy Minimization
  - Markov Random Fields
  - Energy formulation
- Graph cuts for image segmentation
  - Basic idea
  - s-t Mincut algorithm
  - Extension to non-binary case
- **Applications**
  - **Interactive segmentation**

# GraphCut Applications: “GrabCut”

- Interactive Image Segmentation [Boykov & Jolly, ICCV’01]
  - Rough region cues sufficient
  - Segmentation boundary can be extracted from edges
- Procedure
  - User marks foreground and background regions with a brush.
  - This is used to create an initial segmentation which can then be corrected by additional brush strokes.

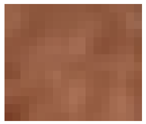




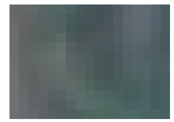

Slide credit: Matthieu Bray

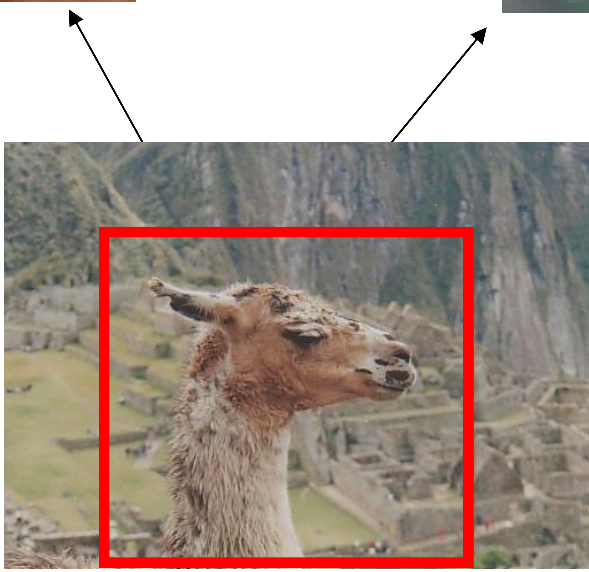
# GrabCut: Data Model

Foreground color



Background color





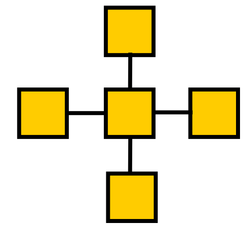
Global optimum of the energy

- **Obtained from interactive user input**
  - User marks foreground and background regions with a brush
  - Alternatively, user can specify a bounding box

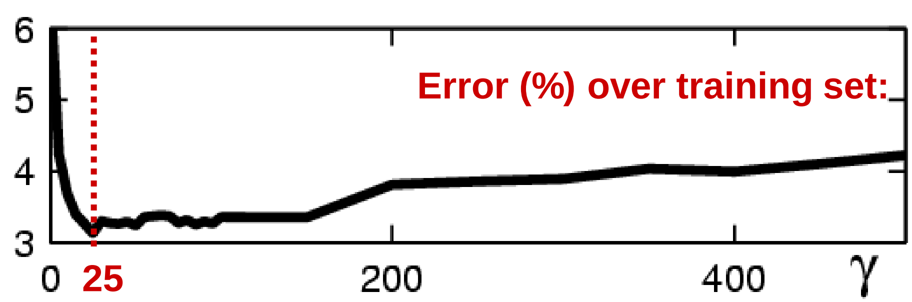
# GrabCut: Coherence Model

- **An object is a coherent set of pixels:**

$$\psi(x,y) = \gamma \sum_{(m,n)\in C} \delta[x_n \neq x_m] e^{-\beta \|y_m - y_n\|^2}$$

