Computer Vision – Lecture 4

Gradients & Edges

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Announcements

- Exercise sheet 2 is available
  - Thresholding, Morphology
  - Gaussian smoothing
  - Image gradients
  - Edge Detection

  $\Rightarrow$ *Deadline: Wednesday night, 05.11. (next week).*

- Reminder
  - You’re encouraged to form teams of up to 3 people!
  - Make it easy for Aljosa & Dora to correct your solutions:
    - Turn in everything as a single zip archive.
    - Use the provided Matlab framework.
    - For each exercise, you need to implement the corresponding `apply` function. If the screen output matches the expected output, you will get the points for the exercise; else, no points.
    - Matlab helps you to find errors (red lines under your code)!

Announcements (2)

- Exam
  - There will be a written exam.
  - I'm currently organizing the exam date…
  - We'll organize a test exam towards the end of the semester.

- Admission requirements
  - Need to reach at least 50% of the exercise points.
  - Points are given
    - for each exercise sheet.
    - for the test exam.
  - Bonus points will be available on several occasions.

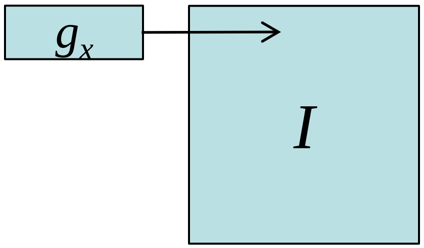
  ⇒ *If you follow the lecture and do the exercises regularly, you won't have to worry about getting admitted.*

Course Outline

- Image Processing Basics
  - Image Formation
  - Binary Image Processing
  - Linear Filters
  - Edge & Structure Extraction
- Segmentation
- Local Features & Matching
- Object Recognition and Categorization
- 3D Reconstruction
- Motion and Tracking

Topics of This Lecture

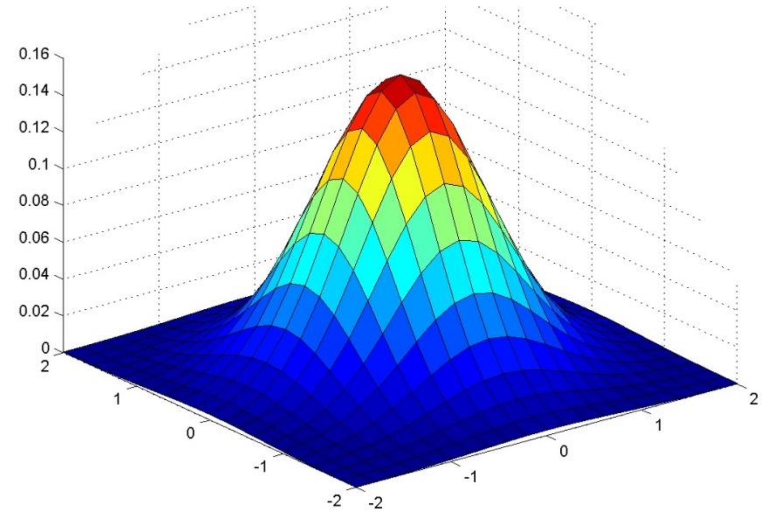
- Recap: Linear Filters
- Multi-Scale representations
  - How to properly rescale an image?
- Filters as templates
  - Correlation as template matching
- Image gradients
  - Derivatives of Gaussian
- Edge detection
  - Canny edge detector



Recap: Gaussian Smoothing

- Gaussian kernel

$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$



- Rotationally symmetric
- Weights nearby pixels more than distant ones
  - This makes sense as 'probabilistic' inference about the signal
- A Gaussian gives a good model of a fuzzy blob

