

# Computer Vision - Lecture 7

## Segmentation as Energy Minimization

13.11.2014

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## Announcements

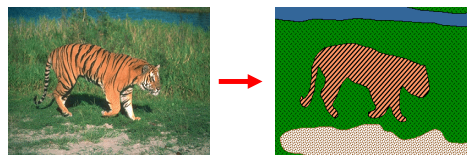
- Please don't forget to register for the exam!
  - On the Campus system

## Course Outline

- Image Processing Basics
- Segmentation
  - Segmentation and Grouping
  - Segmentation as Energy Minimization
- Recognition
  - Global Representations
  - Subspace representations
- Local Features & Matching
- Object Categorization
- 3D Reconstruction
- Motion and Tracking

## Recap: Image Segmentation

- Goal: identify groups of pixels that go together



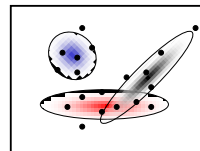
## Recap: K-Means Clustering

- Basic idea; randomly initialize the  $k$  cluster centers, and iterate between the two steps we just saw.
  1. Randomly initialize the cluster centers,  $c_1, \dots, c_k$
  2. Given cluster centers, determine points in each cluster
    - For each point  $p$ , find the closest  $c_i$ . Put  $p$  into cluster  $i$
  3. Given points in each cluster, solve for  $c_i$ 
    - Set  $c_i$  to be the mean of points in cluster  $i$
  4. If  $c_i$  have changed, repeat Step 2
- Properties
  - Will always converge to *some* solution
  - Can be a "local minimum"
    - Does not always find the global minimum of objective function:

$$\sum_{\text{clusters } i} \sum_{\text{points } p \text{ in cluster } i} \|p - c_i\|^2$$



## Recap: Expectation Maximization (EM)



- Goal
  - Find blob parameters  $\theta$  that maximize the likelihood function:

$$p(\text{data}|\theta) = \prod_{n=1}^N p(x_n|\theta)$$

- Approach:
  1. E-step: given current guess of blobs, compute ownership of each point
  2. M-step: given ownership probabilities, update blobs to maximize likelihood function
  3. Repeat until convergence

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## Recap: EM Algorithm

see lecture Machine Learning!

- Expectation-Maximization (EM) Algorithm
  - E-Step: softly assign samples to mixture components
 
$$\gamma_j(x_n) \leftarrow \frac{\pi_j \mathcal{N}(x_n | \mu_j, \Sigma_j)}{\sum_{k=1}^K \pi_k \mathcal{N}(x_n | \mu_k, \Sigma_k)} \quad \forall j = 1, \dots, K, \quad n = 1, \dots, N$$
  - M-Step: re-estimate the parameters (separately for each mixture component) based on the soft assignments
 
$$\hat{N}_j \leftarrow \sum_{n=1}^N \gamma_j(x_n) = \text{soft number of samples labeled } j$$

$$\hat{\pi}_j^{\text{new}} \leftarrow \frac{\hat{N}_j}{N}$$

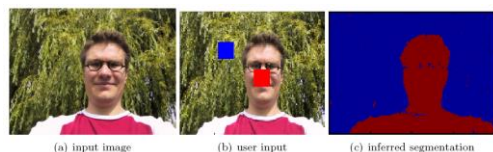
$$\hat{\mu}_j^{\text{new}} \leftarrow \frac{1}{\hat{N}_j} \sum_{n=1}^N \gamma_j(x_n) x_n$$

$$\hat{\Sigma}_j^{\text{new}} \leftarrow \frac{1}{\hat{N}_j} \sum_{n=1}^N \gamma_j(x_n) (x_n - \hat{\mu}_j^{\text{new}})(x_n - \hat{\mu}_j^{\text{new}})^T$$

Slide adapted from Bernt Schiele B. Leibe 7

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## MoG Color Models for Image Segmentation



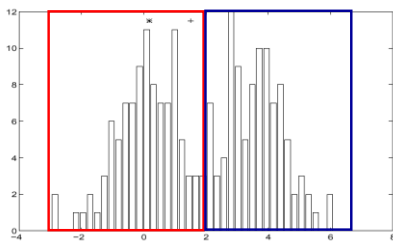
- User assisted image segmentation
  - User marks two regions for foreground and background.
  - Learn a MoG model for the color values in each region.
  - Use those models to classify all other pixels.

