Computer Vision – Lecture 5

Structure Extraction

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Course Outline

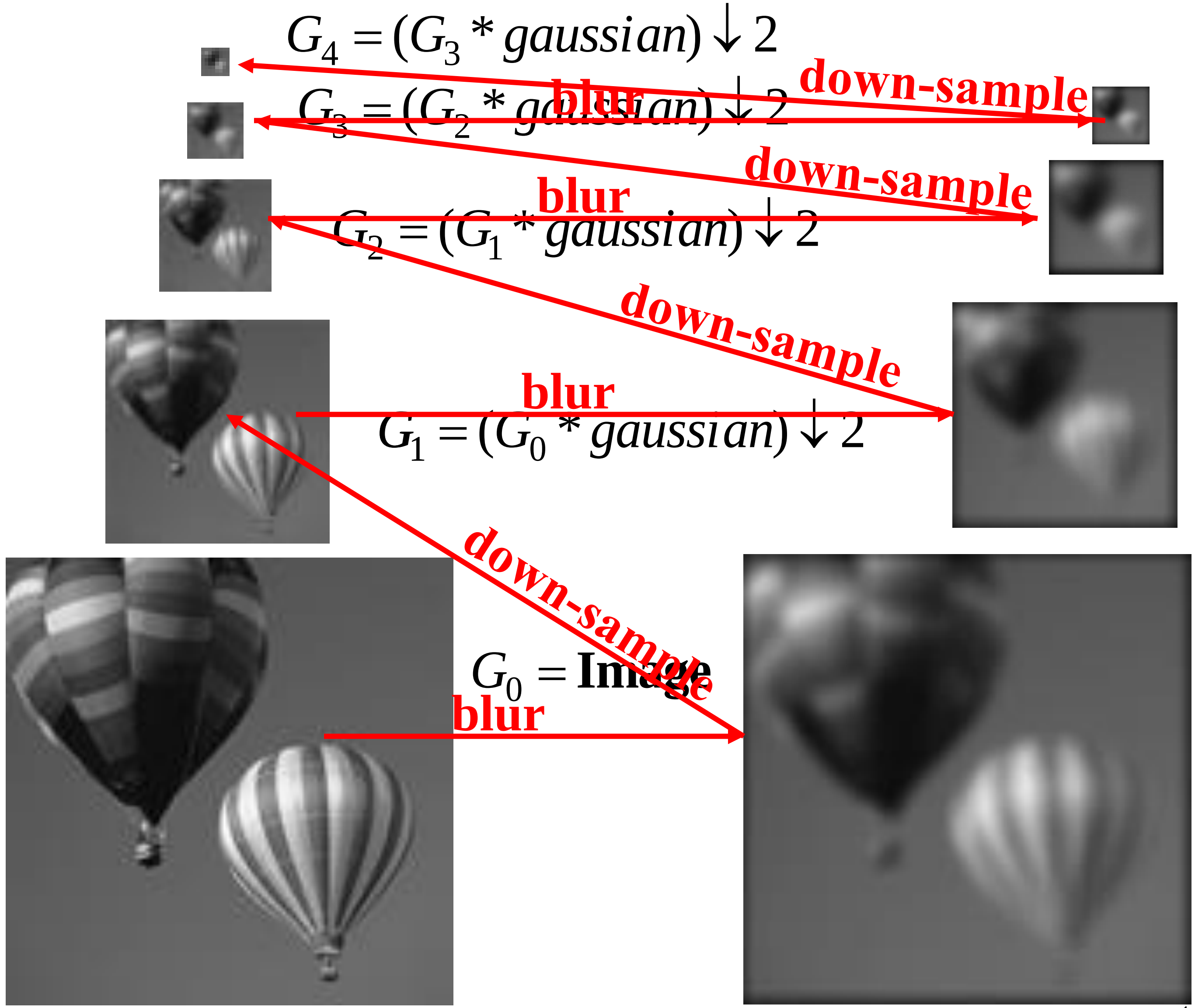
- Image Processing Basics
  - Image Formation
  - Binary Image Processing
  - Linear Filters
  - Edge & Structure Extraction
- Segmentation
- Local Features & Matching
- Object Recognition and Categorization
- 3D Reconstruction
- Motion and Tracking

Topics of This Lecture

- Recap: Edge detection
  - Image gradients
  - Canny edge detector
- Fitting as template matching
  - Distance transform
  - Chamfer matching
  - Application: traffic sign detection
- Fitting as parametric search
  - Line detection
  - Hough transform
  - Extension to circles
  - Generalized Hough transform

Recap: The Gaussian Pyramid

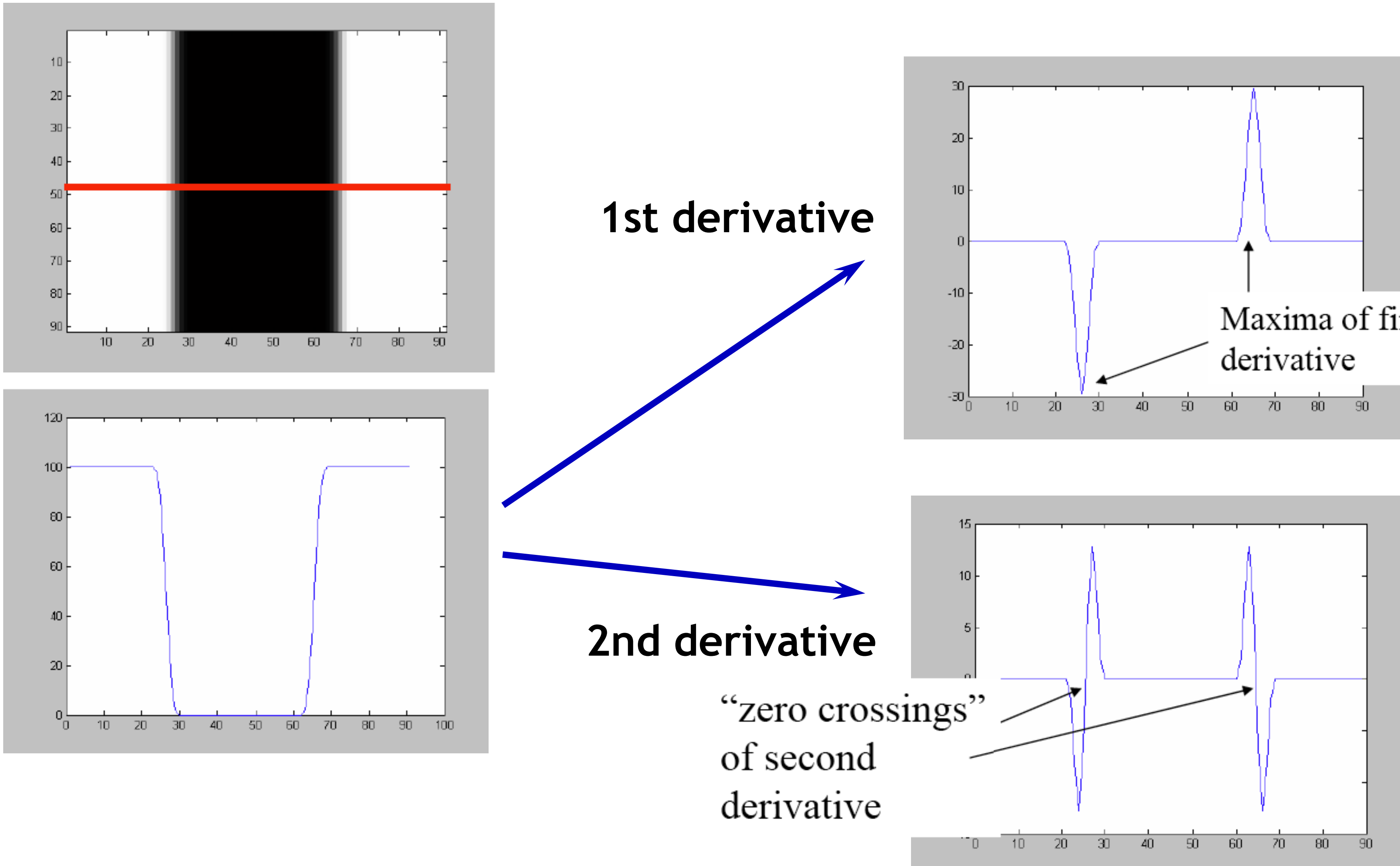
Low resolution



High resolution

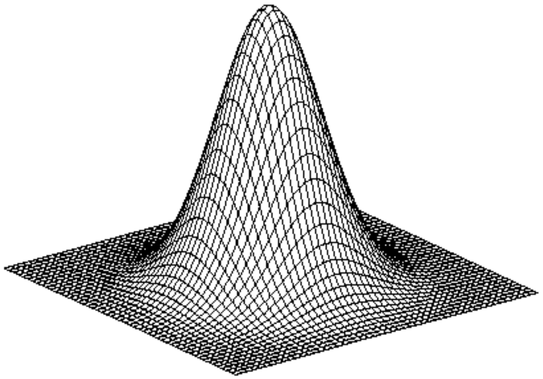
Source: Irani & Basri

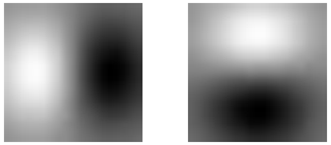
Recap: Derivatives and Edges…

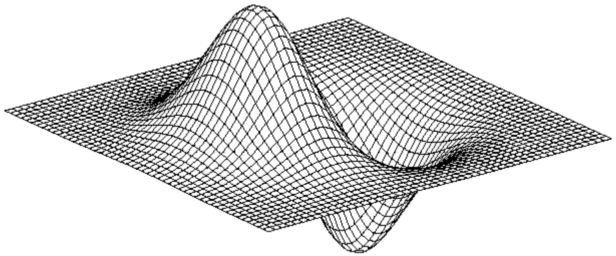


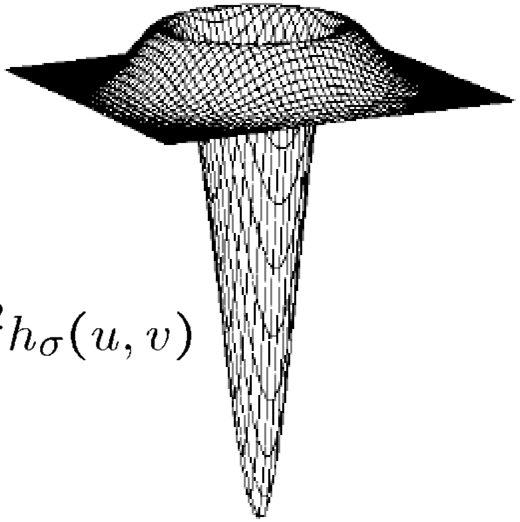
Recap: 2D Edge Detection Filters

see Exercise 2.5!









Laplacian of Gaussian

Gaussian

Derivative of Gaussian

$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$

$$\frac{\partial}{\partial x} h_\sigma(u,v)$$

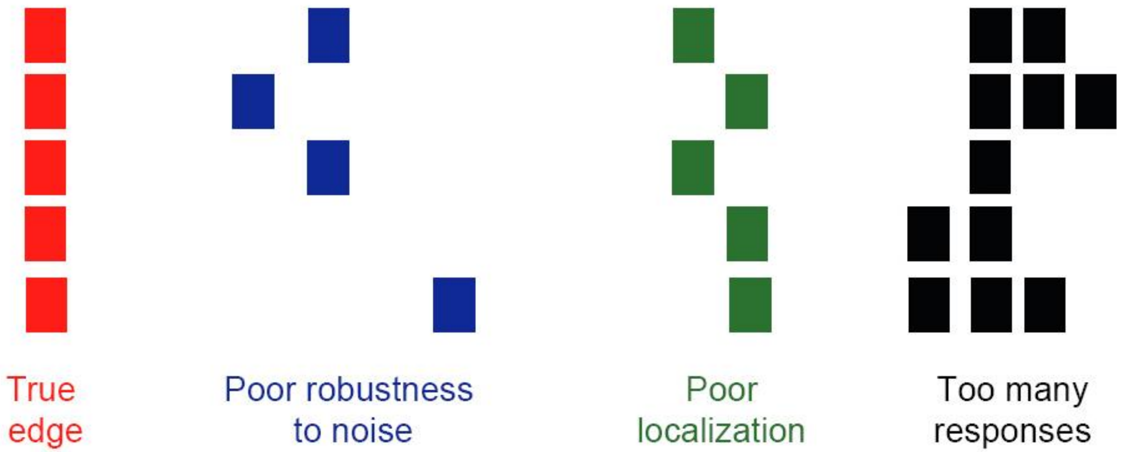
$$\nabla^2 h_\sigma(u,v)$$

- ∇^2 is the Laplacian operator:

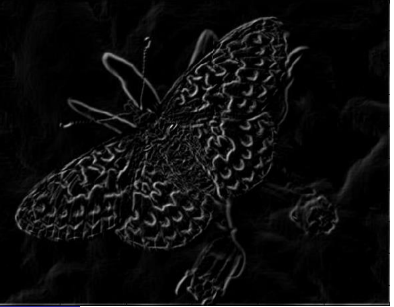
$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Designing an Edge Detector

- Criteria for an "optimal" edge detector:
  - **Good detection**: the optimal detector should minimize the probability of false positives (detecting spurious edges caused by noise), as well as that of false negatives (missing real edges).
  - **Good localization**: the edges detected should be as close as possible to the true edges.
  - **Single response**: the detector should return one point only for each true edge point; that is, minimize the number of local maxima around the true edge.



Source: Li Fei-Fei



Gradients → Edges

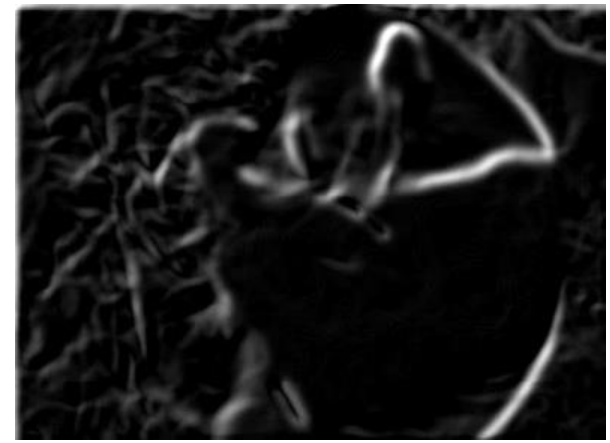
Primary edge detection steps

1. Smoothing: suppress noise
2. Edge enhancement: filter for contrast
3. Edge localization
   - Determine which local maxima from filter output are actually edges vs. noise
   - Thresholding, thinning

- Two issues
  - At what scale do we want to extract structures?
  - How sensitive should the edge extractor be?

Scale: Effect of σ on Derivatives

σ = 1 pixel



σ = 3 pixels

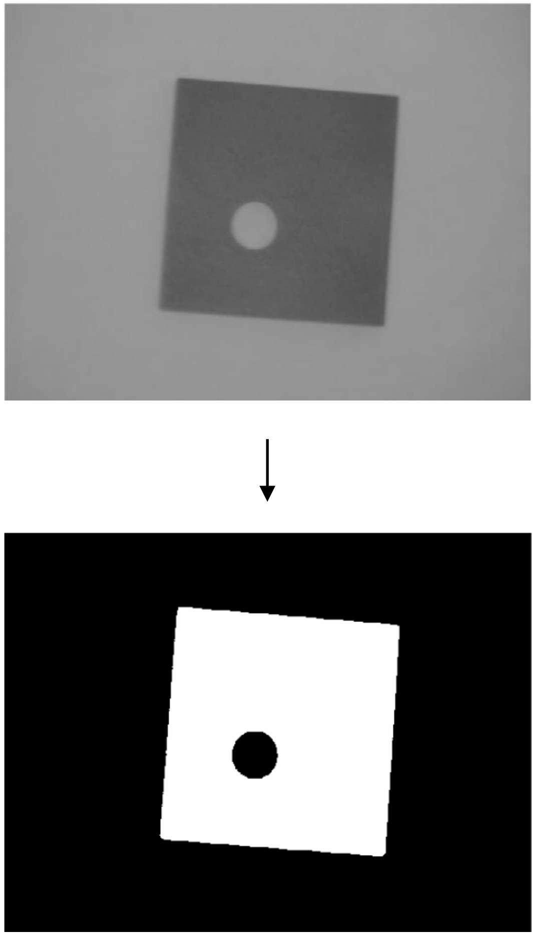
- The apparent structures differ depending on Gaussian's scale parameter.

⇒ Larger values: larger-scale edges detected
⇒ Smaller values: finer features detected

Sensitivity: Recall Thresholding

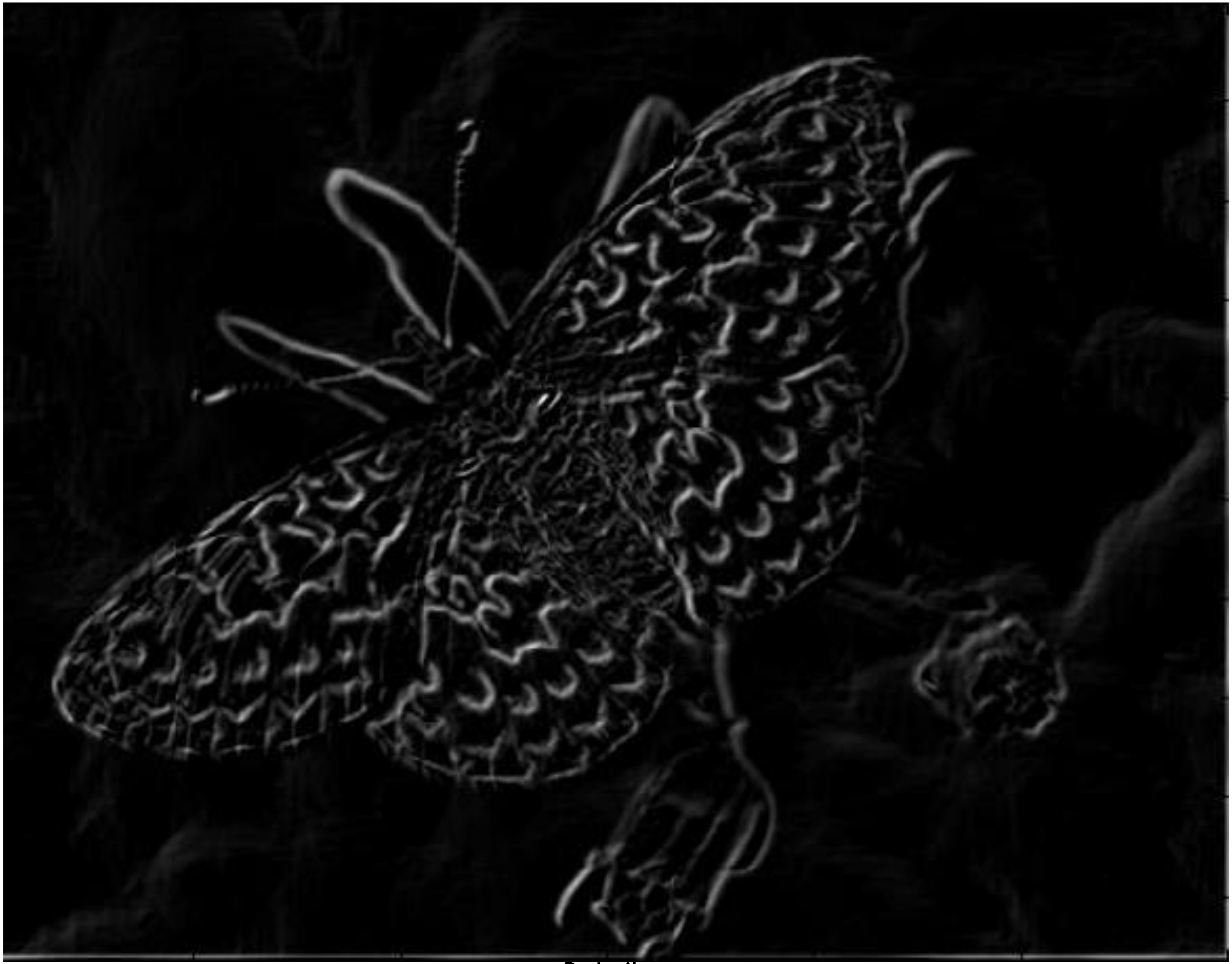
- Choose a threshold t
- Set any pixels less than t to zero (off).
- Set any pixels greater than or equal t to one (on).

$$F_T[i,j] = \begin{cases} 1, & \text{if } F[i,j] \geq t \\ 0, & \text{otherwise} \end{cases}$$



Original Image

Gradient Magnitude Image



Thresholding with a Lower Threshold

Slide credit: Kristen Grauman

Thresholding with a Higher Threshold

Slide credit: Kristen Grauman

Canny Edge Detector

- Probably the most widely used edge detector in computer vision
- Theoretical model: step-edges corrupted by additive Gaussian noise
- Canny has shown that the first derivative of the Gaussian closely approximates the operator that optimizes the product of *signal-to-noise ratio* and localization.