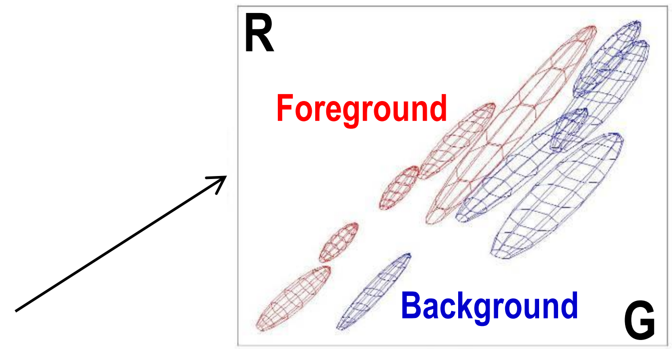


**How to choose $\gamma$?**

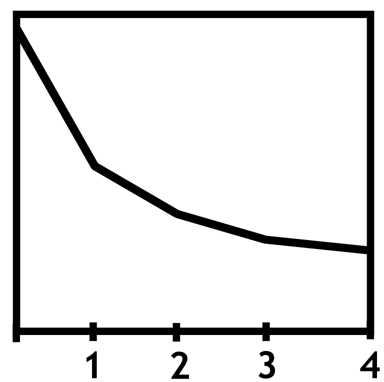


# Iterated Graph Cuts

Result

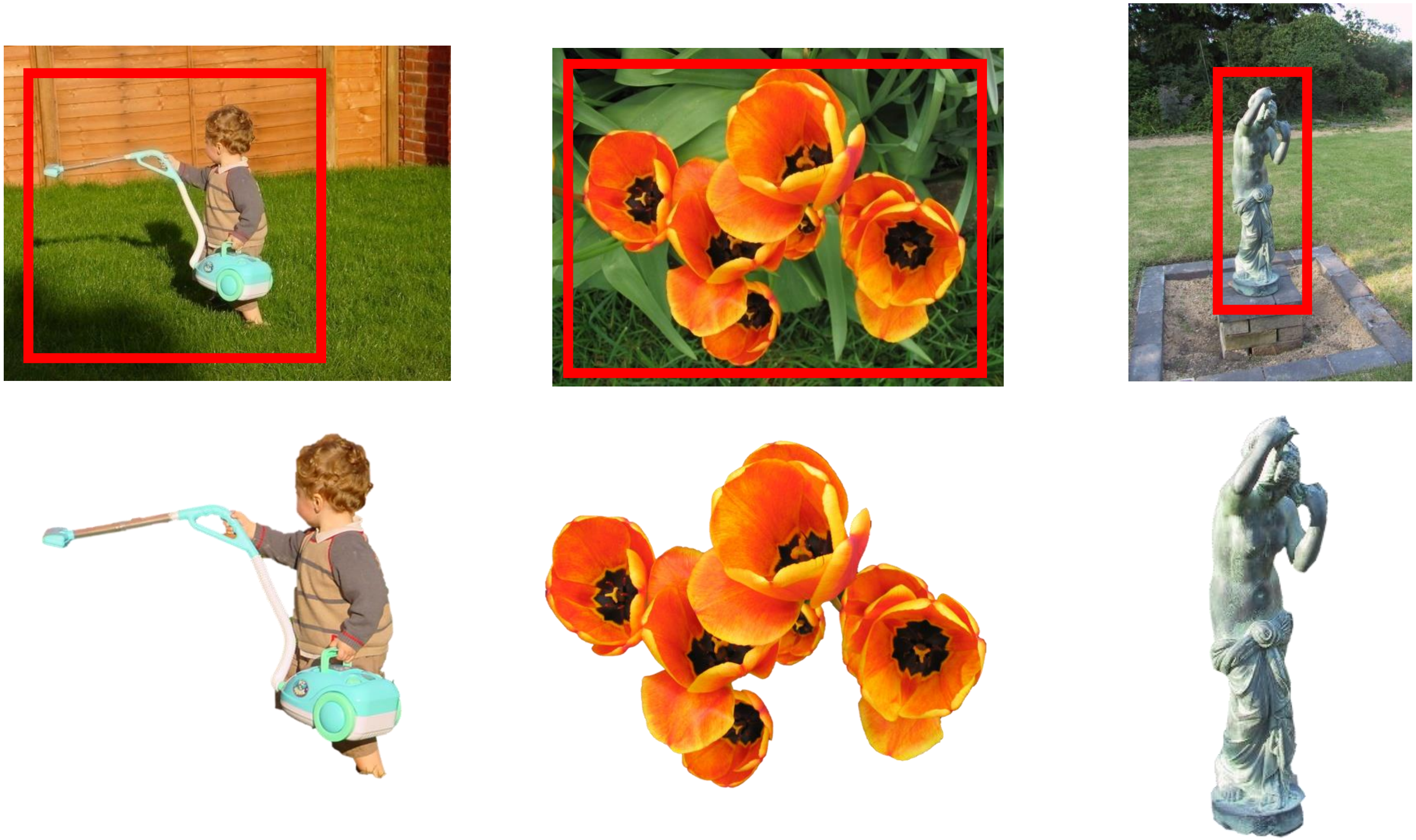



Color model
(Mixture of Gaussians)