⇒ Simple segmentation procedure (building block for more complex applications)

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## Finding Modes in a Histogram



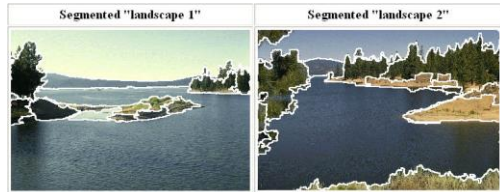
- How many modes are there?
  - Mode = local maximum of the density of a given distribution
  - Easy to see, hard to compute

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## Mean-Shift Segmentation

- An advanced and versatile technique for clustering-based segmentation



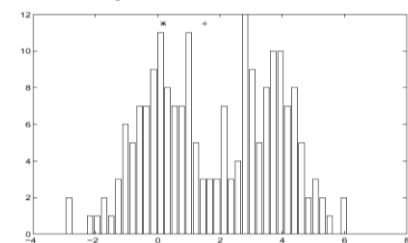
<http://www.caip.rutgers.edu/~comanici/MSPAMI/msPamiResults.html>

D. Comaniciu and P. Meer, Mean Shift: A Robust Approach toward Feature Space Analysis, PAMI 2002.

Slide credit: Svetlana Lazebnik B. Leibe 13

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## Mean-Shift Algorithm

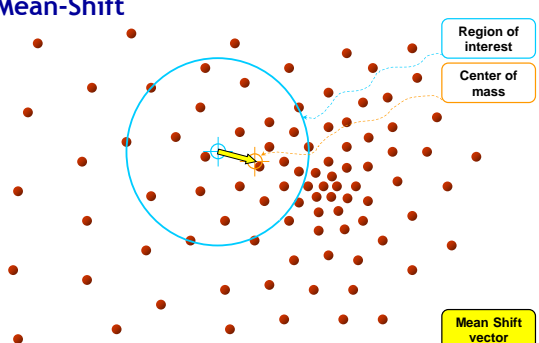


- Iterative Mode Search
  - Initialize random seed, and window W
  - Calculate center of gravity (the "mean") of W:  $\sum_{x \in W} xH(x)$
  - Shift the search window to the mean
  - Repeat Step 2 until convergence