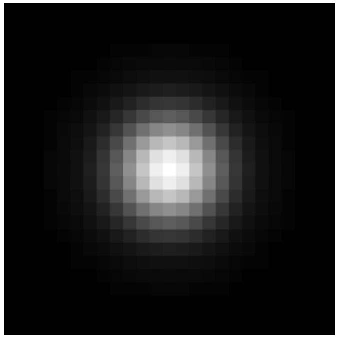
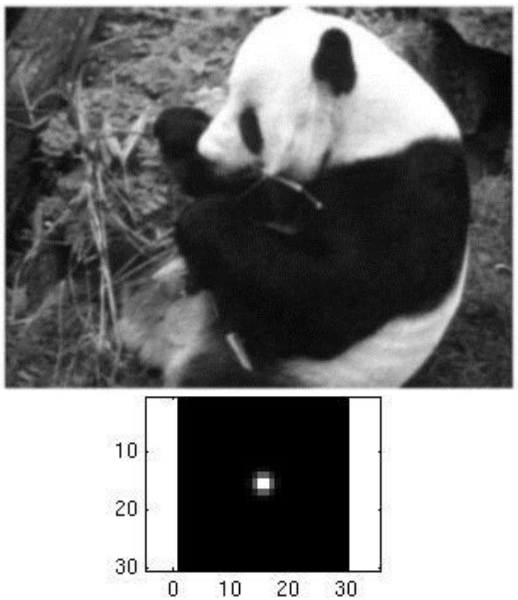
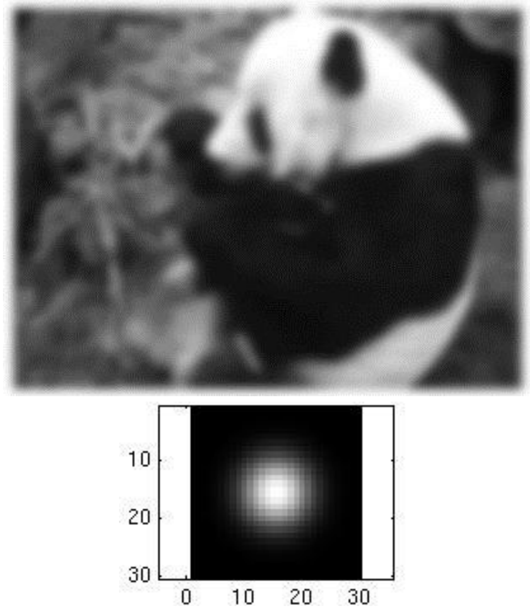


Image Source: Forsyth & Ponce

Recap: Smoothing with a Gaussian

- Parameter σ is the “scale” / “width” / “spread” of the Gaussian kernel and controls the amount of smoothing.





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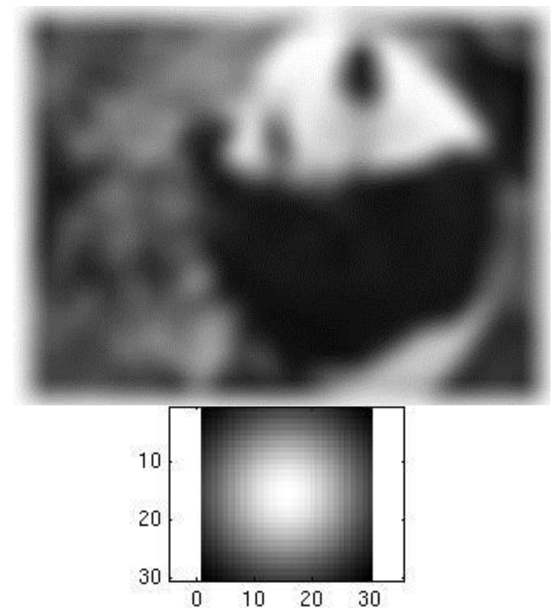


```
for sigma=1:3:10
   h = fspecial('gaussian`, fsize, sigma);
   out = imfilter(im, h);
   imshow(out);
   pause;
end
```