J. Canny, A Computational Approach To Edge Detection, *IEEE Trans. Pattern Analysis and Machine Intelligence*, 8:679-714, 1986.

Source: Li Fei-Fei

Canny Edge Detector

1. Filter image with derivative of Gaussian
2. Find magnitude and orientation of gradient
3. Non-maximum suppression:
   - Thin multi-pixel wide “ridges” down to single pixel width
4. Linking and thresholding (hysteresis):
   - Define two thresholds: low and high
   - Use the high threshold to start edge curves and the low threshold to continue them

- MATLAB:

```
>> edge(image, 'canny');
>> help edge
```


Source: D. Lowe, L. Fei-Fei

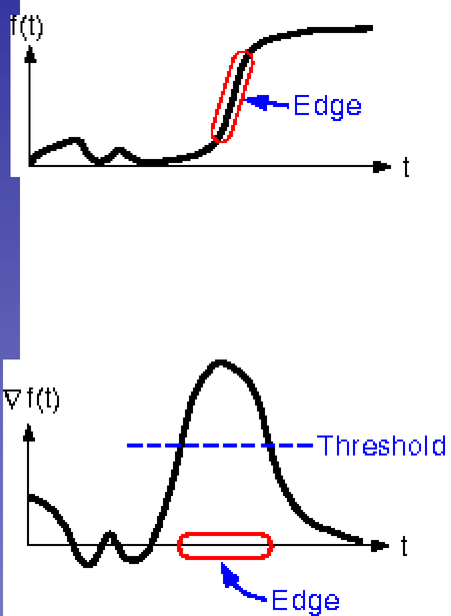
The Canny Edge Detector

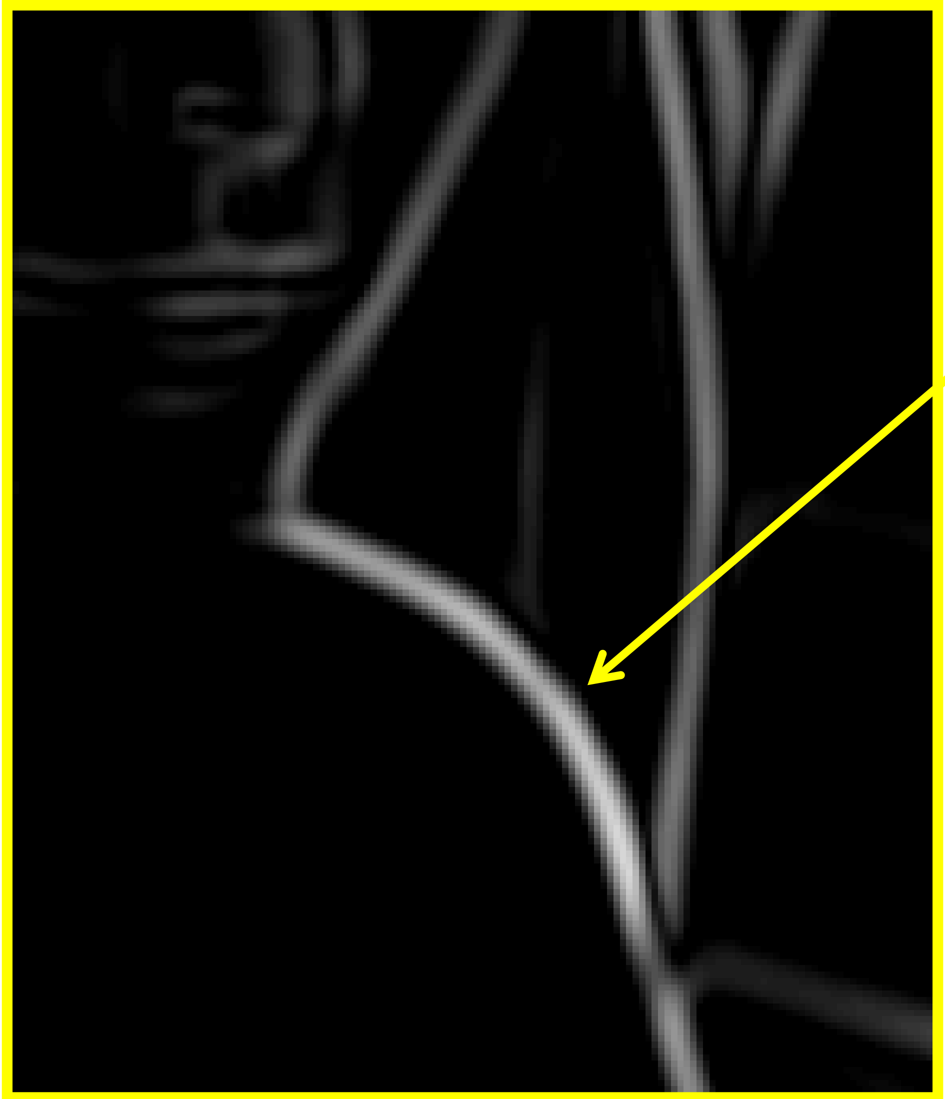
Original image (Lena)

The Canny Edge Detector

Gradient magnitude

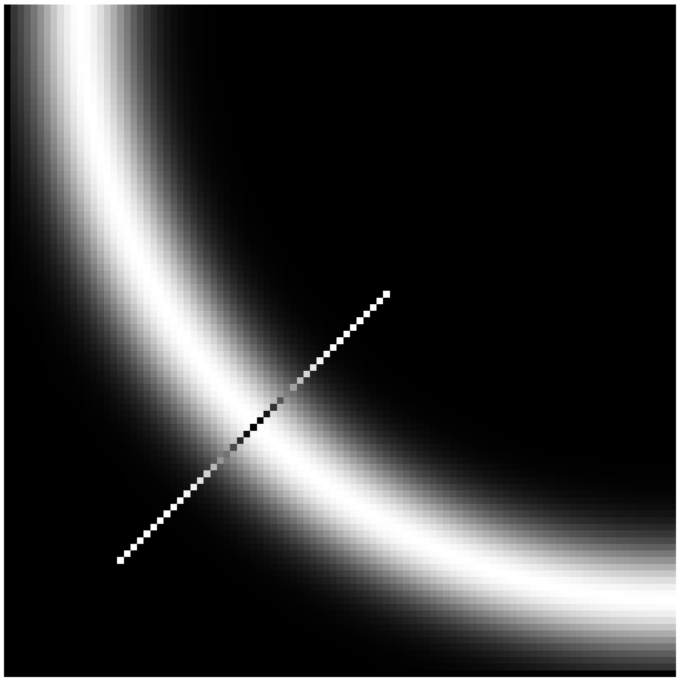
The Canny Edge Detector

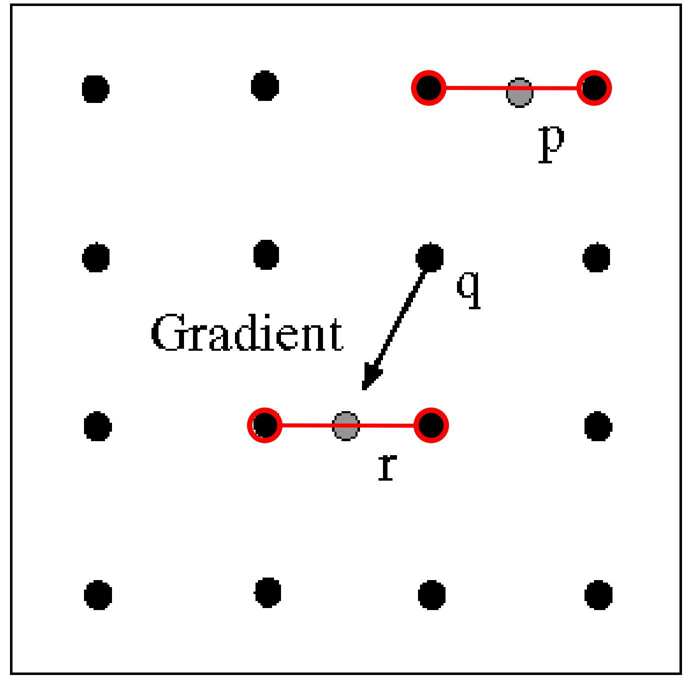




How to turn these thick regions of the gradient into curves?

Non-Maximum Suppression





- **Check if pixel is local maximum along gradient direction, select single max across width of the edge**
  - **Requires checking interpolated pixels p and r**

  ⇒ Linear interpolation based on gradient direction

Source: Forsyth & Ponce

The Canny Edge Detector

Problem: pixels along this edge didn't survive the thresholding.

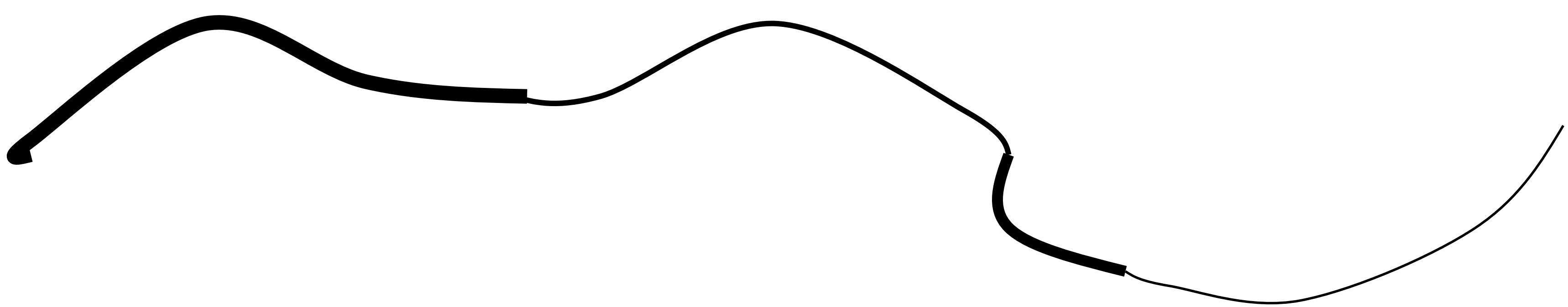
Thinning
(non-maximum suppression)

Slide credit: Kristen Grauman

Solution: Hysteresis Thresholding

- Hysteresis: A lag or momentum factor
- Idea: Maintain two thresholds k_{high} and k_{low}
  - Use k_{high} to find strong edges to start edge chain
  - Use k_{low} to find weak edges which continue edge chain
- Typical ratio of thresholds is roughly

$$k_{high} \;/\; k_{low} = 2$$



Source: D. Lowe, S. Seitz

Hysteresis Thresholding

Original image

High threshold
(strong edges)

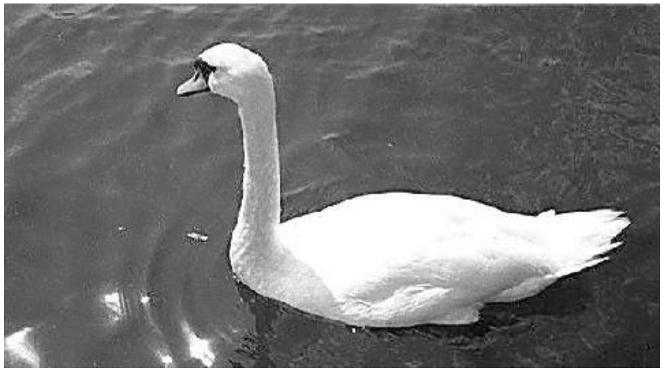
Low threshold
(weak edges)

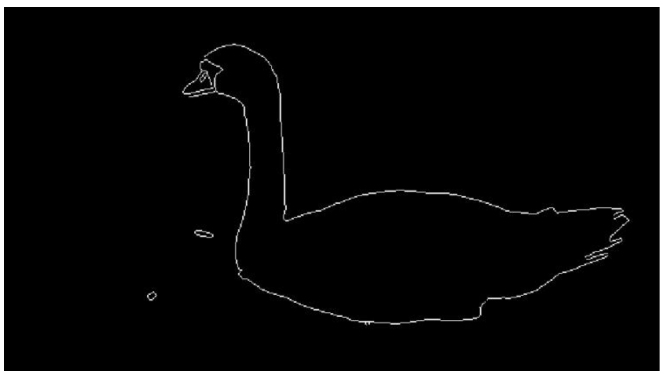
courtesy of G. Loy

Hysteresis threshold

Source: L. Fei-Fei

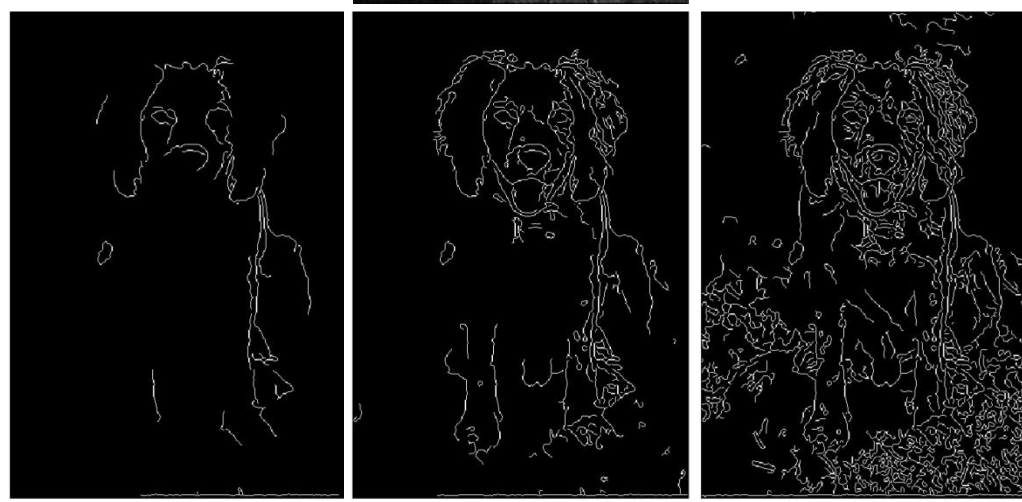
Edges vs. Boundaries





Edges are useful signals to indicate occluding boundaries, shape.

Here the raw edge output is not so bad…



…but quite often boundaries of interest are fragmented, and we have extra “clutter” edge points.

Slide credit: Kristen Grauman

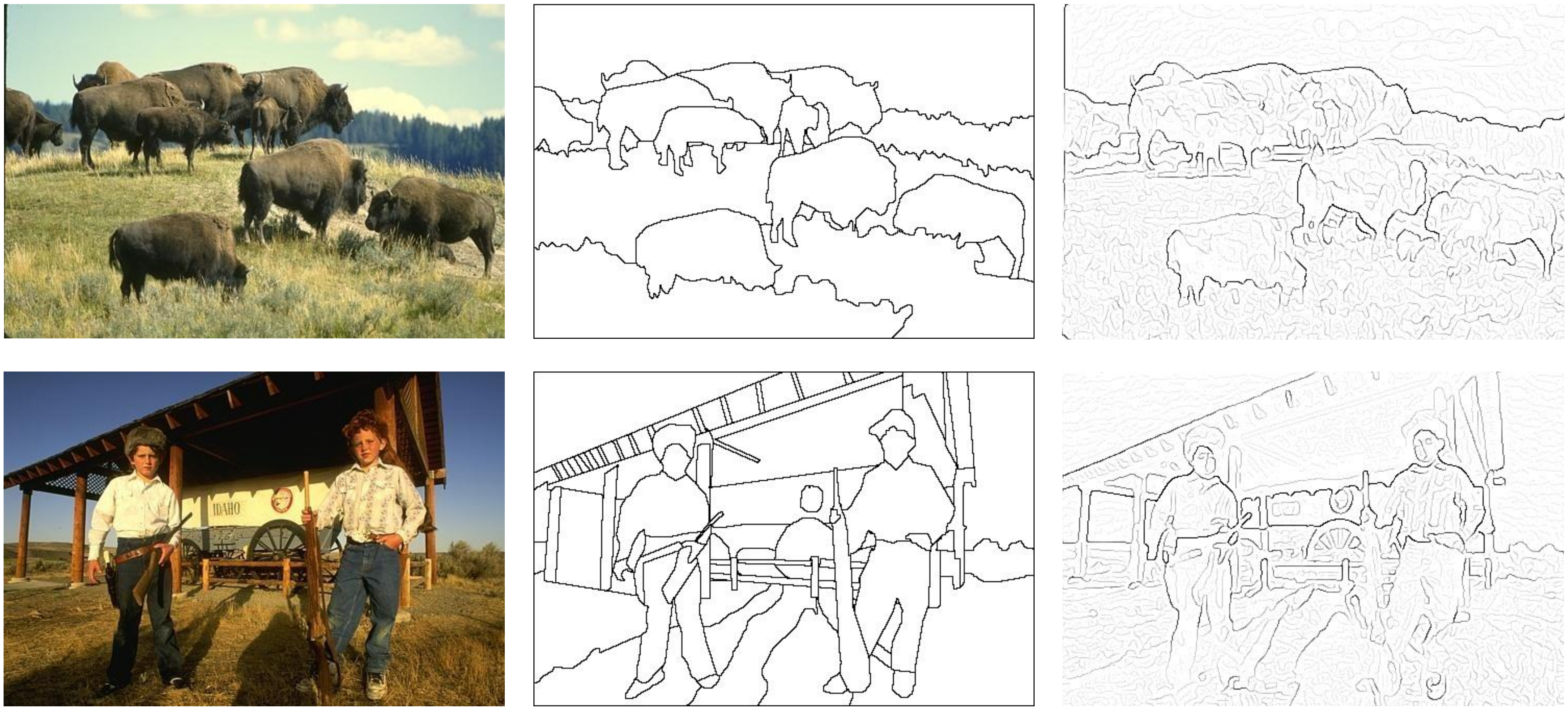
Object Boundaries vs. Edges

Background **Texture** **Shadows**

Slide credit: Kristen Grauman

Edge Detection is Just the Beginning…

Image | Human segmentation | Gradient magnitude

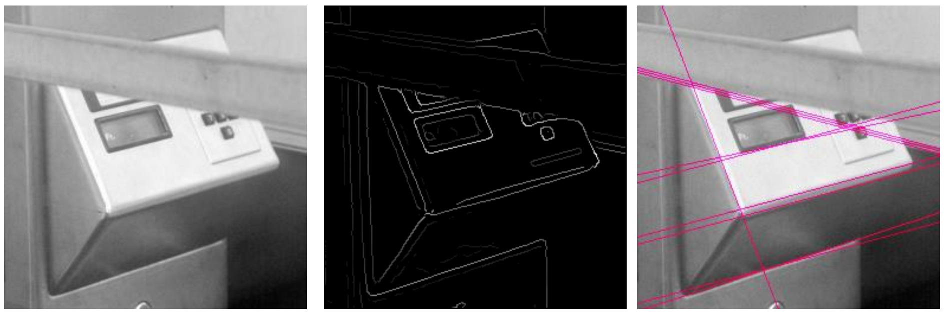


- **Berkeley segmentation database:**
  http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/segbench/

Source: L. Lazebnik

Fitting

- **Want to associate a model with observed features**



[Figure from Marszalek & Schmid, 2007]

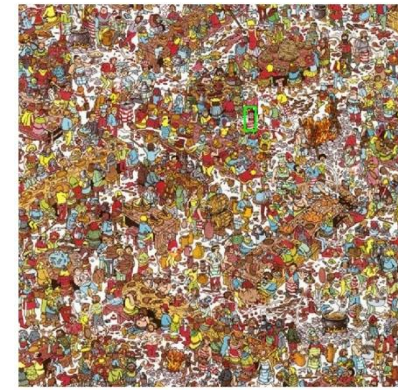
For example, the model could be a line, a circle, or an arbitrary shape.

Topics of This Lecture

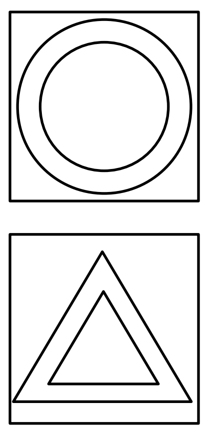
- Recap: Edge detection
  - Image gradients
  - Canny edge detector
- **Fitting as template matching**
  - **Distance transform**
  - **Chamfer matching**
  - **Application: traffic sign detection**
- Fitting as parametric search
  - Line detection
  - Hough transform
  - Extension to circles
  - Generalized Hough transform

Fitting as Template Matching

- We’ve already seen that correlation filtering can be used for template matching in an image.



- Let’s try this idea with “edge templates”.
  - Example: traffic sign detection in (grayvalue) video.