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## Mean-Shift

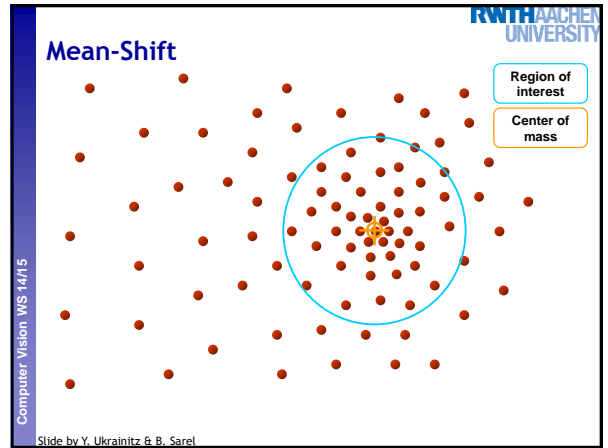
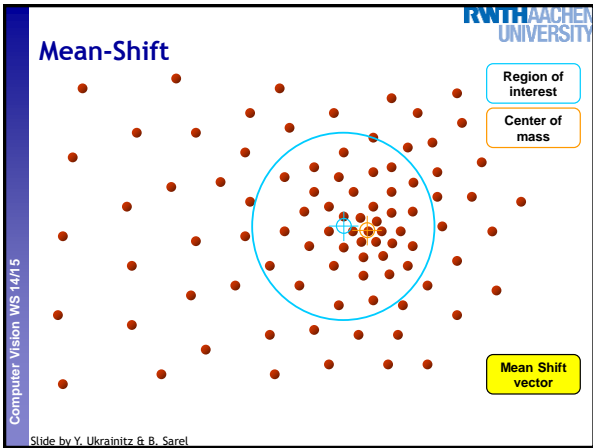
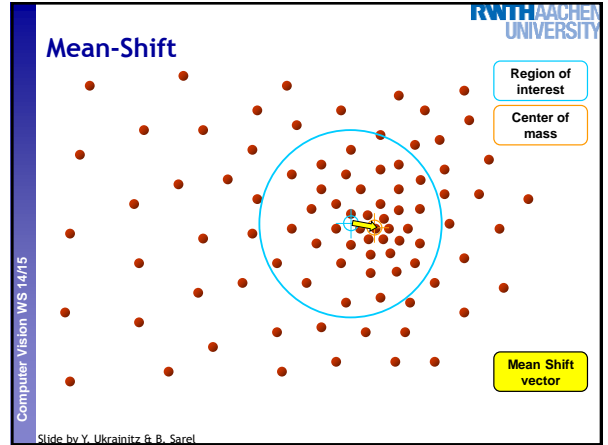
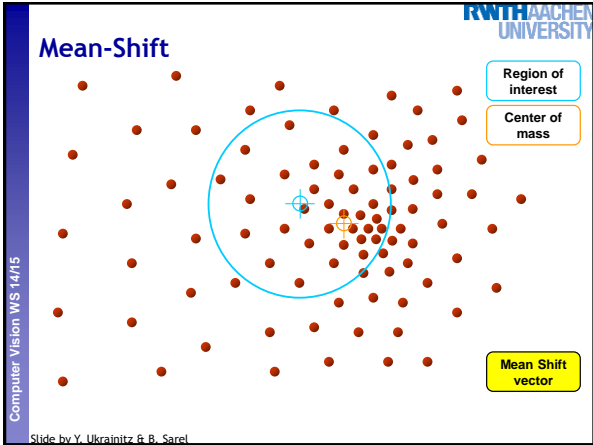
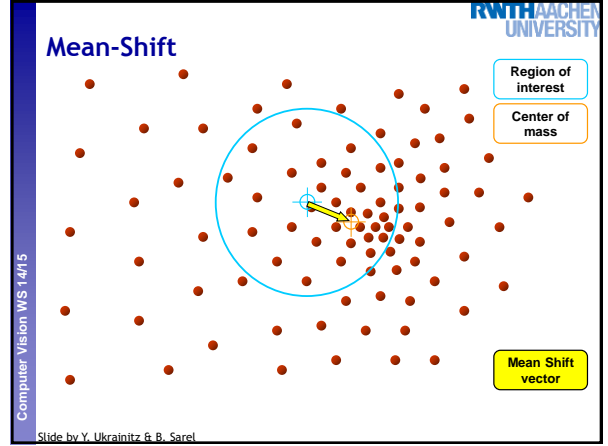
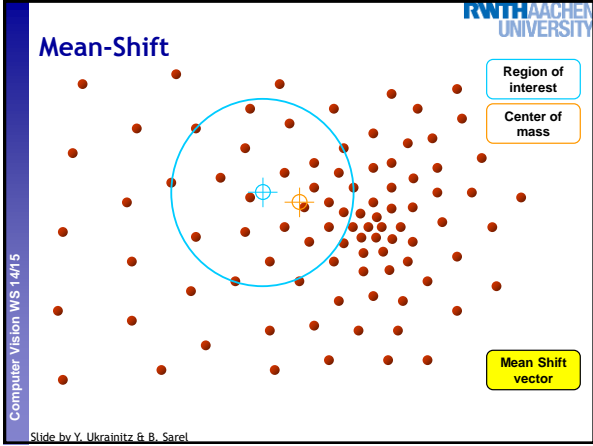


Region of interest

Center of mass

Mean Shift vector

Slide by Y. Ukrainitz & B. Sarel



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## Real Modality Analysis

Tessellate the space with windows      Run the procedure in parallel

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## Real Modality Analysis

The blue data points were traversed by the windows towards the mode.