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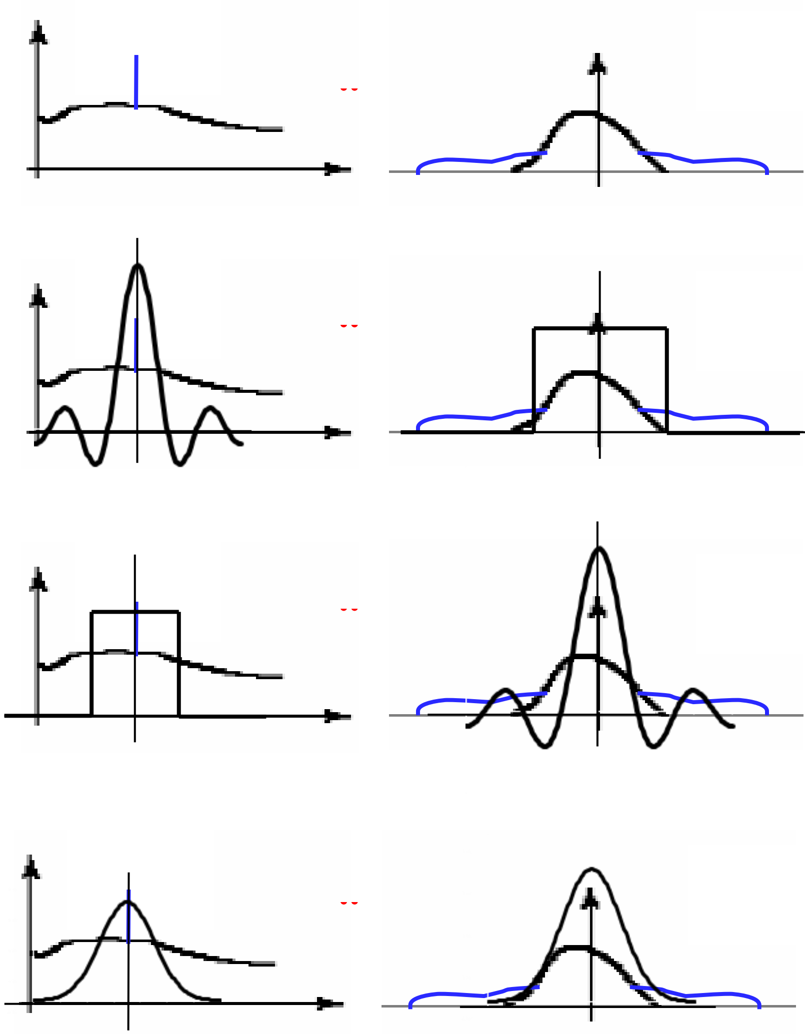
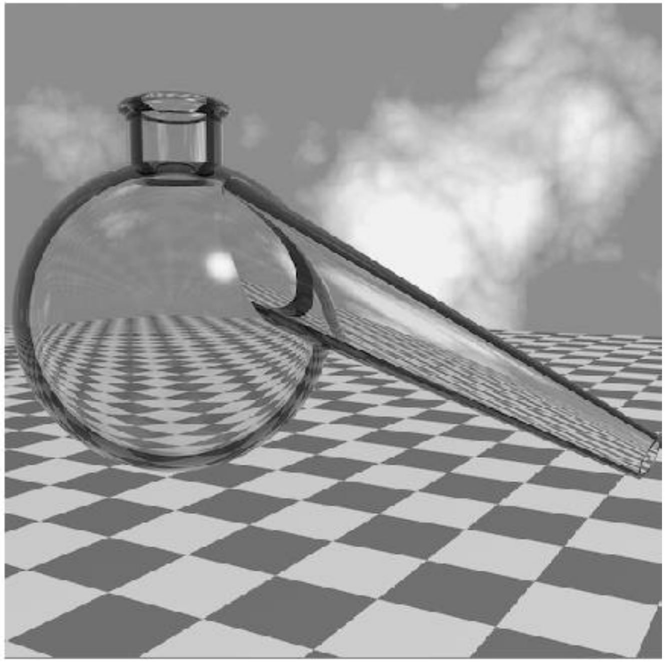
Recap: Effect of Filtering

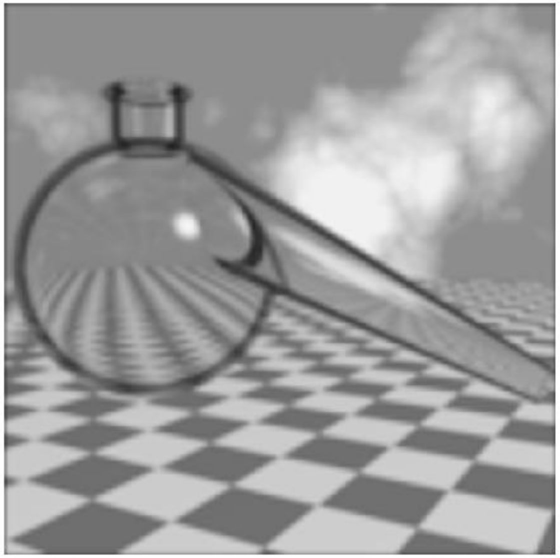
- Noise introduces high frequencies. To remove them, we want to apply a "low-pass" filter.
- The ideal filter shape in the frequency domain would be a box. But this transfers to a spatial sinc, which has infinite spatial support.
- A compact spatial box filter transfers to a frequency sinc, which creates artifacts.
- A Gaussian has compact support in both domains. This makes it a convenient choice for a low-pass filter.