Templates

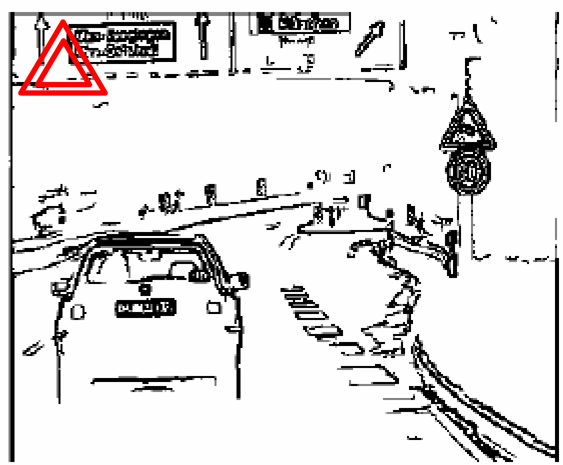
Edge Templates

- **Correlation filtering**
  - **Correlation between edge pixels in template and image**

$$D_{\text{corr}}(x, y) = -\sum_{u,v} T[u, v] I[x + u, y + v]$$

  - **Unfortunately, this doesn't work at all... Why?**

$\Rightarrow$ Zero correlation score if the edge template is 1 pixel off...

B. Leibe

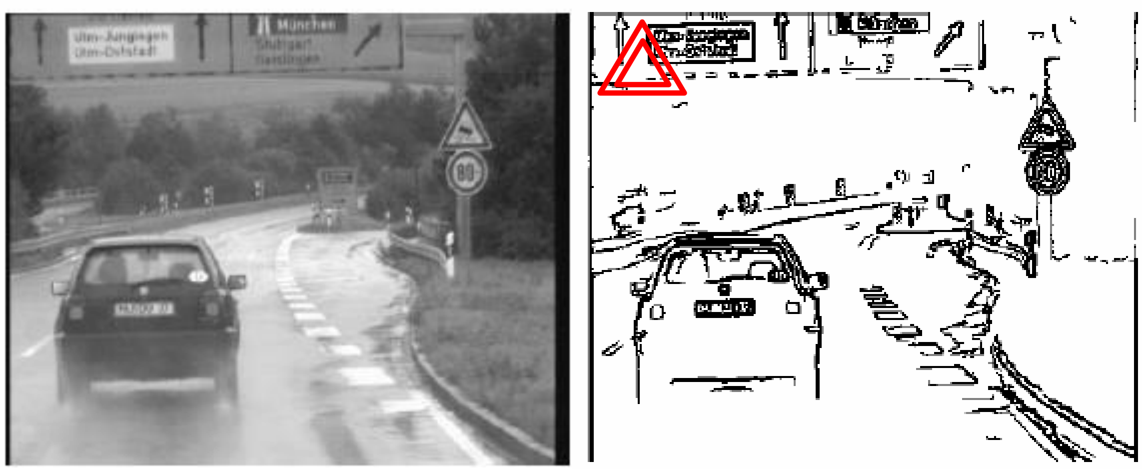
Edge Templates

- **Better: Chamfer Distance**
  - **Average distance to nearest edge pixel**

$$D_{\text{Chamfer}}(x,y) = \frac{1}{|T|} \sum_{u,v:T[u,v]=1} d_t(x+u, y+v)$$

$\Rightarrow$ **More robust to small shifts and size variations.**

- **How can we compute this efficiently?**

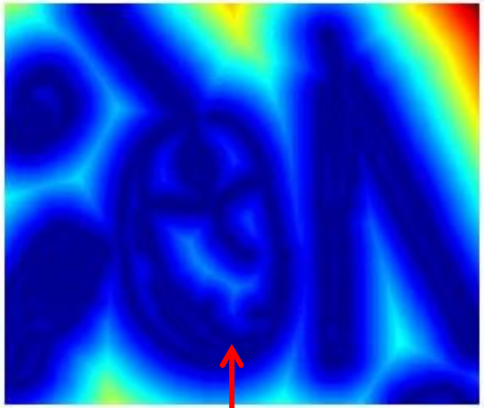


How Can This Be Made Efficient?

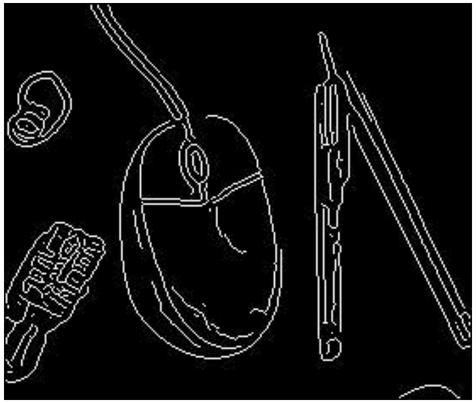
- **Fast edge-based template matching**
  - Distance transform of the edge image

Original

Gradient



Distance transform



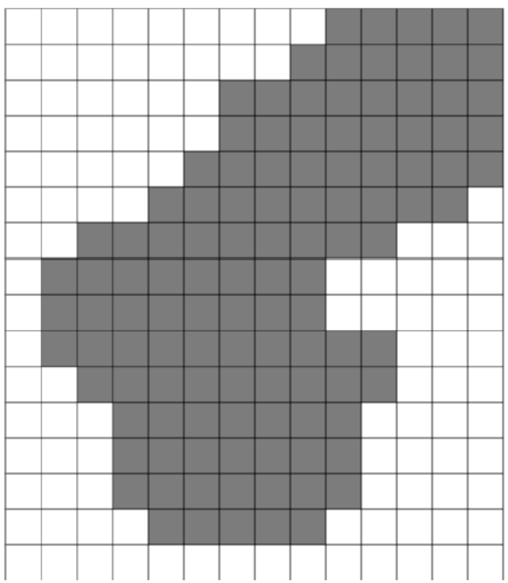
Edges

Value at (x,y) tells how far that position is from the nearest edge point (or other binary image structure)

```
>> help bwdist
```

Distance Transform

- **Image reflecting distance to nearest point in point set (e.g., edge pixels, or foreground pixels).**



									1	1	1	1	1
								1	2	2	2	2	1
						1	1	2	3	3	3	2	1
						1	2	3	4	4	3	2	1
					1	2	3	4	3	3	2	2	1
				1	2	3	4	3	2	2	1	1	
		1	1	2	3	4	3	2	1	1			
	1	2	2	3	4	3	2	1					
	1	2	3	4	4	3	2	1					
	1	2	3	4	5	4	3	2	1	1			
		1	2	3	4	5	4	3	2	1			
			1	2	3	4	3	2	1				
			1	2	3	3	3	2	1				
			1	2	2	2	2	2	1				
				1	1	1	1	1					

4-connected adjacency

									1	1	1	1	1
								1	1	2	2	2	1
						1	1	1	2	2	3	2	1
						1	2	2	2	3	2	2	1
					1	1	2	3	2	2	2	1	1
				1	1	2	2	2	2	1	1	1	
		1	1	1	2	2	2	1	1	1			
	1	1	2	2	2	3	2	1					
	1	2	2	3	3	3	2	1					
	1	1	2	2	3	3	2	1	1	1			
		1	1	2	3	3	2	2	1	1			
			1	2	3	3	3	2	1				
			1	2	2	3	2	2	1				
			1	1	2	2	2	1	1				
				1	1	1	1	1					

8-connected adjacency

Distance Transform Algorithm (1D)

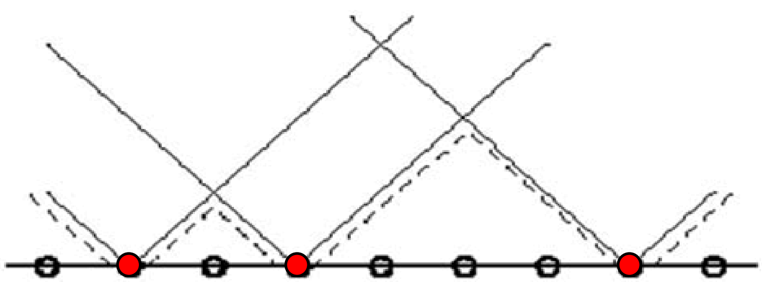
- Two-pass O(n) algorithm for 1D L_1 norm

1. **Initialize:** For all j
   - $D[j] \leftarrow 1_P[j]$ // 0 if j is in P, infinity otherwise
2. **Forward:** For j from 1 up to n-1
   - $D[j] \leftarrow \min(D[j], D[j-1]+1)$

+1	0

3. **Backward:** For j from n-2 down to 0
   - $D[j] \leftarrow \min(D[j], D[j+1]+1)$

0	+1



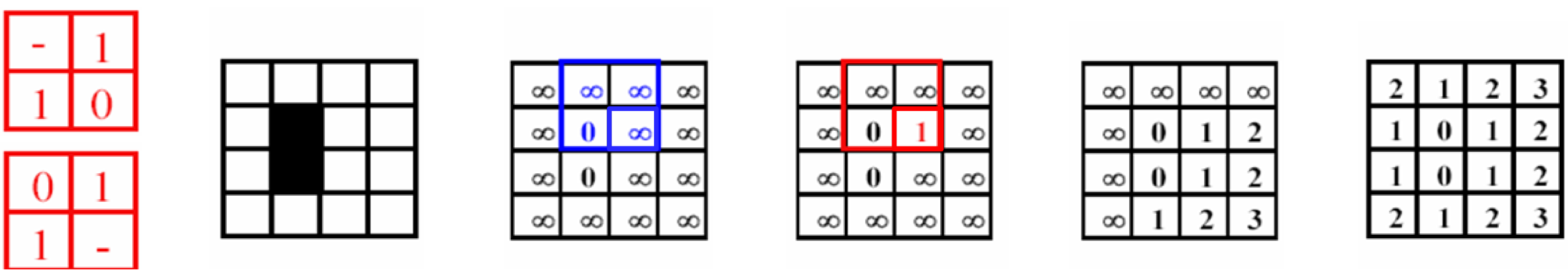
∞	0	∞	0	∞	∞	∞	0	∞

∞	0	1	0	1	2	3	0	1

1	0	1	0	1	2	1	0	1

Distance Transform Algorithm (2D)

- 2D case analogous to 1D
  - Initialization
  - Forward and backward pass
    - Fwd pass finds closest above and to the left
    - Bwd pass finds closest below and to the right

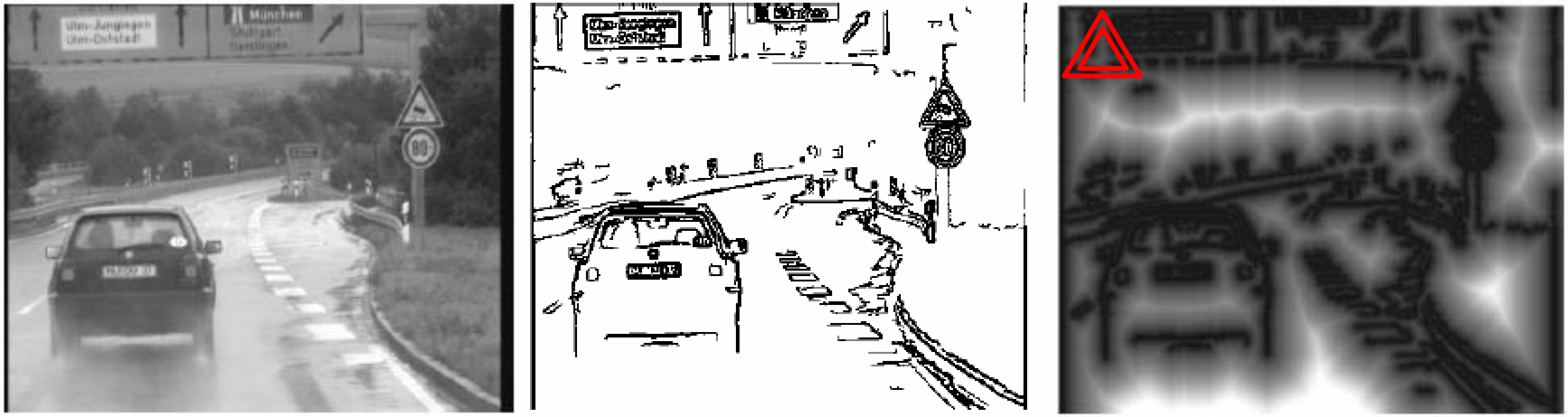


Chamfer Matching

- Chamfer Distance
  - Average distance to nearest feature

$$D_{chamfer}(T,I) \equiv \frac{1}{|T|} \sum_{t \in T} d_I(t)$$

  - This can be computed efficiently by correlating the edge template with the distance-transformed image



Edge image

Distance transform image

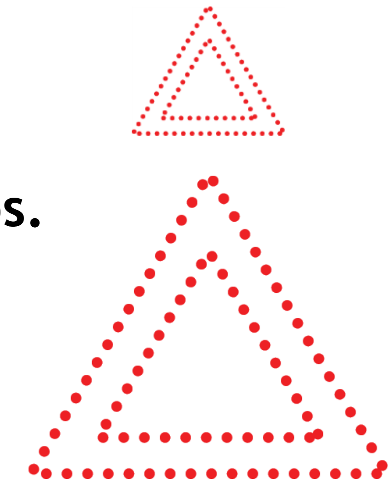
[D. Gavrila, DAGM'99]

Chamfer Matching

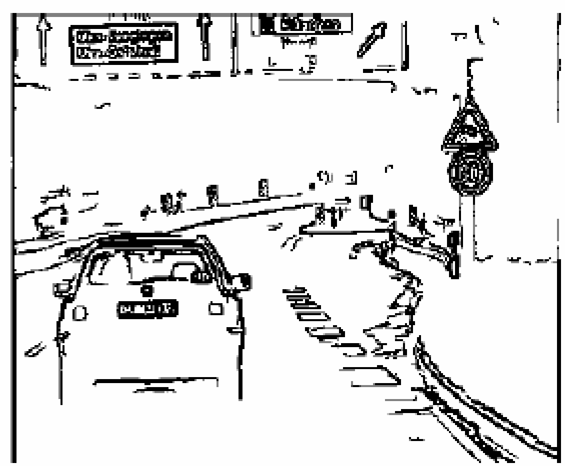
- **Efficient implementation**
  - **Instead of correlation, sample fixed number of points on template contour.**

  ⇒ Chamfer score boils down to series of DT lookups.

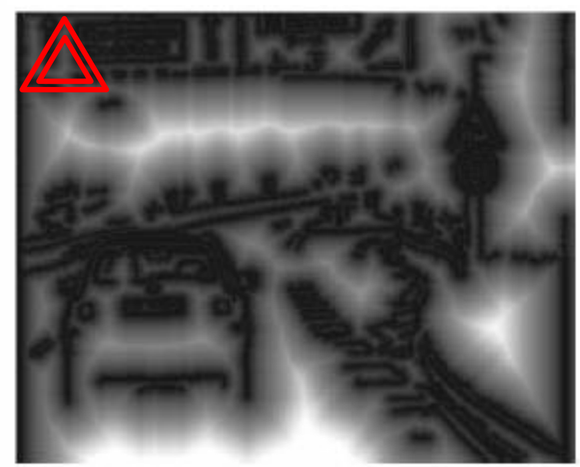
$$D_{chamfer}(T, I) \equiv \frac{1}{|T|} \sum_{t \in T} d_I(t)$$



⇒ Computational effort independent of scale.



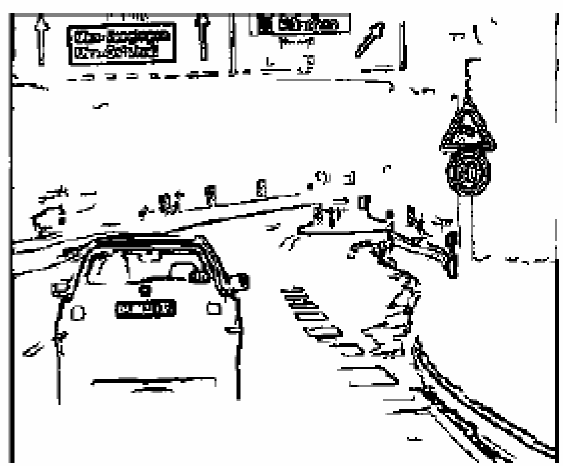
Edge image



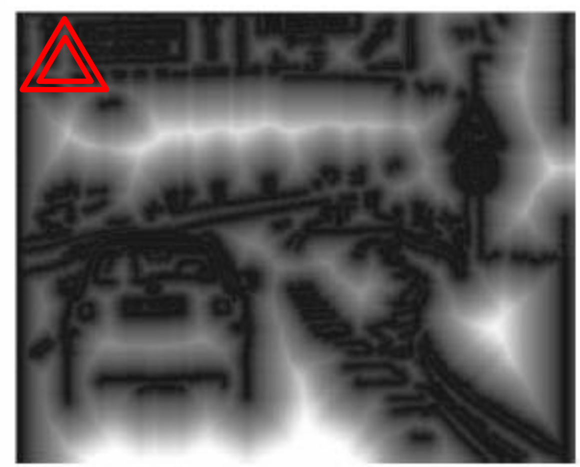
Distance transform image

[D. Gavrila, DAGM'99]

Chamfer Matching Results



Edge image

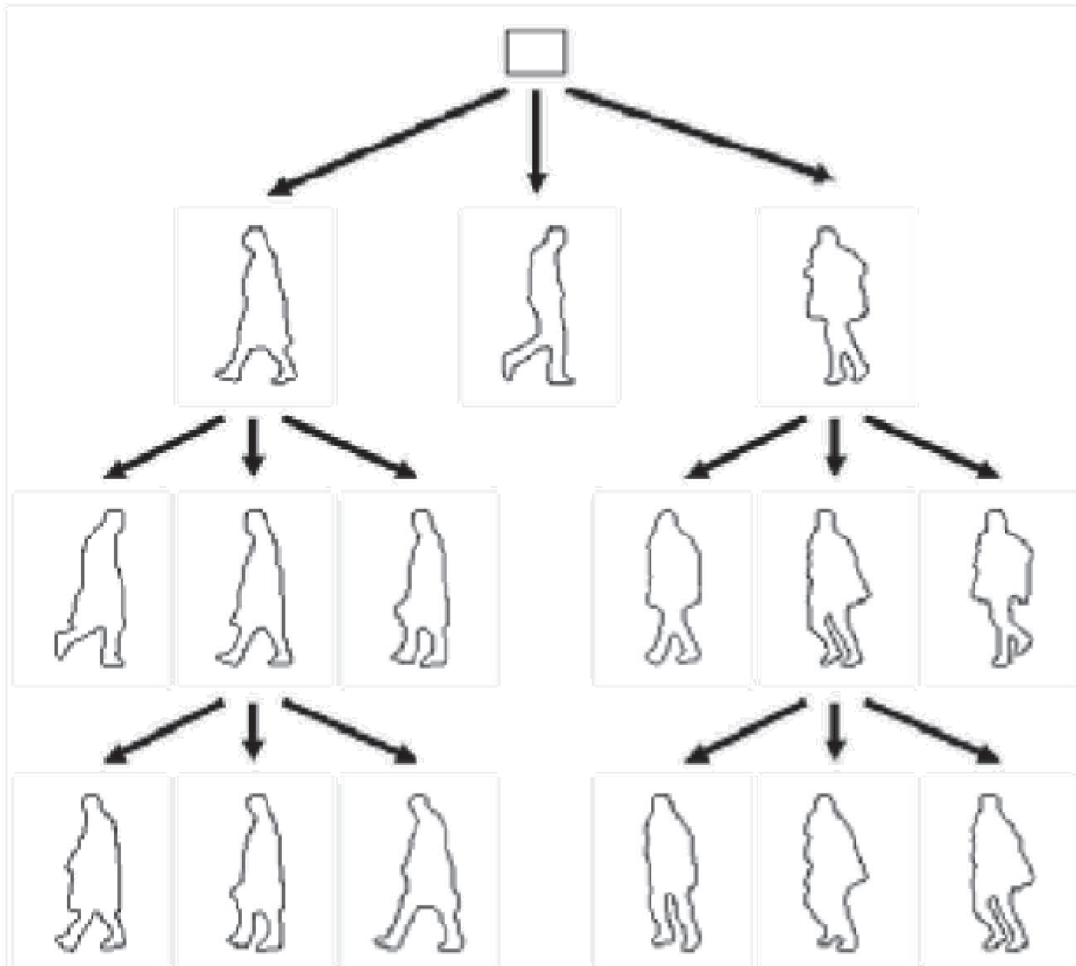


Distance transform image

[D. Gavrila, DAGM’99]

Chamfer Matching for Pedestrian Detection

- **Organize templates in tree structure for fast matching**



[D. Gavrila, V. Philomin, ICCV'99, PAMI'07]

Chamfer Matching for Pedestrian Detection

[D. Gavrila, V. Philomin, ICCV'99, PAMI'07]

Summary Chamfer Matching

- Pros
  - Fast and simple method for matching edge-based templates.
  - Works well for matching upright shapes with little intra-class variation.
  - Good method for finding candidate matches in a longer recognition pipeline.
- Cons
  - Chamfer score averages over entire contour, not very discriminative in practice.
    $\Rightarrow$ Further verification needed.
  - Low matching cost in cluttered regions with many edges.
    $\Rightarrow$ Many false positive detections.
  - In order to detect rotated & rescaled shapes, need to match with rotated & rescaled templates $\Rightarrow$ can get very expensive.