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## Mean-Shift Clustering

- Cluster: all data points in the attraction basin of a mode
- Attraction basin: the region for which all trajectories lead to the same mode

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## Mean-Shift Clustering/Segmentation

- Find features (color, gradients, texture, etc)
- Initialize windows at individual pixel locations
- Perform mean shift for each window until convergence
- Merge windows that end up near the same "peak" or mode

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## Mean-Shift Segmentation Results

<http://www.caip.rutgers.edu/~comanici/MSPAMI/msPamiResults.html>

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## More Results

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## More Results

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## Problem: Computational Complexity

- Need to shift many windows...
- Many computations will be redundant.

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## Speedups: Basin of Attraction

1. Assign all points within radius  $r$  of end point to the mode.

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## Speedups

2. Assign all points within radius  $r/c$  of the search path to the mode.

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## Summary Mean-Shift

- **Pros**
  - General, application-independent tool
  - Model-free, does not assume any prior shape (spherical, elliptical, etc.) on data clusters
  - Just a single parameter (window size  $h$ )
    - $h$  has a physical meaning (unlike k-means)
  - Finds variable number of modes
  - Robust to outliers
- **Cons**
  - Output depends on window size
  - Window size (bandwidth) selection is not trivial
  - Computationally (relatively) expensive ( $\sim 2s/\text{image}$ )
  - Does not scale well with dimension of feature space

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## Back to the Image Segmentation Problem...

- Goal: identify groups of pixels that go together

- Up to now, we have focused on ways to group pixels into image segments based on their appearance...
  - Segmentation as clustering.
- We also want to enforce region constraints.
  - Spatial consistency
  - Smooth borders

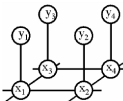
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## Topics of This Lecture

- Segmentation as Energy Minimization
  - Markov Random Fields
  - Energy formulation
- Graph cuts for image segmentation
  - Basic idea
  - s-t Mincut algorithm
  - Extension to non-binary case
- Applications
  - Interactive segmentation

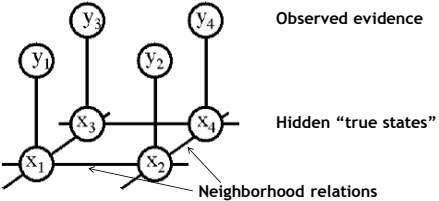


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## Markov Random Fields

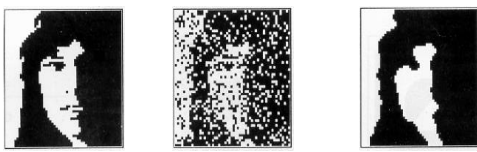
- Allow rich probabilistic models for images
- But built in a local, modular way
  - Learn local effects, get global effects out



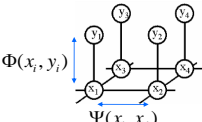
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## MRF Nodes as Pixels



Original image      Degraded image      Reconstruction from MRF modeling pixel neighborhood statistics



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## Network Joint Probability

$$p(\mathbf{x}, \mathbf{y}) = \prod_i \Phi(x_i, y_i) \prod_{i,j} \Psi(x_i, x_j)$$

Scene Image      Image-scene compatibility function      Scene-scene compatibility function

Local observations      Neighboring scene nodes

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## Energy Formulation

- Joint probability
 
$$p(\mathbf{x}, \mathbf{y}) = \prod_i \Phi(x_i, y_i) \prod_{i,j} \Psi(x_i, x_j)$$
- Maximizing the joint probability is the same as minimizing the negative log
 
$$-\log p(\mathbf{x}, \mathbf{y}) = -\sum_i \log \Phi(x_i, y_i) - \sum_{i,j} \log \Psi(x_i, x_j)$$

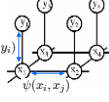
$$E(\mathbf{x}, \mathbf{y}) = \sum_i \phi(x_i, y_i) + \sum_{i,j} \psi(x_i, x_j)$$
- This is similar to free-energy problems in statistical mechanics (spin glass theory). We therefore draw the analogy and call  $E$  an *energy function*.
- $\phi$  and  $\psi$  are called *potentials*.