Energy after
each iteration

# GrabCut: Example Results

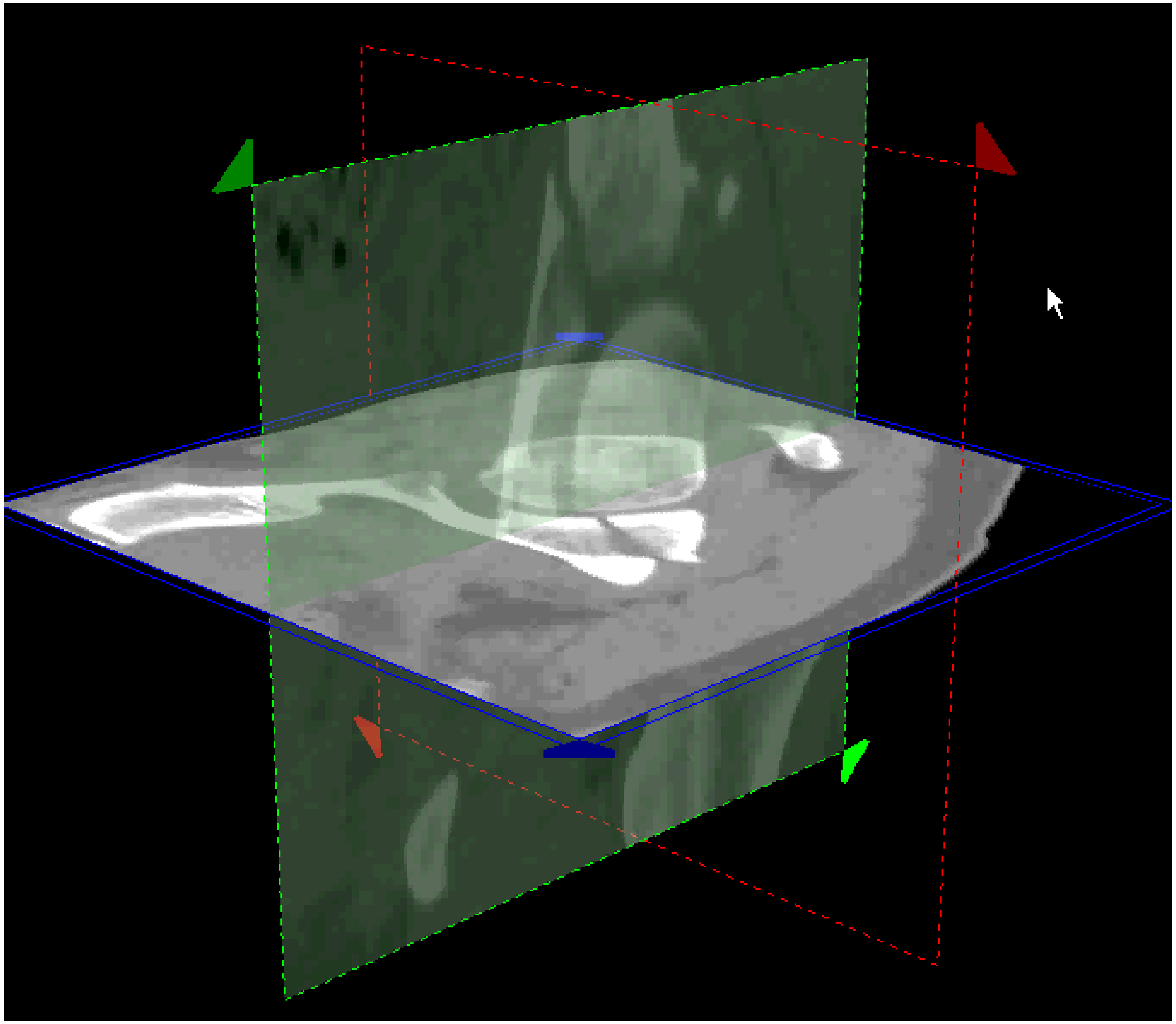


- ***This is included in the newest version of MS Office!***



Image source: Carsten Rother

# Applications: Interactive 3D Segmentation



# Summary: Graph Cuts Segmentation

- **Pros**
  - Powerful technique, based on probabilistic model (MRF).
  - Applicable for a wide range of problems.
  - Very efficient algorithms available for vision problems.
  - Becoming a de-facto standard for many segmentation tasks.

- **Cons/Issues**
  - Graph cuts can only solve a limited class of models
    - Submodular energy functions
    - Can capture only part of the expressiveness of MRFs
  - Only approximate algorithms available for multi-label case

# References and Further Reading

- A gentle introduction to Graph Cuts can be found in the following paper:
  - Y. Boykov, O. Veksler, Graph Cuts in Vision and Graphics: Theories and Applications. In *Handbook of Mathematical Models in Computer Vision*, edited by N. Paragios, Y. Chen and O. Faugeras, Springer, 2006.
- Read how the interactive segmentation is realized in MS Office 2010
  - C. Rother, V. Kolmogorov, Y. Boykov, A. Blake, Interactive Foreground Extraction using Graph Cut, Microsoft Research Tech Report MSR-TR-2011-46, March 2011
- Try the GraphCut implementation at

  http://www.cs.ucl.ac.uk/staff/V.Kolmogorov/software.html