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  - Application: traffic sign detection
- **Fitting as parametric search**
  - **Line detection**
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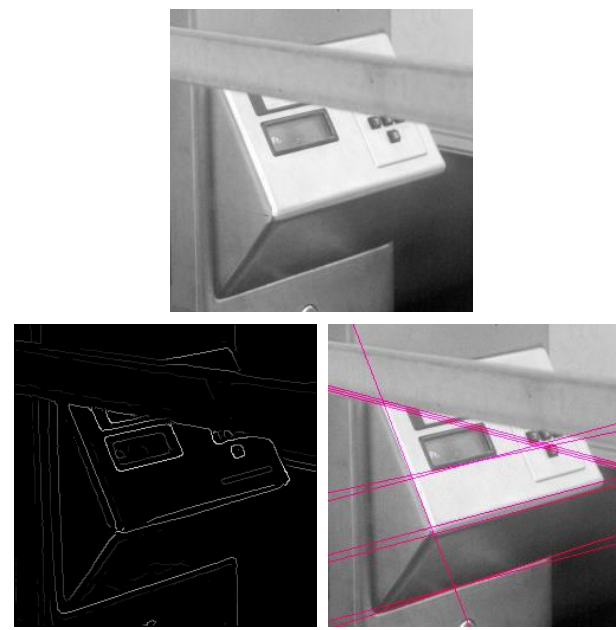


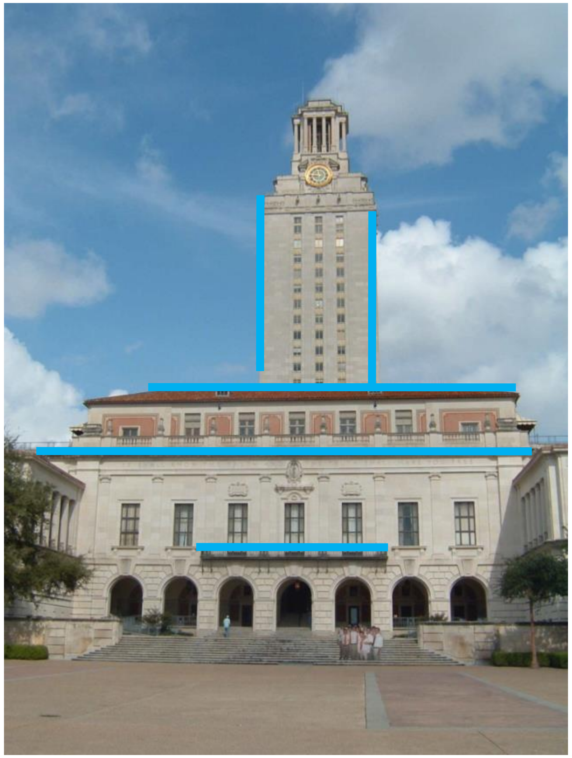
Fitting as Search in Parametric Space

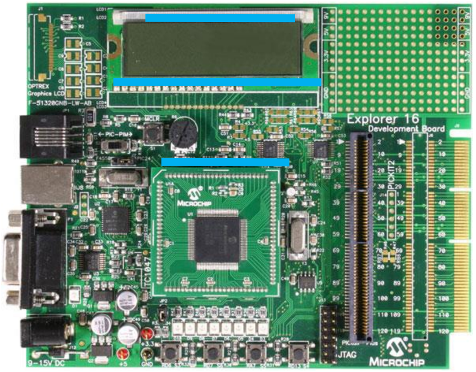
- Choose a parametric model to represent a set of features
- Membership criterion is not local
  - Can’t tell whether a point belongs to a given model just by looking at that point.
- Three main questions:
  - What model represents this set of features best?
  - Which of several model instances gets which feature?
  - How many model instances are there?
- Computational complexity is important
  - It is infeasible to examine every possible set of parameters and every possible combination of features

Source: L. Lazebnik

Example: Line Fitting

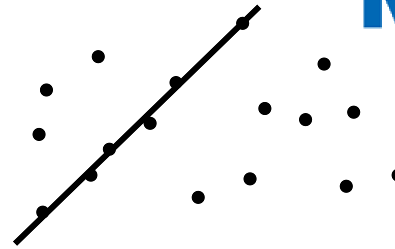
- Why fit lines?
  - Many objects are characterized by presence of straight lines

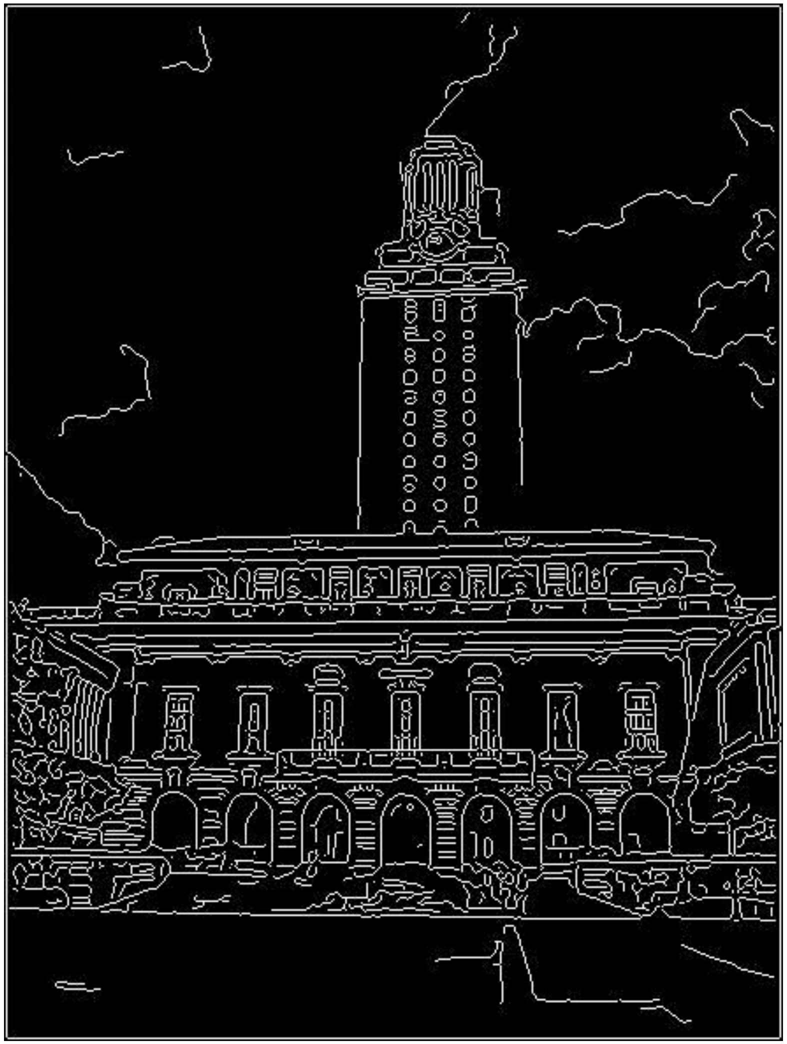




- Wait, why aren't we done just by running edge detection?

Difficulty of Line Fitting





- Extra edge points (clutter), multiple models:
  - Which points go with which line, if any?
- Only some parts of each line detected, and some parts are missing:
  - How to find a line that bridges missing evidence?
- Noise in measured edge points, orientations:
  - How to detect true underlying parameters?

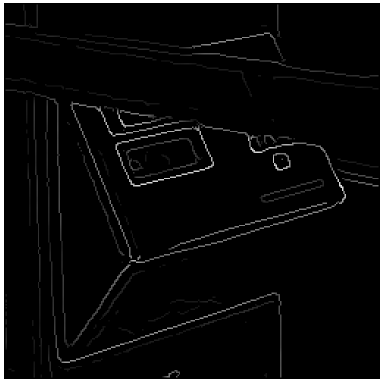
Voting

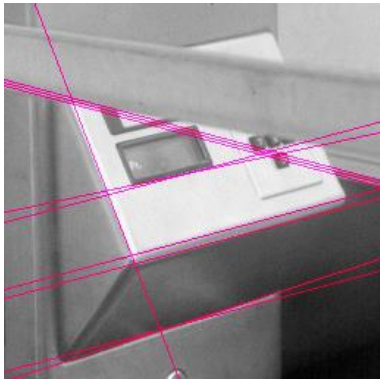
- It's not feasible to check all combinations of features by fitting a model to each possible subset.
- Voting is a general technique where we let the features vote for all models that are compatible with it.
  - Cycle through features, cast votes for model parameters.
  - Look for model parameters that receive a lot of votes.
- Noise & clutter features will cast votes too, *but* typically their votes should be inconsistent with the majority of "good" features.
- Ok if some features not observed, as model can span multiple fragments.

Fitting Lines

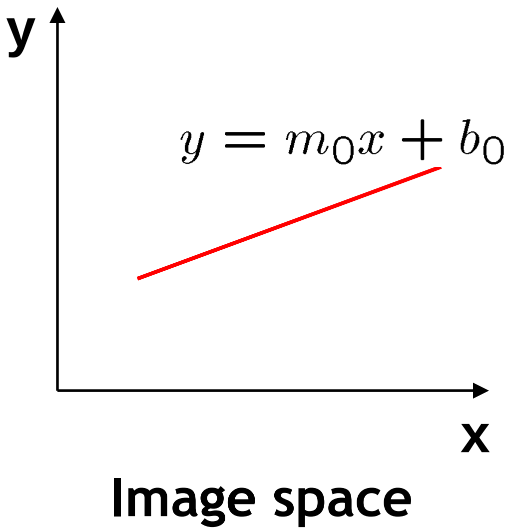
- Given points that belong to a line, what is the line?
- How many lines are there?
- Which points belong to which lines?

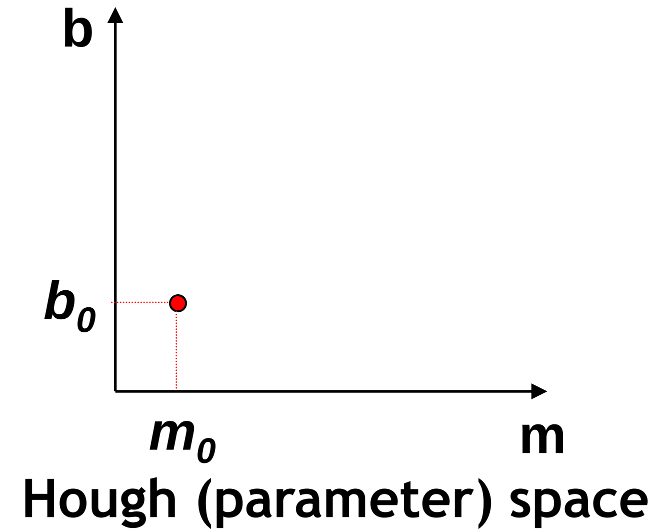
- The *Hough Transform* is a voting technique that can be used to answer all of these
- Main idea:
  1. Vote for all possible lines on which each edge point could lie.
  2. Look for lines that get many votes.





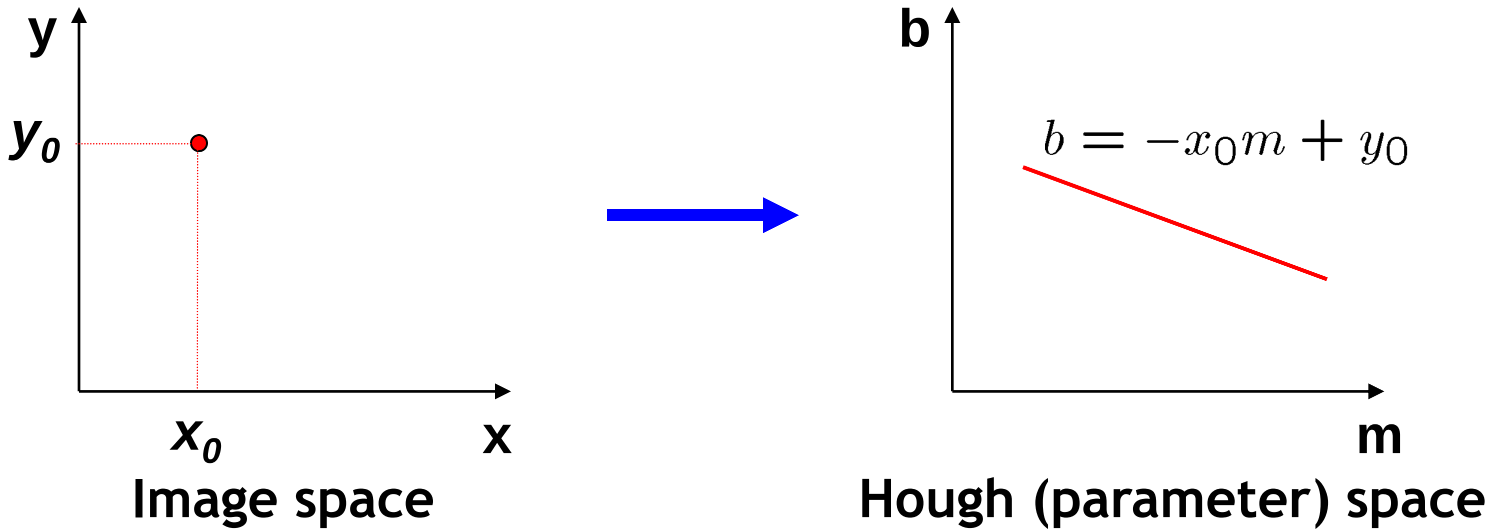
Finding Lines in an Image: Hough Space





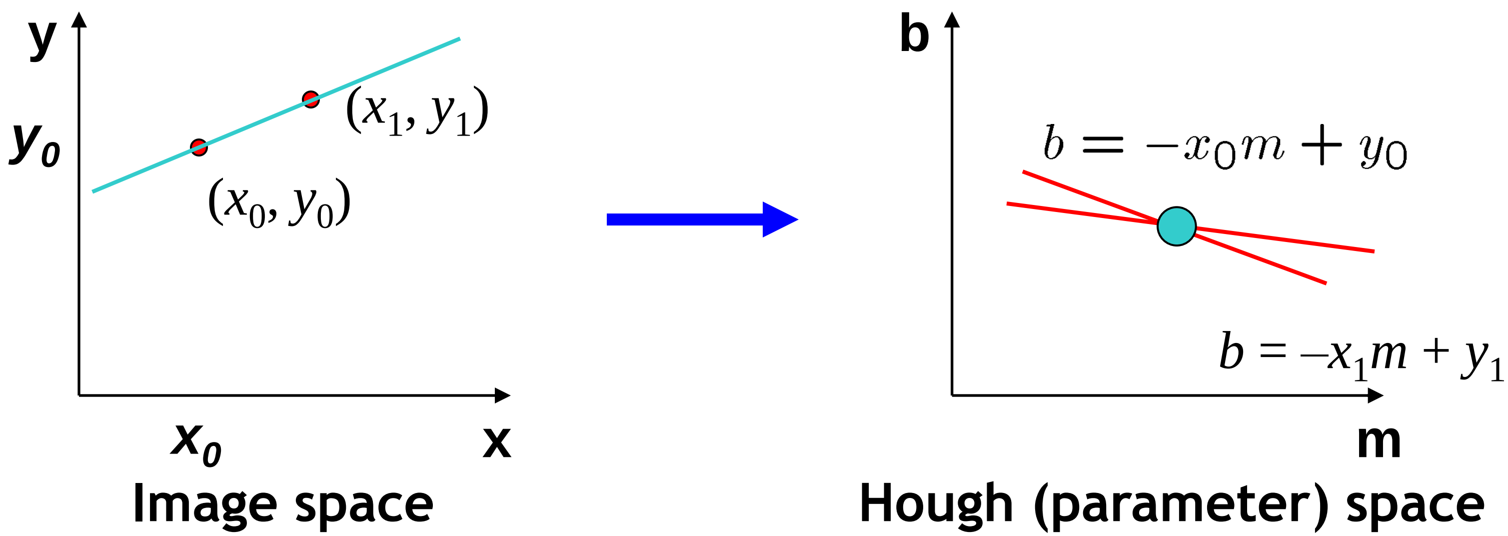
- Connection between image (x,y) and Hough (m,b) spaces
  - A line in the image corresponds to a point in Hough space.
  - To go from image space to Hough space:
    - Given a set of points (x,y), find all (m,b) such that $y = mx + b$

Finding Lines in an Image: Hough Space



- Connection between image (x,y) and Hough (m,b) spaces
  - A line in the image corresponds to a point in Hough space.
  - To go from image space to Hough space:
    - Given a set of points (x,y), find all (m,b) such that $y = mx + b$
  - What does a point (x_0, y_0) in the image space map to?
    - Answer: the solutions of $b = -x_0 m + y_0$
    - This is a line in Hough space.

Finding Lines in an Image: Hough Space



- **What are the line parameters for the line that contains both (x_0, y_0) and (x_1, y_1)?**
  - **It is the intersection of the lines**
    $b = -x_0 m + y_0$ **and**
    $b = -x_1 m + y_1$

Finding Lines in an Image: Hough Space

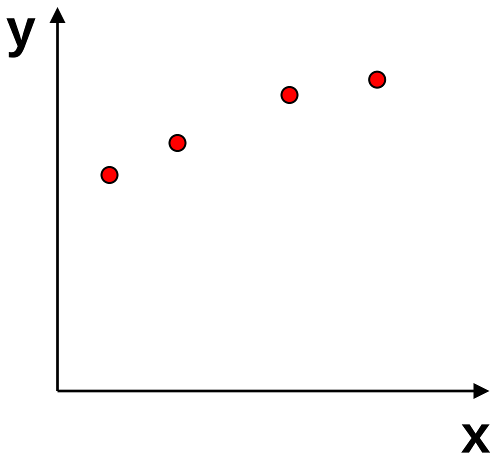
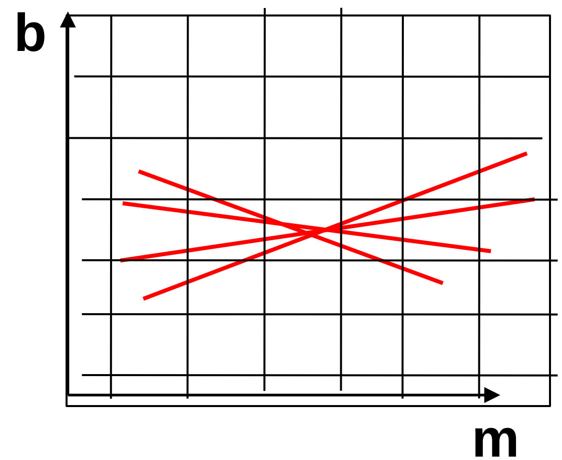


Image space

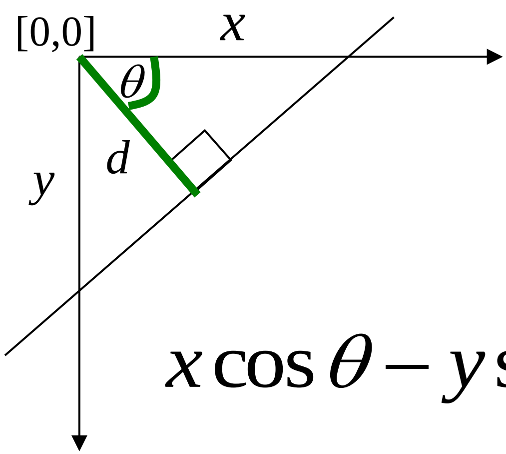


Hough (parameter) space

- How can we use this to find the most likely parameters (m,b) for the most prominent line in the image space?
  - Let each edge point in image space *vote* for a set of possible parameters in Hough space.
  - Accumulate votes in discrete set of bins; parameters with the most votes indicate line in image space.

Polar Representation for Lines

- Issues with usual (m,b) parameter space: can take on infinite values, undefined for vertical lines.