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## Energy Formulation

- Energy function
 
$$E(\mathbf{x}, \mathbf{y}) = \sum_i \underbrace{\phi(x_i, y_i)}_{\text{Single-node potentials}} + \sum_{i,j} \underbrace{\psi(x_i, x_j)}_{\text{Pairwise potentials}}$$
- Single-node potentials  $\phi$  (“unary potentials”)
  - Encode local information about the given pixel/patch
  - How likely is a pixel/patch to belong to a certain class (e.g. foreground/background)?
- Pairwise potentials  $\psi$ 
  - Encode neighborhood information
  - How different is a pixel/patch’s label from that of its neighbor? (e.g. based on intensity/color/texture difference, edges)



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**Energy Minimization**

- **Goal:**
  - Infer the optimal labeling of the MRF.
- Many inference algorithms are available, e.g.
  - Gibbs sampling, simulated annealing
  - Iterated conditional modes (ICM)
  - Variational methods
  - Belief propagation
  - Graph cuts
- Recently, Graph Cuts have become a popular tool
  - Only suitable for a certain class of energy functions
  - But the solution can be obtained very fast for typical vision problems (~1MPixel/sec).

see lecture Machine Learning!

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**Topics of This Lecture**

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- Applications
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**Graph Cuts for Optimal Boundary Detection**

- Idea: convert MRF into source-sink graph

hard constraint

a cut

hard constraint

$$w_{ij} = \exp\left\{-\frac{\Delta_{ij}}{2\sigma^2}\right\}$$

Minimum cost cut can be computed in polynomial time (max-flow/min-cut algorithms)

Slide credit: Yuri Boykov

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[Boykov & Jolly, ICCV'01]

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**Simple Example of Energy**

$$E(\mathbf{x}, \mathbf{y}) = \sum_i \phi_i(x_i) + \sum_{i,j} w_{ij} \cdot \delta(x_i \neq x_j)$$

Unary terms      Pairwise terms

$$w_{ij} = \exp\left\{-\frac{\Delta_{ij}}{2\sigma^2}\right\}$$

$x \in \{s, t\}$   
(binary object segmentation)

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**Adding Regional Properties**

Regional bias example

Suppose  $I^s$  and  $I^t$  are given "expected" intensities of object and background

$$\phi_i(s) \propto \exp\left(-\|I_i - I^s\|^2 / 2\sigma^2\right)$$

$$\phi_i(t) \propto \exp\left(-\|I_i - I^t\|^2 / 2\sigma^2\right)$$

**NOTE: hard constrains are not required, in general.**

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[Boykov & Jolly, ICCV'01]

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**Adding Regional Properties**

"expected" intensities of object and background  $I^s$  and  $I^t$  can be re-estimated

**EM-style optimization**

$$\phi_i(s) \propto \exp\left(-\|I_i - I^s\|^2 / 2\sigma^2\right)$$

$$\phi_i(t) \propto \exp\left(-\|I_i - I^t\|^2 / 2\sigma^2\right)$$

Slide credit: Yuri Boykov

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[Boykov & Jolly, ICCV'01]

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## Adding Regional Properties

- More generally, regional bias can be based on any intensity models of object and background

$\phi_i(L_i) = -\log p(I_i | L_i)$

given object and background intensity histograms

Slide credit: Yuri Boykov B. Leibe [Boykov & Jolly, ICCV'01]

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## How to Set the Potentials? Some Examples

- Color potentials
  - e.g., modeled with a Mixture of Gaussians
 
$$\phi(x_i, y_i; \theta_\phi) = \log \sum_k \theta_\phi(x_i, k) p(k | x_i) \mathcal{N}(y_i; \mu_k, \Sigma_k)$$
- Edge potentials
  - E.g., a "contrast sensitive Potts model"
 
$$\psi(x_i, x_j, g_{ij}(\mathbf{y}); \theta_\psi) = -\theta_\psi g_{ij}(\mathbf{y}) \delta(x_i \neq x_j)$$
 where
 
$$g_{ij}(\mathbf{y}) = e^{-\beta \|y_i - y_j\|^2} \quad \beta = \frac{1}{2} (\text{avg}(\|y_i - y_j\|^2))^{-1}$$
- Parameters  $\theta_\phi, \theta_\psi$  need to be learned, too!

Slide credit: B. Leibe [Shotton & Winn, ECCV'06]

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## Example: MRF for Image Segmentation

- MRF structure

Data (D)    Unary likelihood    Pair-wise Terms    MAP Solution

Slide adapted from Phil Torr

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## Topics of This Lecture

- Segmentation as Energy Minimization
  - Markov Random Fields
  - Energy formulation
- Graph cuts for image segmentation
  - Basic idea
  - s-t Mincut algorithm
  - Extension to non-binary case
- Applications
  - Interactive segmentation

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## How Does it Work? The s-t-Mincut Problem

**Graph (V, E, C)**  
 Vertices  $V = \{v_1, v_2, \dots, v_n\}$   
 Edges  $E = \{(v_1, v_2), \dots\}$   
 Costs  $C = \{c_{(1,2)}, \dots\}$

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## The s-t-Mincut Problem

What is an st-cut?

An st-cut (S,T) divides the nodes between source and sink.

What is the cost of a st-cut?

Sum of cost of all edges going from S to T

**5 + 2 + 9 = 16**

Slide credit: Pushmeet Kohli B. Leibe



**The s-t-Mincut Problem**

What is an st-cut?

An st-cut (S,T) divides the nodes between source and sink.

What is the cost of a st-cut?

Sum of cost of all edges going from S to T

What is the st-mincut?

st-cut with the minimum cost

$2 + 1 + 4 = 7$

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**How to Compute the s-t-Mincut?**

Solve the dual maximum flow problem

Compute the maximum flow between Source and Sink

Constraints

Edges: Flow < Capacity

Nodes: Flow in = Flow out

Min-cut/Max-flow Theorem

In every network, the maximum flow equals the cost of the st-mincut

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**History of Maxflow Algorithms**

Augmenting Path and Push-Relabel

year	discoverer(s)	bound
1951	Dantzig	$O(n^2 m U)$
1955	Ford & Fulkerson	$O(m^2 U)$
1970	Dinitz	$O(n^2 m)$
1972	Edmonds & Karp	$O(m^2 \log U)$
1973	Dinitz	$O(nm \log U)$
1974	Karzanov	$O(n^3)$
1977	Cherkassky	$O(n^2 m \log^2 U)$
1980	Gall & Naamad	$O(nm \log^2 n)$
1983	Sleator & Tarjan	$O(nm \log n)$
1986	Goldberg & Tarjan	$O(nm \log(n^2/m))$
1987	Ahuja & Orlin	$O(nm + n^2 \log U)$
1987	Ahuja et al.	$O(nm \log(n \sqrt{U}/m))$
1989	Cheriyán & Hagerup	$E(nm + n^2 \log^2 n)$
1990	Cheriyán et al.	$O(n^2 \log n)$
1990	Alon	$O(nm + n^{2.5} \log n)$
1992	King et al.	$O(nm + n^{2.5})$
1993	Phillips & Westbrook	$O(nm(\log_{m/n} n + \log^{2.5} n))$
1994	King et al.	$O(nm \log_{m/n} n)$
1997	Goldberg & Rao	$O(n^{2.5} \log(n^2/m) \log U)$
		$O(n^{2.5} m \log(n^2/m) \log U)$

$n$ : #nodes  
 $m$ : #edges  
 $U$ : maximum edge weight

Algorithms assume non-negative edge weights

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Slide credit: Andrew Goldberg

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**Maxflow Algorithms**

Flow = 0

Augmenting Path Based Algorithms

1. Find path from source to sink with positive capacity
2. Push maximum possible flow through this path
3. Repeat until no path can be found

Algorithms assume non-negative capacity

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**Maxflow Algorithms**

Flow = 0

Augmenting Path Based Algorithms

1. Find path from source to sink with positive capacity
2. Push maximum possible flow through this path
3. Repeat until no path can be found

Algorithms assume non-negative capacity

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**Maxflow Algorithms**

Flow = 0 + 2

Augmenting Path Based Algorithms

1. Find path from source to sink with positive capacity
2. Push maximum possible flow through this path
3. Repeat until no path can be found