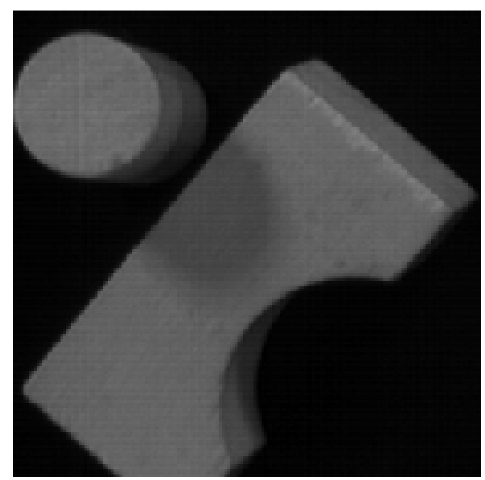


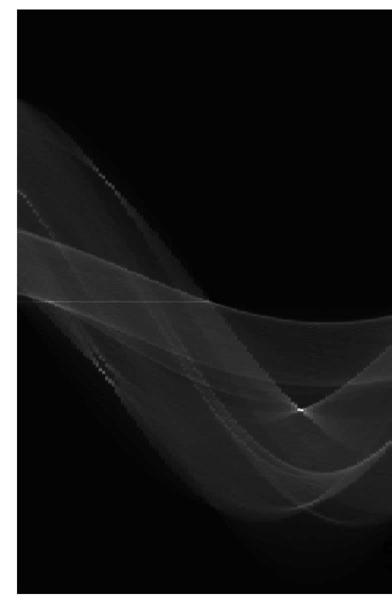
$$x\cos\theta - y\sin\theta = d$$

d : perpendicular distance from line to origin

θ : angle the perpendicular makes with the x-axis

- Point in image space
  $\Rightarrow$ Sinusoid segment in Hough space

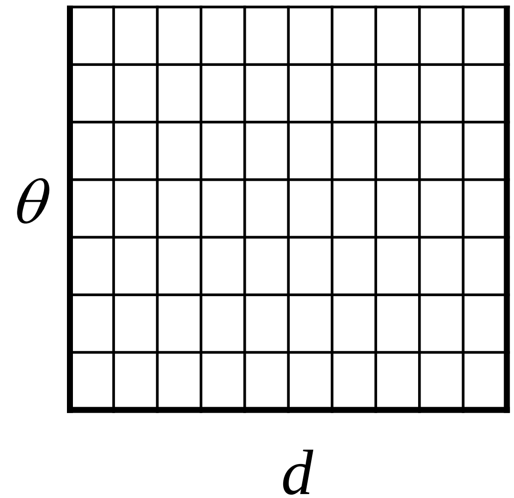




Slide adapted from Steve Seitz

Hough Transform Algorithm

H: accumulator array (votes)



Using the polar parameterization:

$$x\cos\theta - y\sin\theta = d$$

Basic Hough transform algorithm

1. **Initialize** $H[d,\theta] = 0.$
2. **For each edge point** (x,y) **in the image**

   for $\theta = 0$ **to** 180 // some quantization

   $$d = x\cos\theta - y\sin\theta$$

   H[d, θ] += 1

3. **Find the value(s) of** (d,θ) **where** $H[d,\theta]$ **is maximal.**
4. **The detected line in the image is given by** $d = x\cos\theta - y\sin\theta$

[Hough line demo]

- **Time complexity (in terms of number of votes)?**

Example: HT for Straight Lines

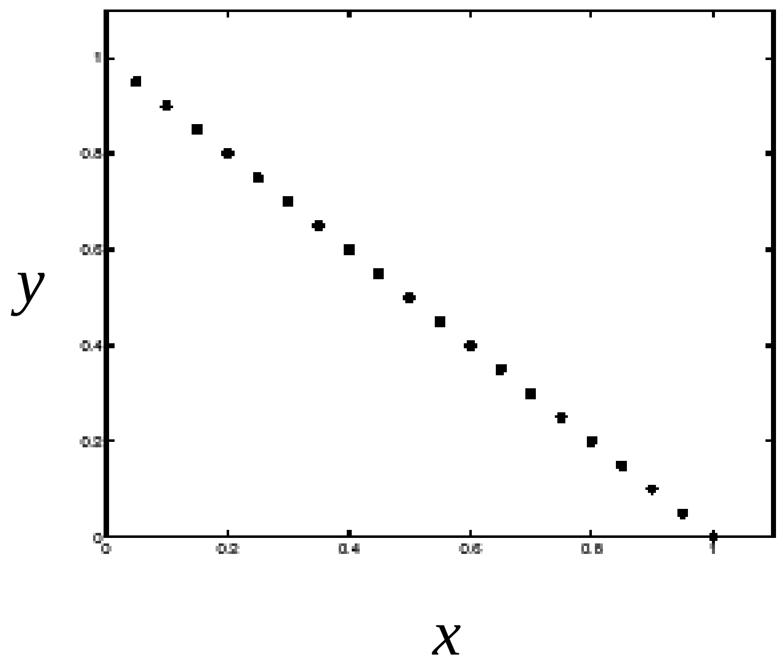
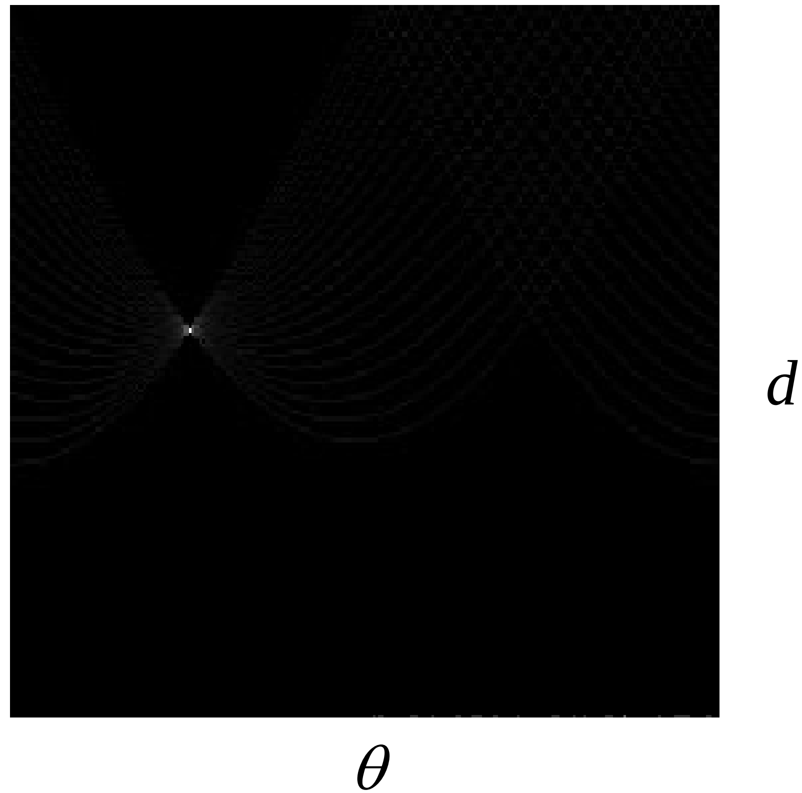


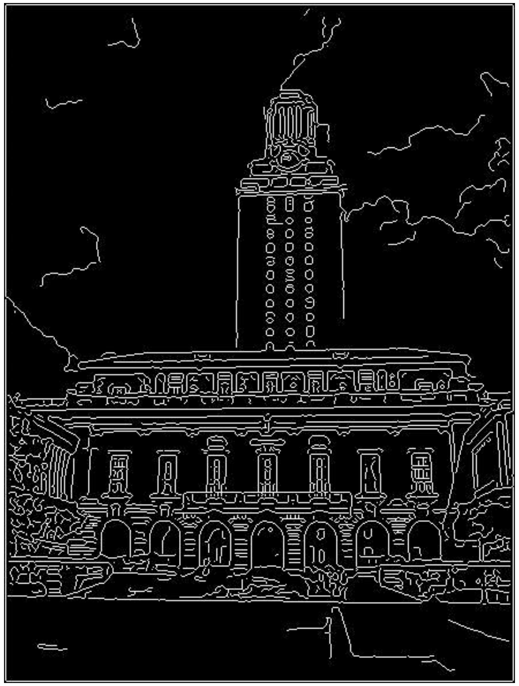
Image space edge coordinates

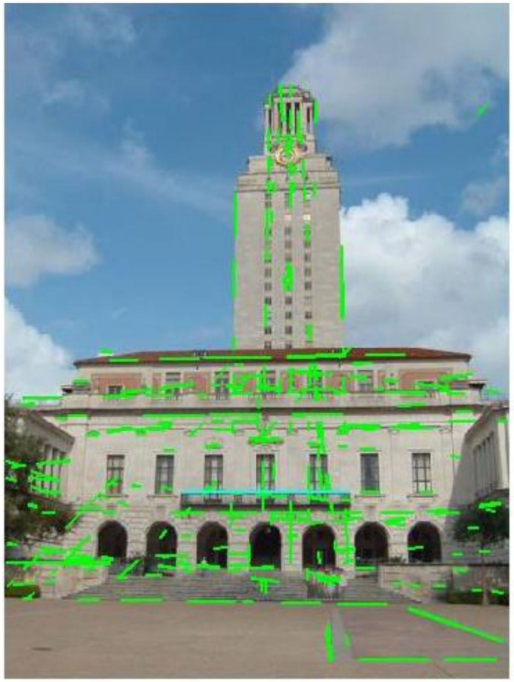


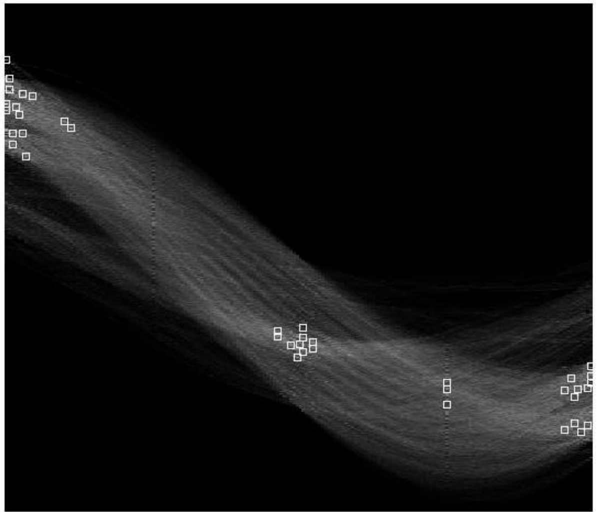
Votes

Bright value = high vote count
Black = no votes

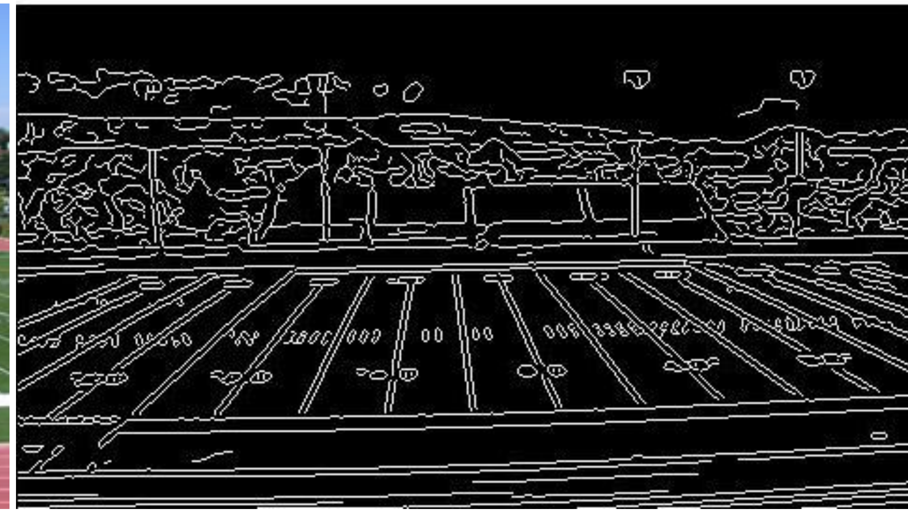
Real-World Examples

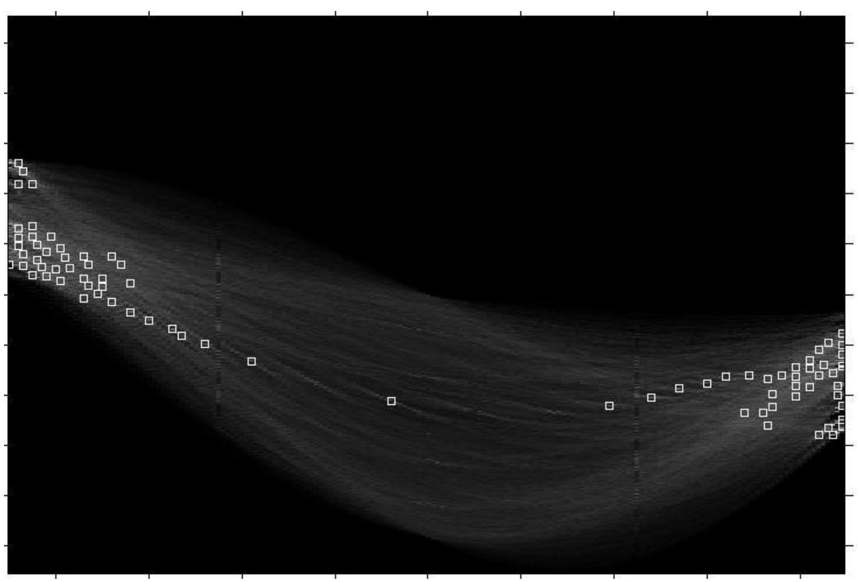


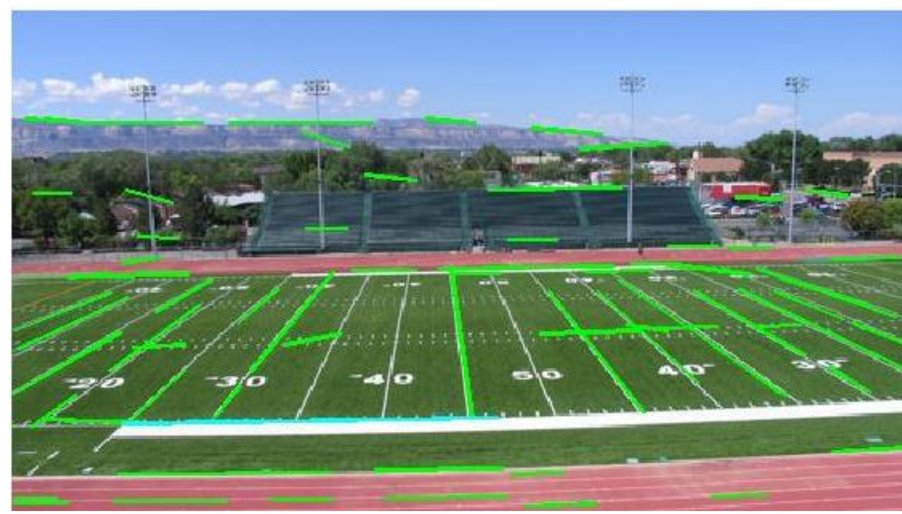




Slide credit: Kristen Grauman

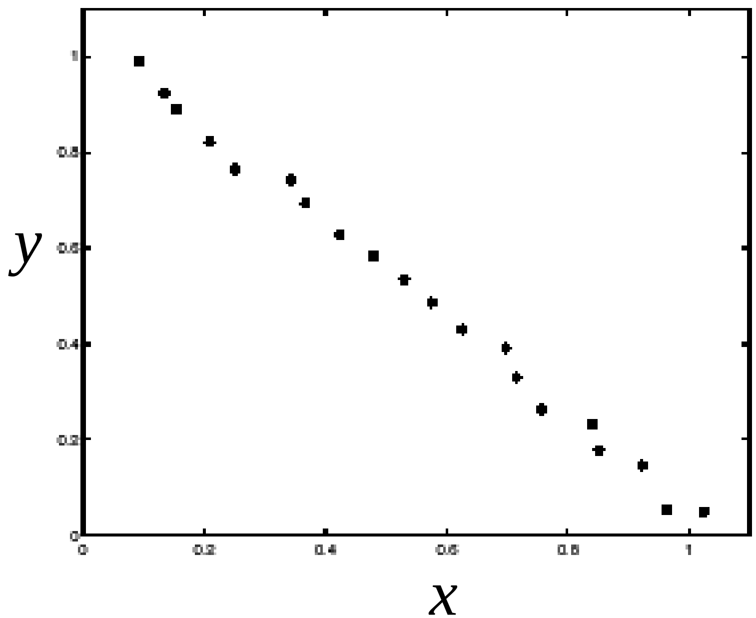






Showing longest segments found

Impact of Noise on Hough Transform



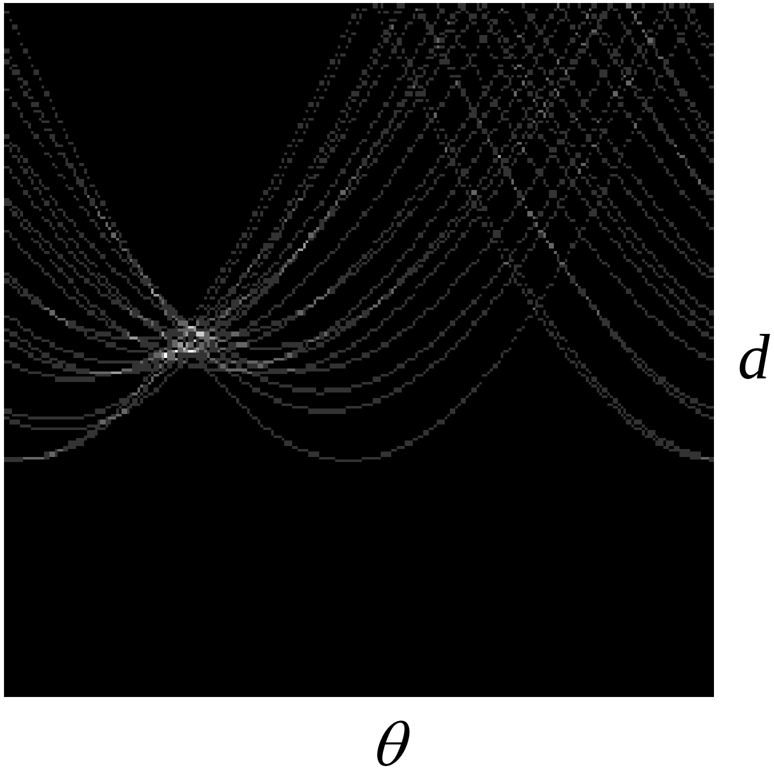


Image space edge coordinates

Votes

What difficulty does this present for an implementation?

Impact of Noise on Hough Transform

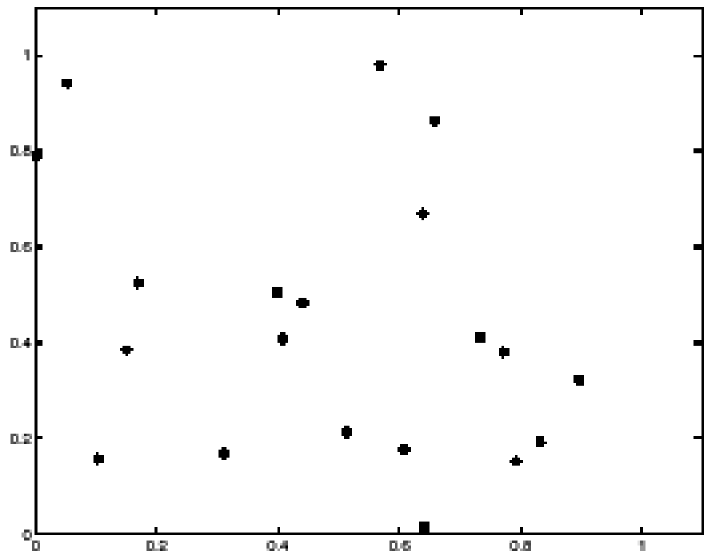
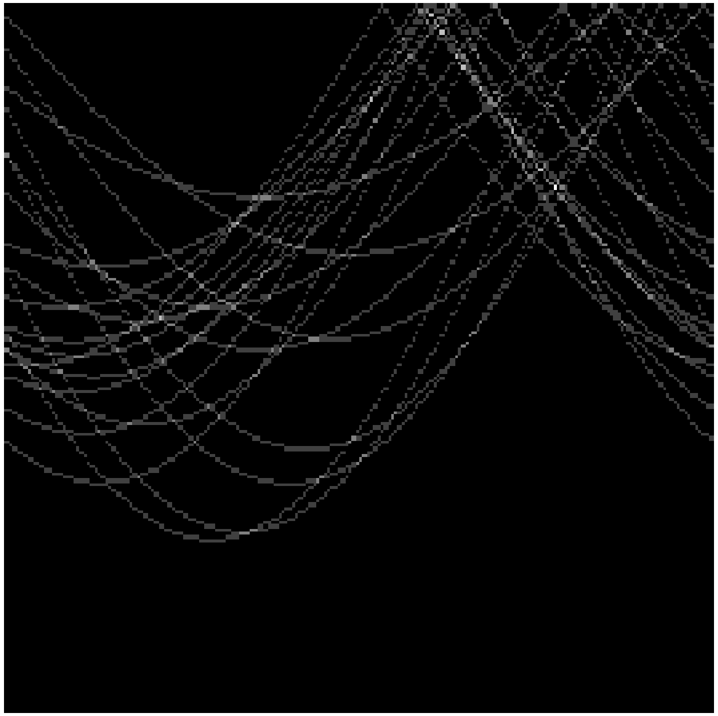


Image space
edge coordinates

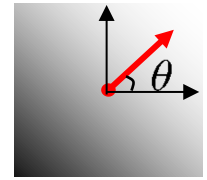


Votes

Here, everything appears to be "noise", or random edge points, but we still see peaks in the vote space.

Slide credit: David Lowe

Extensions



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} \Big/ \frac{\partial f}{\partial x}\right)$$

Extension 1: Use the image gradient

1. **same**
2. **for each edge point** I[x,y] **in the image**

   θ = **gradient at** (x,y)

   $$d = x\cos\theta - y\sin\theta$$

   H[d, θ] += 1
3. **same**
4. **same**

(Reduces degrees of freedom)

Extensions

Extension 1: Use the image gradient

1. same
2. for each edge point $I[x,y]$ in the image

   compute unique (d,θ) based on image gradient at (x,y)

   $H[d,\theta] \mathrel{+}= 1$
3. same
4. same

(Reduces degrees of freedom)

Extension 2

- Give more votes for stronger edges (use magnitude of gradient)

Extension 3

- Change the sampling of (d,θ) to give more/less resolution

Extension 4

- The same procedure can be used with circles, squares, or any other shape...

Slide credit: Kristen Grauman

Hough Transform for Circles

- **Circle: center (a,b) and radius r**

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

- **For a fixed radius r, unknown gradient direction**

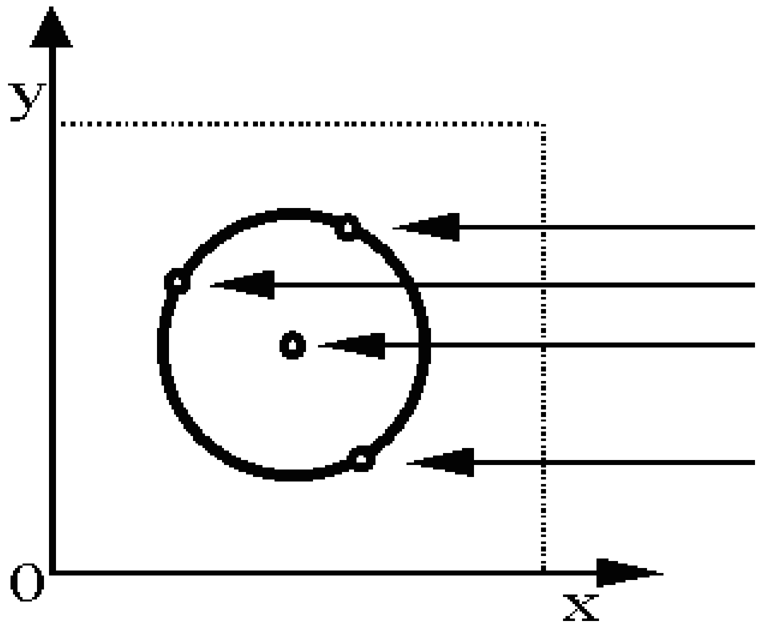
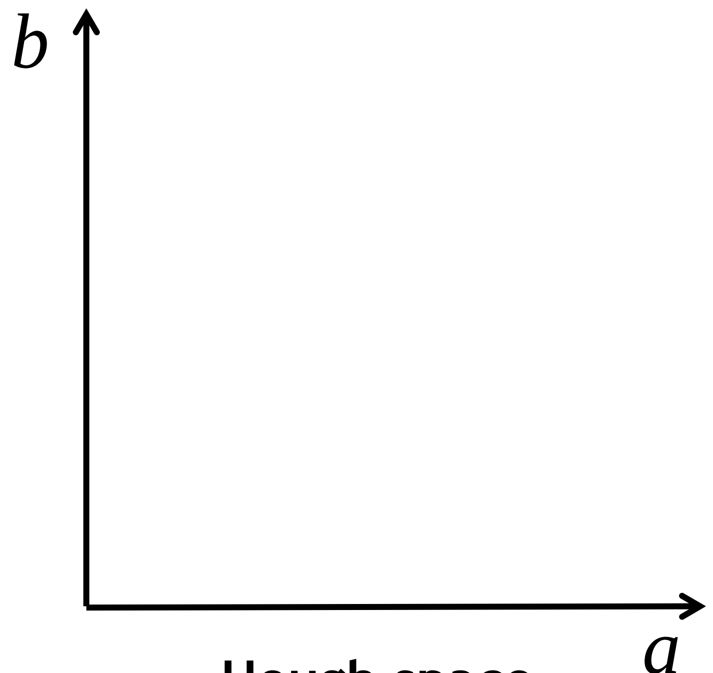


Image space



Hough space

Hough Transform for Circles

- Circle: center (a,b) and radius r

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

- For a fixed radius r, unknown gradient direction

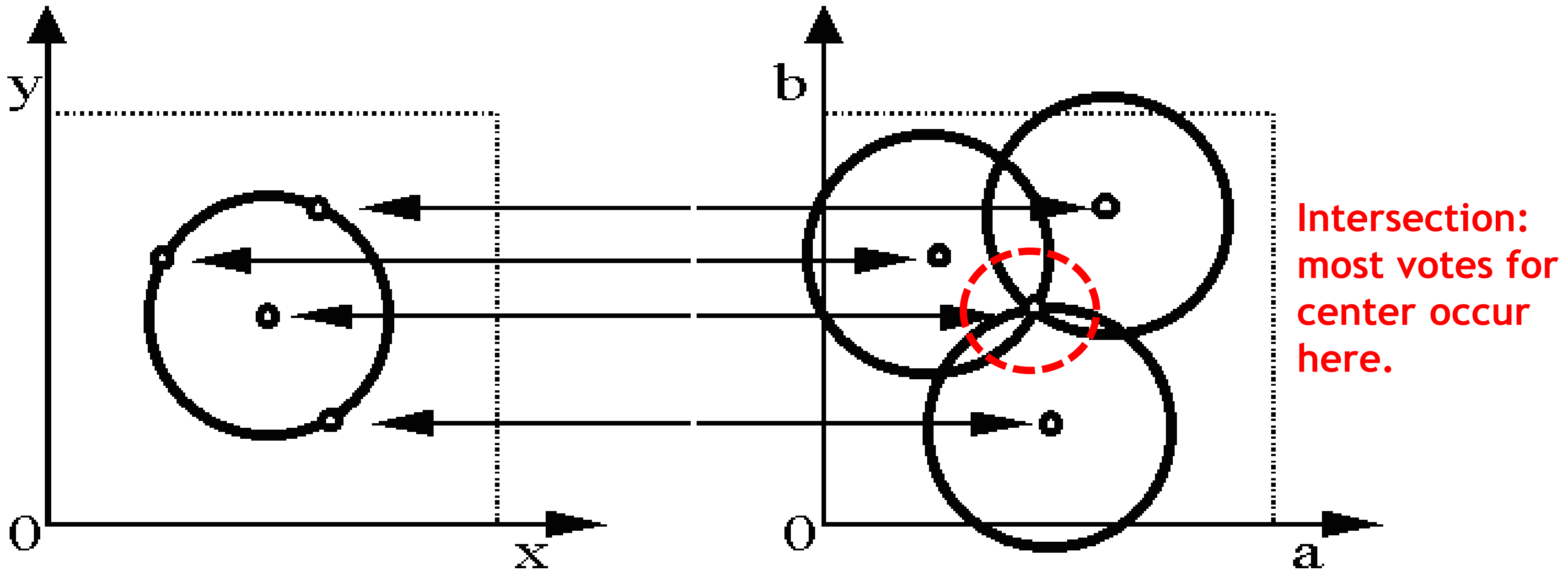


Image space

Hough space

Slide credit: Kristen Grauman

Hough Transform for Circles

- **Circle: center (a,b) and radius r**

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

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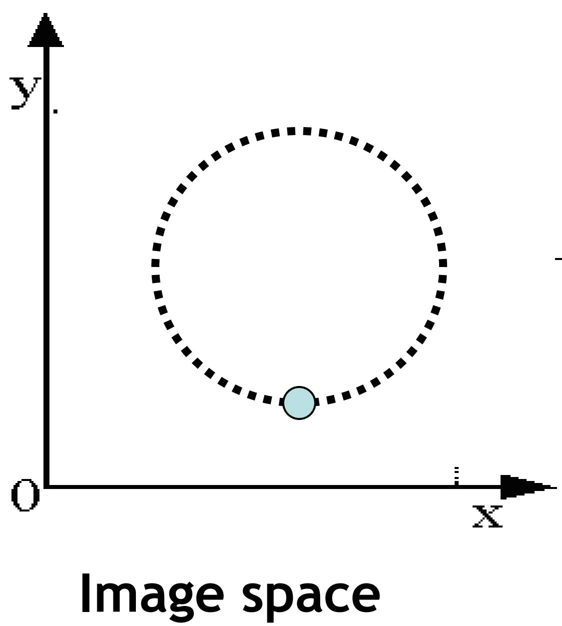
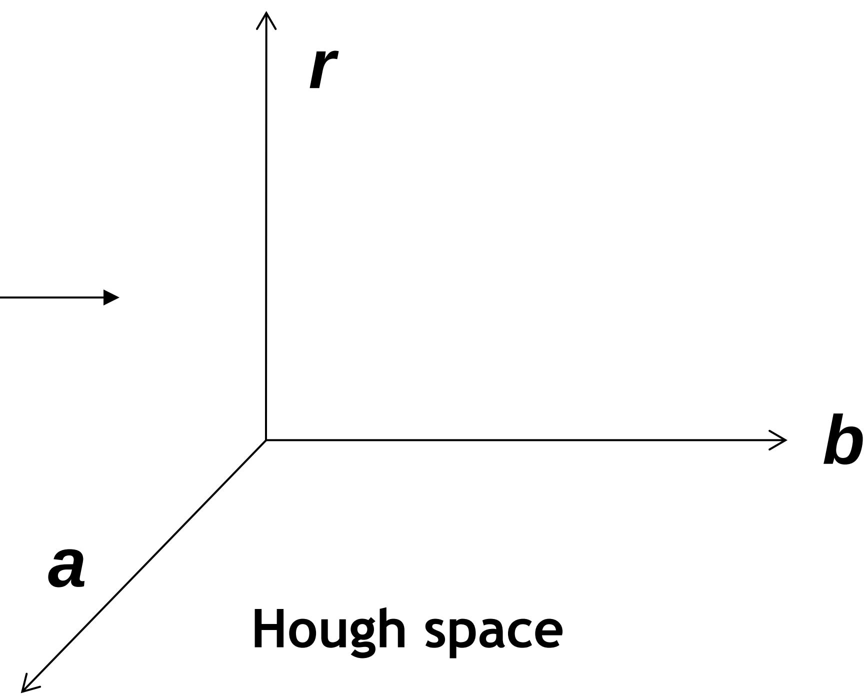


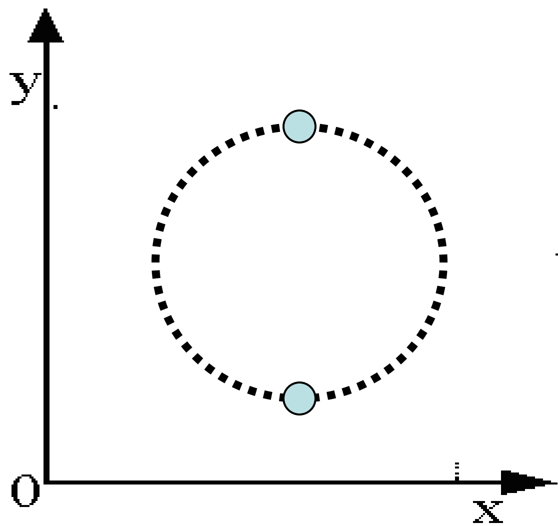


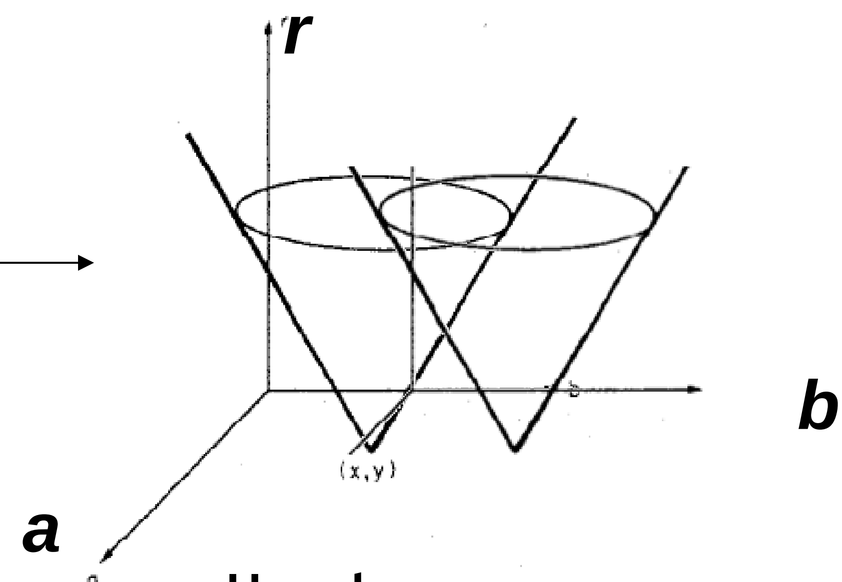
Hough Transform for Circles

- **Circle: center (a,b) and radius r**

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

- **For an unknown radius r, unknown gradient direction**





Hough Transform for Circles

- **Circle: center (a,b) and radius r**

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

- **For an unknown radius r, *known* gradient direction**

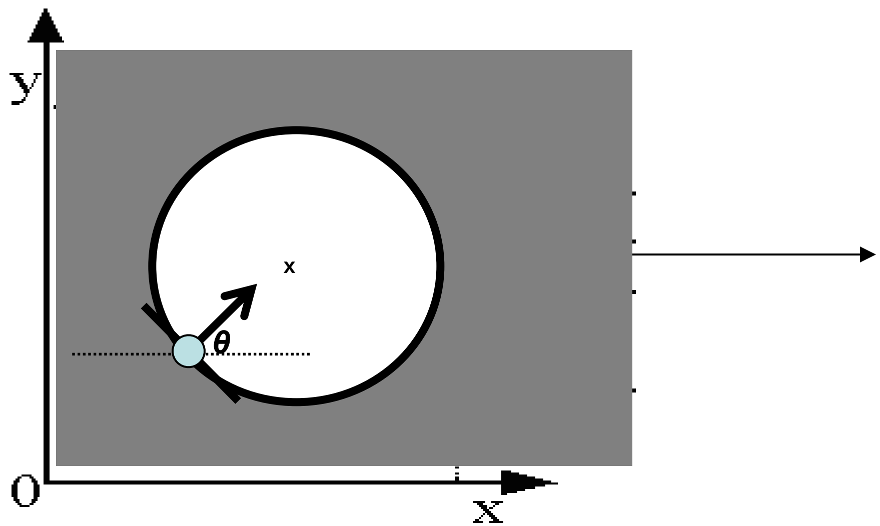
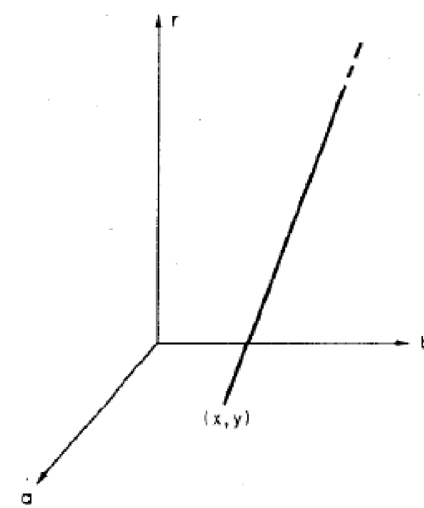


Image space



Hough space

Hough Transform for Circles

For every edge pixel (x,y) :

For each possible radius value r:

For each possible gradient direction θ:
// or use estimated gradient

$$a = x - r\cos(\theta)$$

$$b = y + r\sin(\theta)$$

$$H[a,b,r] \mathrel{+}= 1$$

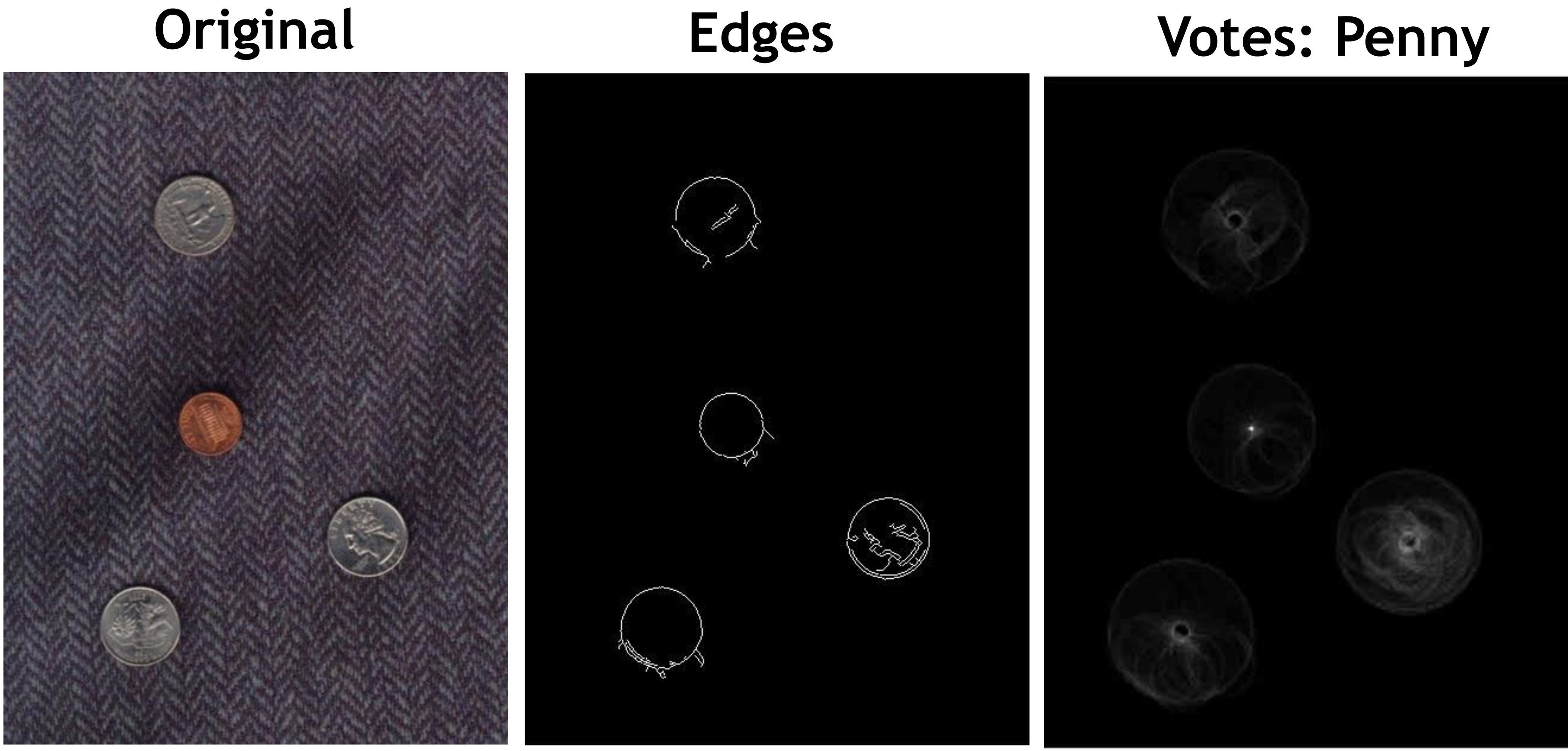
end

end

Example: Detecting Circles with Hough

Crosshair indicates results of Hough transform, bounding box found via motion differencing.