Algorithms assume non-negative capacity

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## Maxflow Algorithms

Flow = 2

Augmenting Path Based Algorithms

1. Find path from source to sink with positive capacity
2. Push maximum possible flow through this path
3. Repeat until no path can be found

Algorithms assume non-negative capacity

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## Maxflow Algorithms

Flow = 2

Augmenting Path Based Algorithms

1. Find path from source to sink with positive capacity
2. Push maximum possible flow through this path
3. Repeat until no path can be found

Algorithms assume non-negative capacity

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## Maxflow Algorithms

Flow = 2

Augmenting Path Based Algorithms

1. Find path from source to sink with positive capacity
2. Push maximum possible flow through this path
3. Repeat until no path can be found

Algorithms assume non-negative capacity

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## Maxflow Algorithms

Flow = 2 + 4

Augmenting Path Based Algorithms

1. Find path from source to sink with positive capacity
2. Push maximum possible flow through this path
3. Repeat until no path can be found

Algorithms assume non-negative capacity

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## Maxflow Algorithms

Flow = 6

Augmenting Path Based Algorithms

1. Find path from source to sink with positive capacity
2. Push maximum possible flow through this path
3. Repeat until no path can be found

Algorithms assume non-negative capacity

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## Maxflow Algorithms

Flow = 6

Augmenting Path Based Algorithms

1. Find path from source to sink with positive capacity
2. Push maximum possible flow through this path
3. Repeat until no path can be found

Algorithms assume non-negative capacity

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## Maxflow Algorithms

Flow = 6 + 1

Augmenting Path Based Algorithms

1. Find path from source to sink with positive capacity
2. Push maximum possible flow through this path
3. Repeat until no path can be found

Algorithms assume non-negative capacity

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## Maxflow Algorithms

Flow = 7

Augmenting Path Based Algorithms

1. Find path from source to sink with positive capacity
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Algorithms assume non-negative capacity

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## Maxflow Algorithms

Flow = 7

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Algorithms assume non-negative capacity

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## Applications: Maxflow in Computer Vision

- Specialized algorithms for vision problems
  - Grid graphs
  - Low connectivity ( $m - O(n)$ )
- Dual search tree augmenting path algorithm [Boykov and Kolmogorov PAMI 2004]
  - Finds approximate shortest augmenting paths efficiently.
  - High worst-case time complexity.
  - Empirically outperforms other algorithms on vision problems.
  - Efficient code available on the web <http://www.cs.ucl.ac.uk/staff/V.Kolmogorov/software.html>

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## When Can s-t Graph Cuts Be Applied?

$$E(L) = \sum_p E_p(L_p) + \sum_{p,q \in N} E(L_p, L_q)$$

Regional term
Boundary term  
t-links
n-links

$L_p \in \{s, t\}$

- s-t graph cuts can only globally minimize **binary energies** that are **submodular**. [Boros & Hummer, 2002, Kolmogorov & Zabih, 2004]

$E(L) \text{ can be minimized by s-t graph cuts} \iff E(s, s) + E(t, t) \leq E(s, t) + E(t, s)$ 

Submodularity ("convexity")

- Non-submodular cases can still be addressed with some optimality guarantees.
  - Current research topic

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## Topics of This Lecture

- Segmentation as Energy Minimization
  - Markov Random Fields
  - Energy formulation
- Graph cuts for image segmentation
  - Basic idea
  - s-t Mincut algorithm
  - Extension to non-binary case
- Applications
  - Interactive segmentation

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## Dealing with Non-Binary Cases

- Limitation to binary energies is often a nuisance.
  - ⇒ E.g. binary segmentation only...
- We would like to solve also multi-label problems.
  - The bad news: Problem is NP-hard with 3 or more labels!
- There exist some approximation algorithms which extend graph cuts to the multi-label case:
  - $\alpha$ -Expansion
  - $\alpha\beta$ -Swap
- They are no longer guaranteed to return the globally optimal result.
  - But  $\alpha$ -Expansion has a guaranteed approximation quality (2-approx) and converges in a few iterations.

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## $\alpha$ -Expansion Move

- Basic idea:
  - Break multi-way cut computation into a sequence of binary s-t cuts.