Example: Detecting Circles with Hough



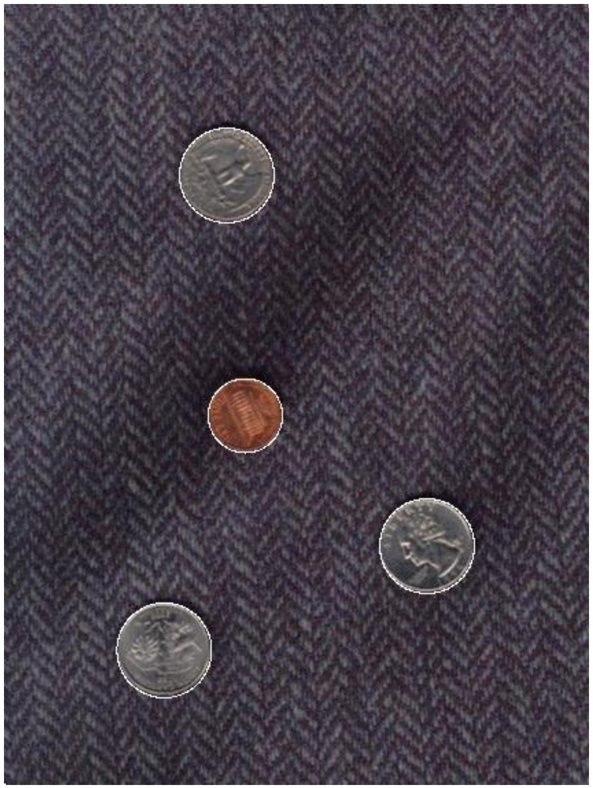
Note: a different Hough transform (with separate accumulators) was used for each circle radius (quarters vs. penny).

Coin finding sample images from: Vivek Kwatra

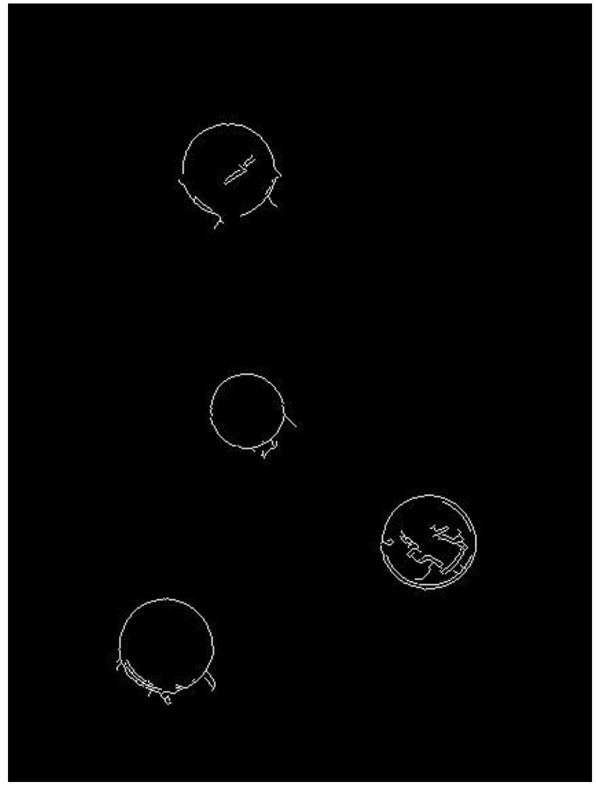
Example: Detecting Circles with Hough

Original

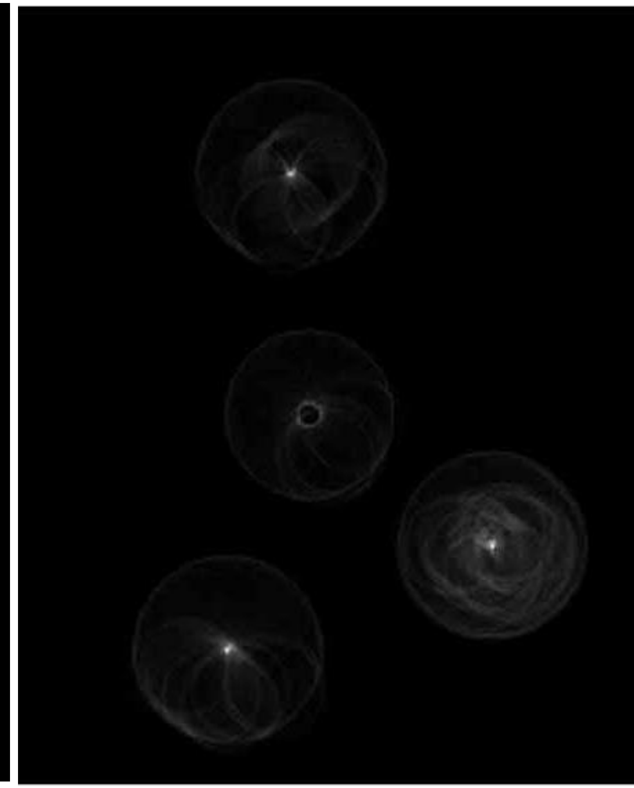
Combined detections



Edges



Votes: Quarter



Coin finding sample images from: Vivek Kwatra

Voting: Practical Tips

- Minimize irrelevant tokens first (take edge points with significant gradient magnitude)
- Choose a good grid / discretization
  - Too coarse: large votes obtained when too many different lines correspond to a single bucket
  - Too fine: miss lines because some points that are not exactly collinear cast votes for different buckets
- Vote for neighbors, also (smoothing in accumulator array)
- Utilize direction of edge to reduce free parameters by 1
- To read back which points voted for "winning" peaks, keep tags on the votes.

Hough Transform: Pros and Cons

Pros

- All points are processed independently, so can cope with occlusion
- Some robustness to noise: noise points unlikely to contribute consistently to any single bin
- Can detect multiple instances of a model in a single pass

Cons

- Complexity of search time increases exponentially with the number of model parameters
- Non-target shapes can produce spurious peaks in parameter space
- Quantization: hard to pick a good grid size

Generalized Hough Transform

- What if want to detect arbitrary shapes defined by boundary points and a reference point?

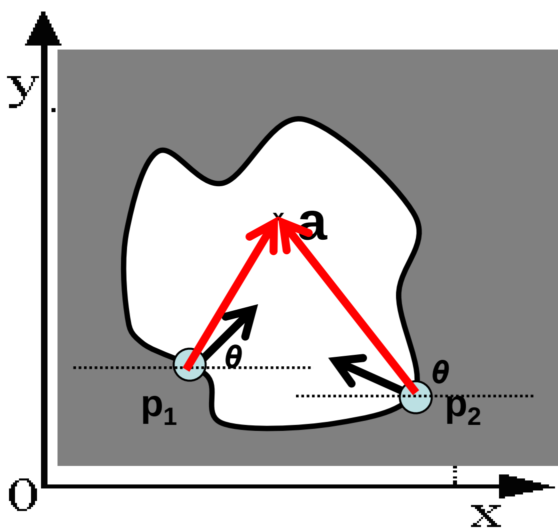


Image space

At each boundary point, compute displacement vector: $r = a - p_i$.

For a given model shape: store these vectors in a table indexed by gradient orientation θ.

[Dana H. Ballard, Generalizing the Hough Transform to Detect Arbitrary Shapes, 1980]

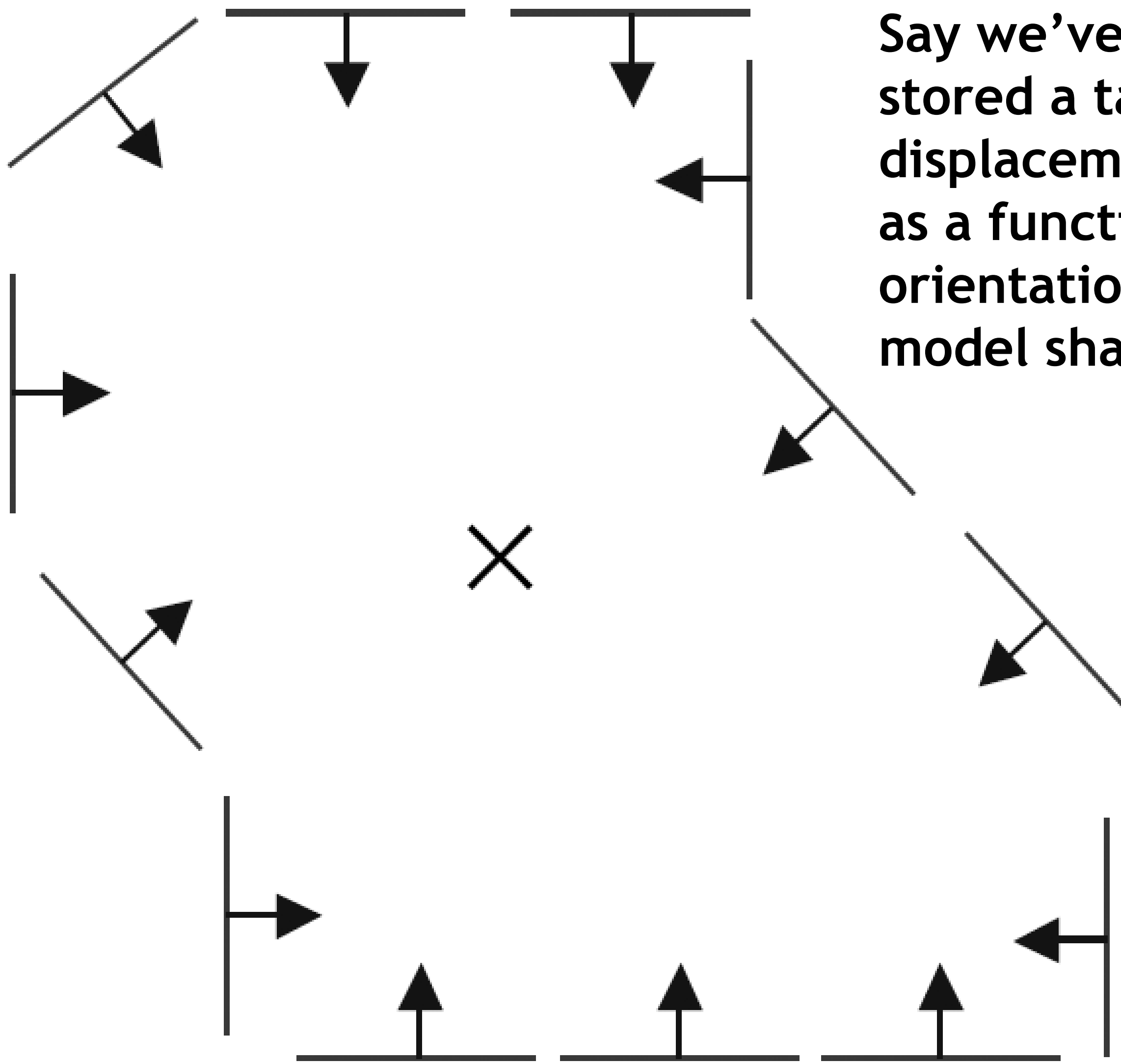
Generalized Hough Transform

To *detect* the model shape in a new image:

- For each edge point
  - Index into table with its gradient orientation θ
  - Use retrieved r vectors to vote for position of reference point
- Peak in this Hough space is reference point with most supporting edges

Assuming translation is the only transformation here, i.e., orientation and scale are fixed.

Example: Generalized Hough Transform

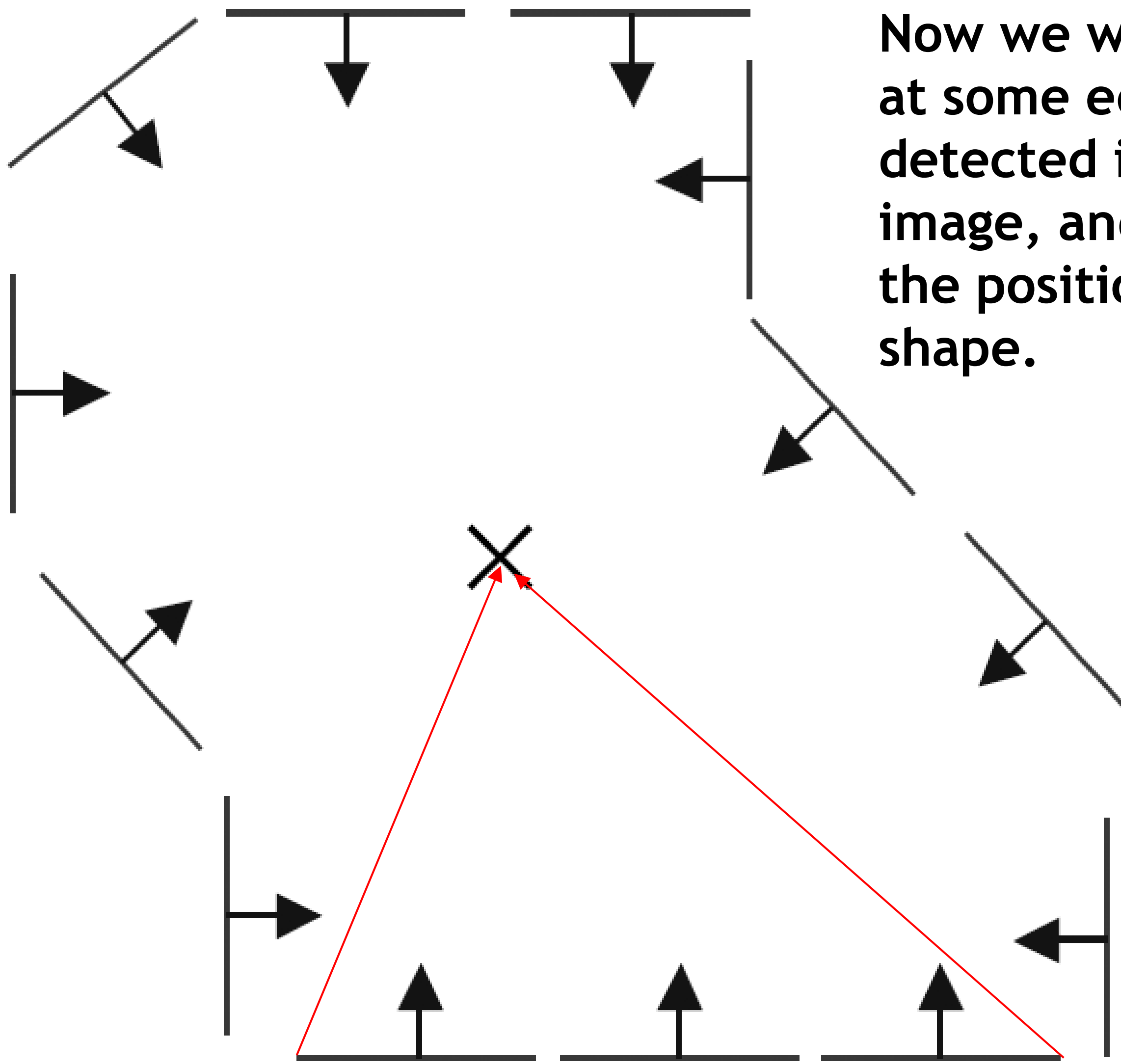


Model shape

Say we've already stored a table of displacement vectors as a function of edge orientation for this model shape.

Slide credit: Svetlana Lazebnik

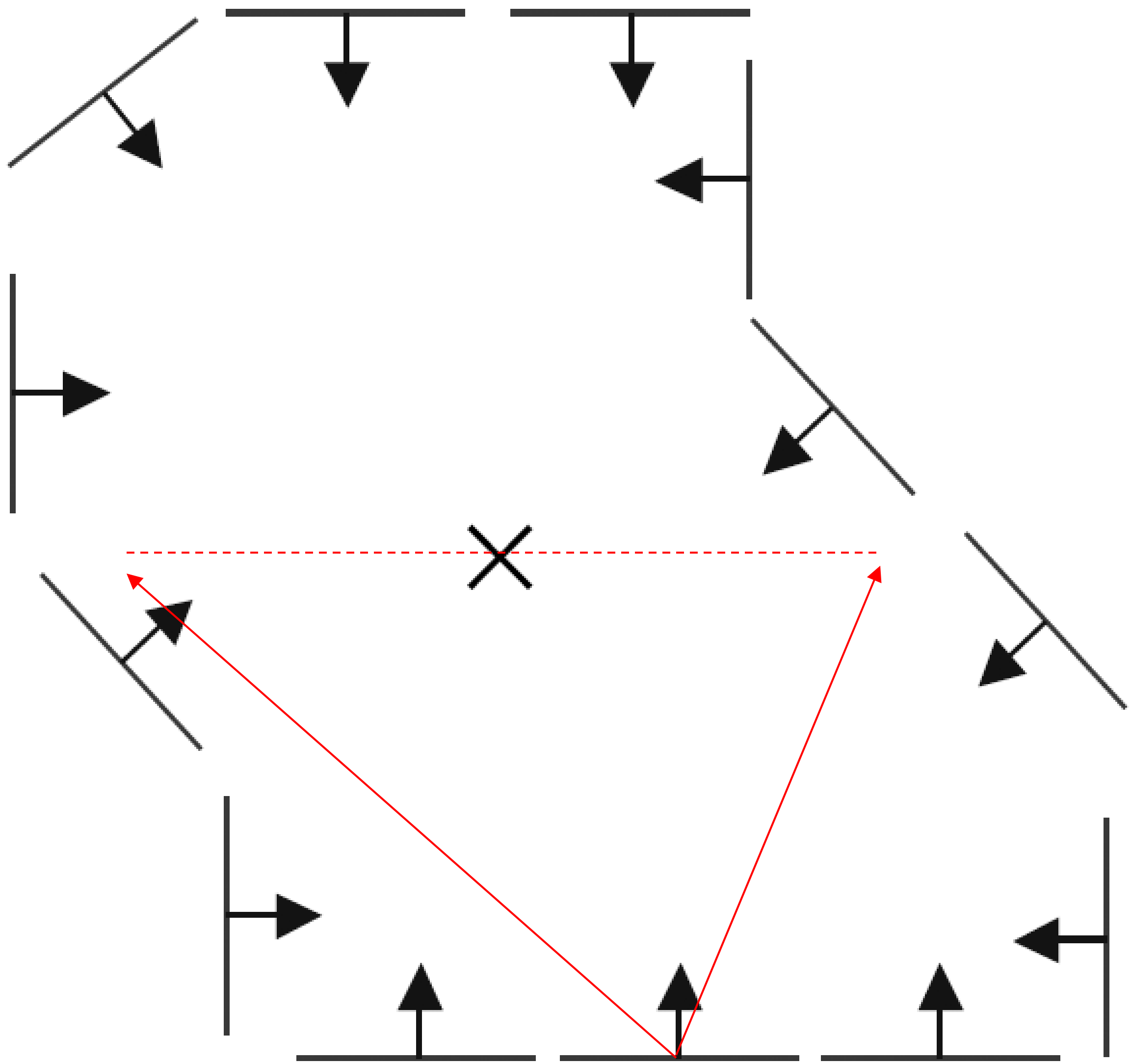
Example: Generalized Hough Transform



Now we want to look at some edge points detected in a *new* image, and vote on the position of that shape.