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## $\alpha$ -Expansion Algorithm

1. Start with any initial solution
2. For each label " $\alpha$ " in any (e.g. random) order:
  1. Compute optimal  $\alpha$ -expansion move (s-t graph cuts).
  2. Decline the move if there is no energy decrease.
3. Stop when no expansion move would decrease energy.

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## Example: Stereo Vision

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## $\alpha$ -Expansion Moves

- In each  $\alpha$ -expansion a given label " $\alpha$ " grabs space from other labels

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For each move, we choose the expansion that gives the largest decrease in the energy: ⇒ binary optimization problem

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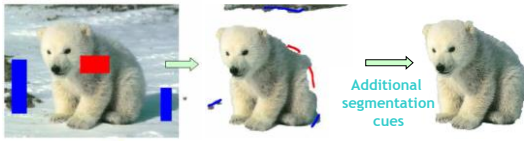
80

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## GraphCut Applications: "GrabCut"

- Interactive Image Segmentation [Boykov & Jolly, ICCV'01]
  - Rough region cues sufficient
  - Segmentation boundary can be extracted from edges
- Procedure
  - User marks foreground and background regions with a brush.
  - This is used to create an initial segmentation which can then be corrected by additional brush strokes.



User segmentation cues


Additional segmentation cues

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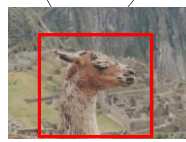

## GrabCut: Data Model

Foreground color



Background color



Global optimum of the energy

- Obtained from interactive user input
  - User marks foreground and background regions with a brush
  - Alternatively, user can specify a bounding box


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
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## GrabCut: Coherence Model

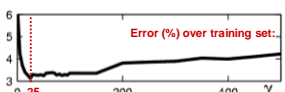
- An object is a coherent set of pixels:

$$\psi(x, y) = \gamma \sum_{(m,n) \in C} \delta[x_n \neq x_m] e^{-\beta \|y_m - y_n\|^2}$$






How to choose  $\gamma$ ?



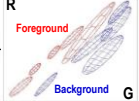
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## Iterated Graph Cuts



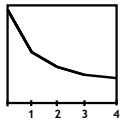
R



G

Color model (Mixture of Gaussians)




Result






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## GrabCut: Example Results

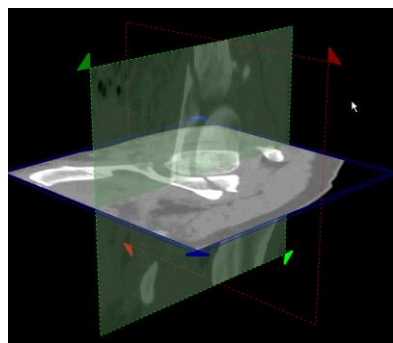




- This is included in the newest version of MS Office!

Computer Vision WS 14/15 B. Leibe Image source: Carsten Rother 85

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## Applications: Interactive 3D Segmentation



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## Summary: Graph Cuts Segmentation

- Pros
  - Powerful technique, based on probabilistic model (MRF).
  - Applicable for a wide range of problems.
  - Very efficient algorithms available for vision problems.
  - Becoming a de-facto standard for many segmentation tasks.
- Cons/Issues
  - Graph cuts can only solve a limited class of models
    - Submodular energy functions
    - Can capture only part of the expressiveness of MRFs
  - Only approximate algorithms available for multi-label case

## References and Further Reading

- A gentle introduction to Graph Cuts can be found in the following paper:
  - Y. Boykov, O. Veksler, [Graph Cuts in Vision and Graphics: Theories and Applications](#). In *Handbook of Mathematical Models in Computer Vision*, edited by N. Paragios, Y. Chen and O. Faugeras, Springer, 2006.
- Read how the interactive segmentation is realized in MS Office 2010
  - C. Rother, V. Kolmogorov, Y. Boykov, A. Blake, [Interactive Foreground Extraction using Graph Cut](#), Microsoft Research Tech Report MSR-TR-2011-46, March 2011
- Try the GraphCut implementation at <http://www.cs.ucl.ac.uk/staff/V.Kolmogorov/software.html>