Displacement vectors for model points

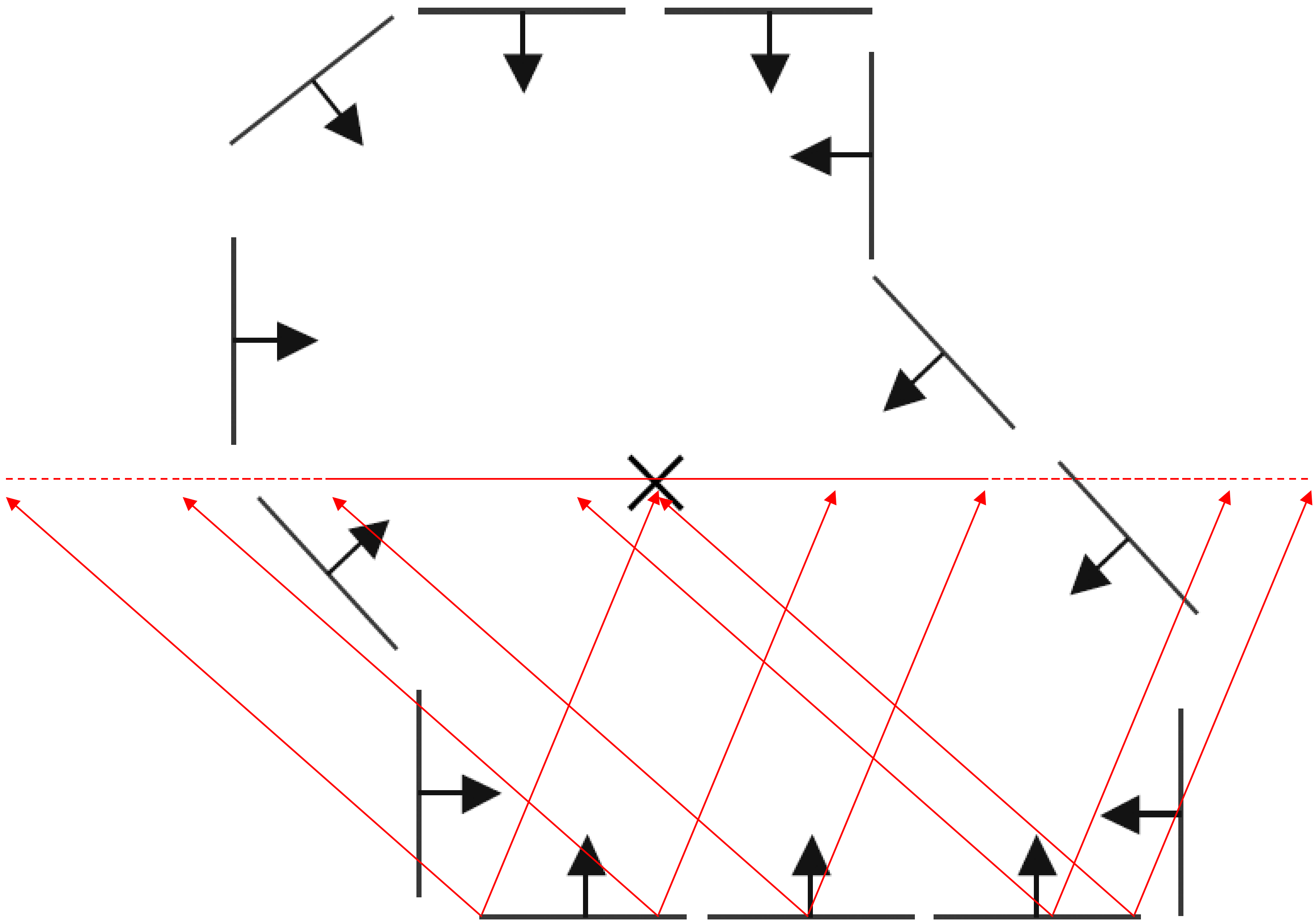
Example: Generalized Hough Transform



Range of voting locations for test point

Slide credit: Svetlana Lazebnik

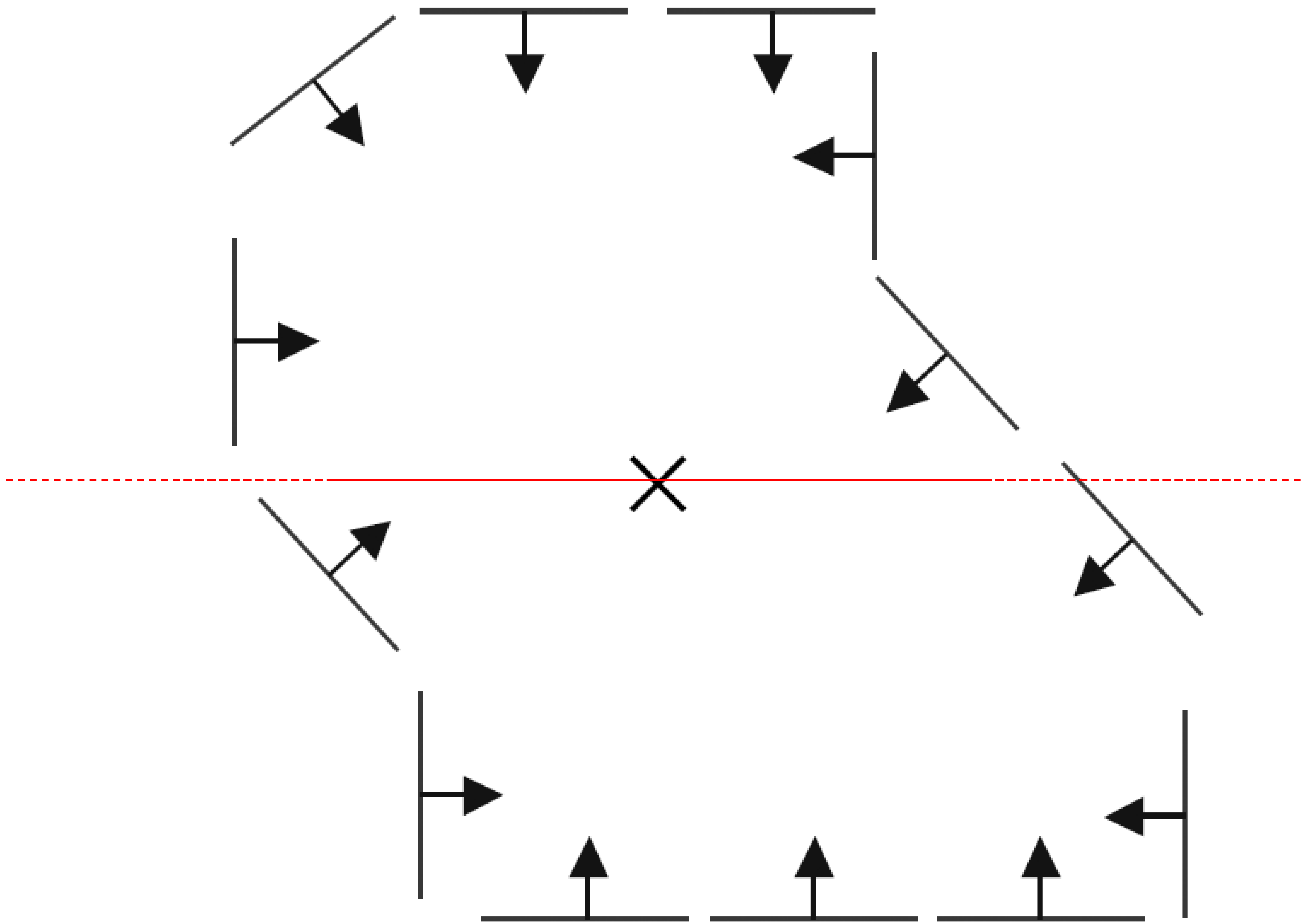
Example: Generalized Hough Transform



Range of voting locations for test point

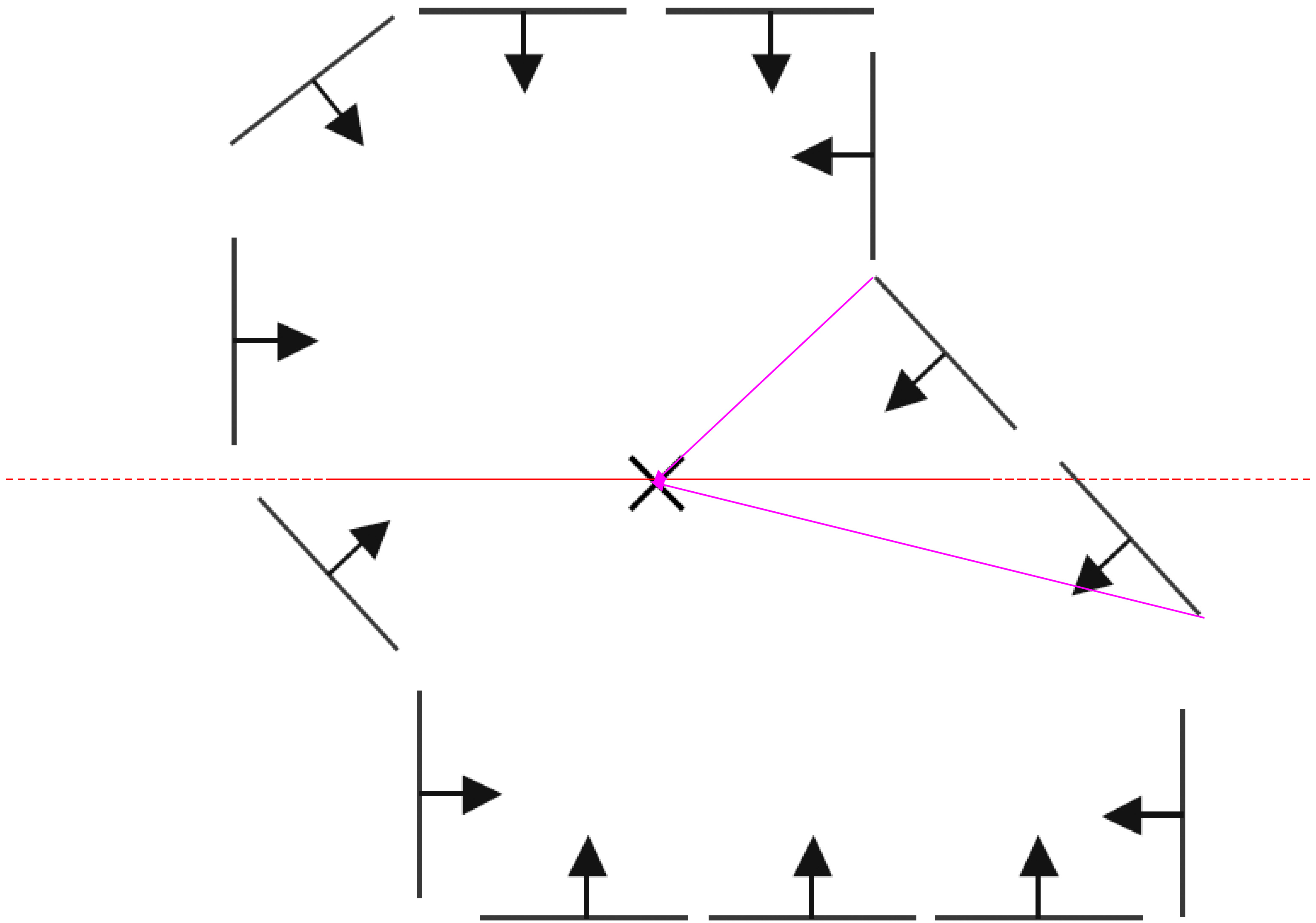
Slide credit: Svetlana Lazebnik

Example: Generalized Hough Transform



Votes for points with $\theta = \uparrow$

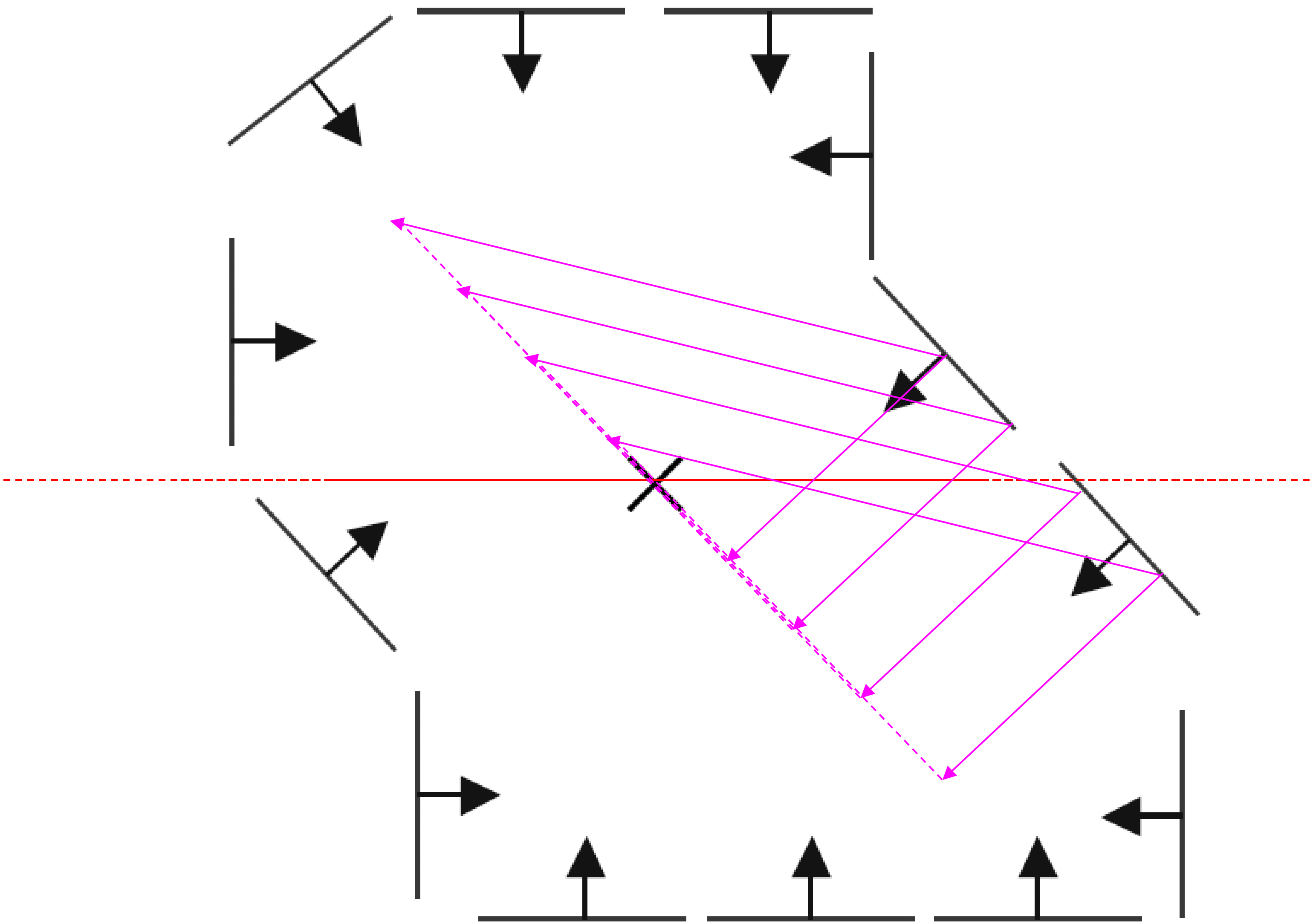
Example: Generalized Hough Transform



Displacement vectors for model points

Slide credit: Svetlana Lazebnik

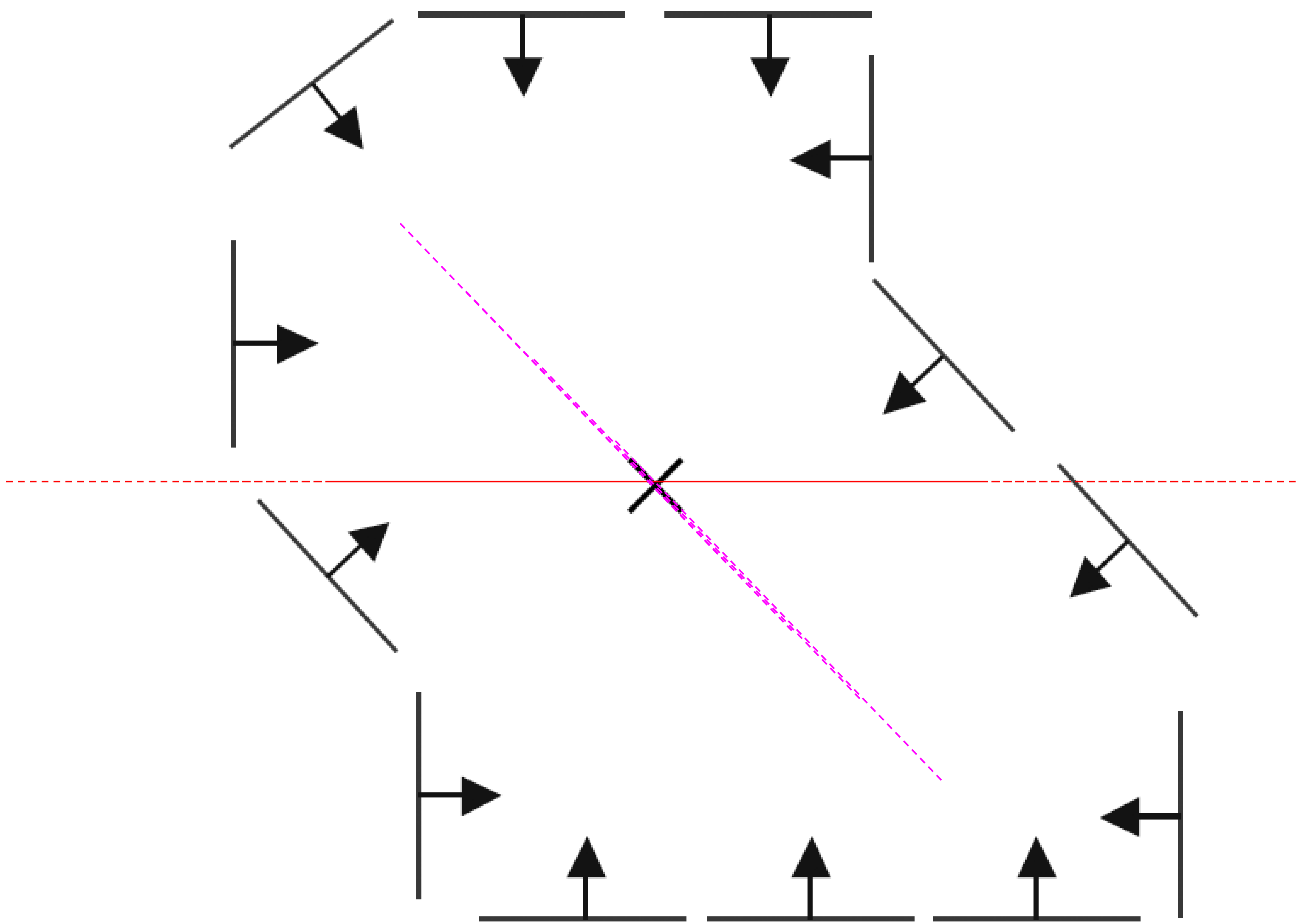
Example: Generalized Hough Transform



Range of voting locations for test point

Slide credit: Svetlana Lazebnik

Example: Generalized Hough Transform

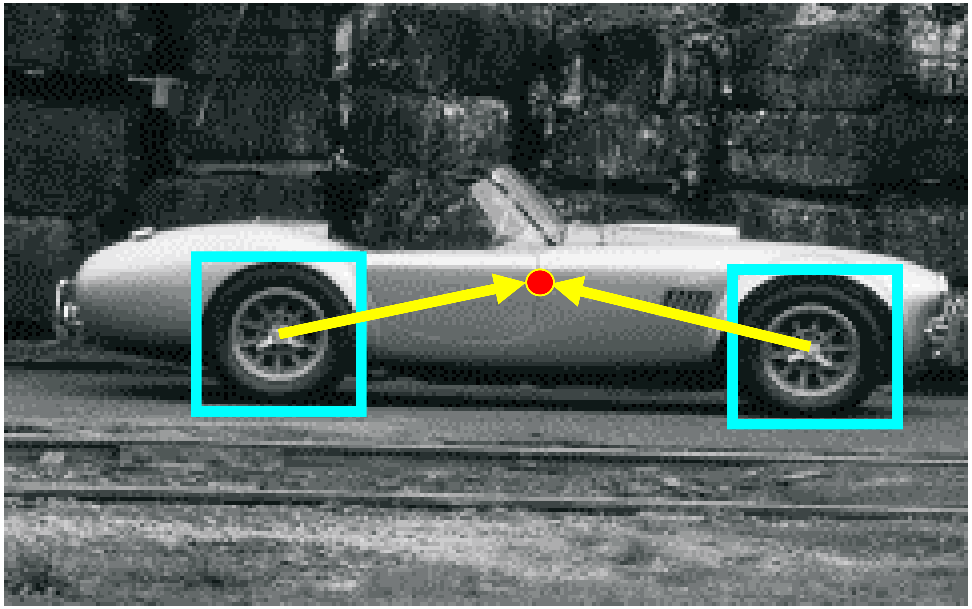


Votes for points with θ = ↙

Slide credit: Svetlana Lazebnik

Application in Recognition

- Instead of indexing displacements by gradient orientation, index by "visual codeword".



Training image

Visual codeword with displacement vectors

B. Leibe, A. Leonardis, and B. Schiele, Robust Object Detection with Interleaved Categorization and Segmentation, International Journal of Computer Vision, Vol. 77(1-3), 2008.

Application in Recognition

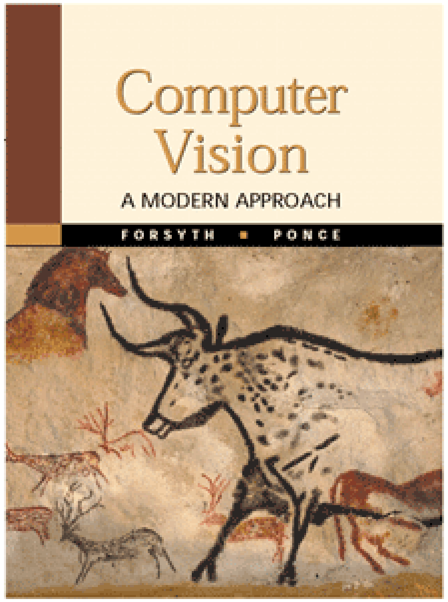
- Instead of indexing displacements by gradient orientation, index by "visual codeword".

Test image

- We'll hear more about this method in lecture 14…

References and Further Reading

- Background information on edge detection can be found in Chapter 8 of
  - D. Forsyth, J. Ponce, *Computer Vision - A Modern Approach*. Prentice Hall, 2003



- Read Ballard & Brown's description of the Generalized Hough Transform in Chapter 4.3 of
  - D.H. Ballard & C.M. Brown, *Computer Vision*, Prentice Hall, 1982 (available from the class homepage)
- Try the Hough Transform demo at http://www.dis.uniroma1.it/~iocchi/slides/icra2001/java/hough.html