Recap: Low-Pass vs. High-Pass



Original image



Low-pass filtered

High-pass filtered

Image Source: S. Chenney

Topics of This Lecture

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  - How to properly rescale an image?
- Filters as templates
  - Correlation as template matching
- Image gradients
  - Derivatives of Gaussian
- Edge detection
  - Canny edge detector

Motivation: Fast Search Across Scales

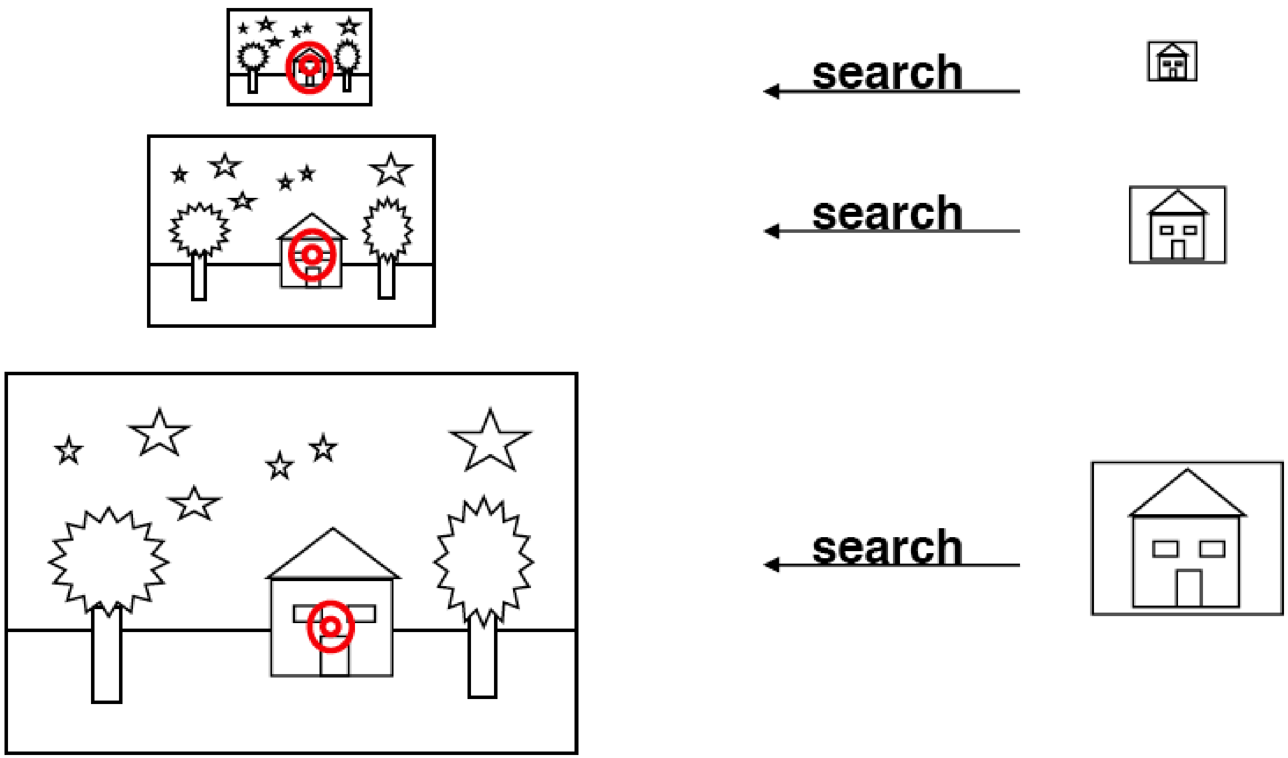


Image Source: Irani & Basri

Recap: Sampling and Aliasing

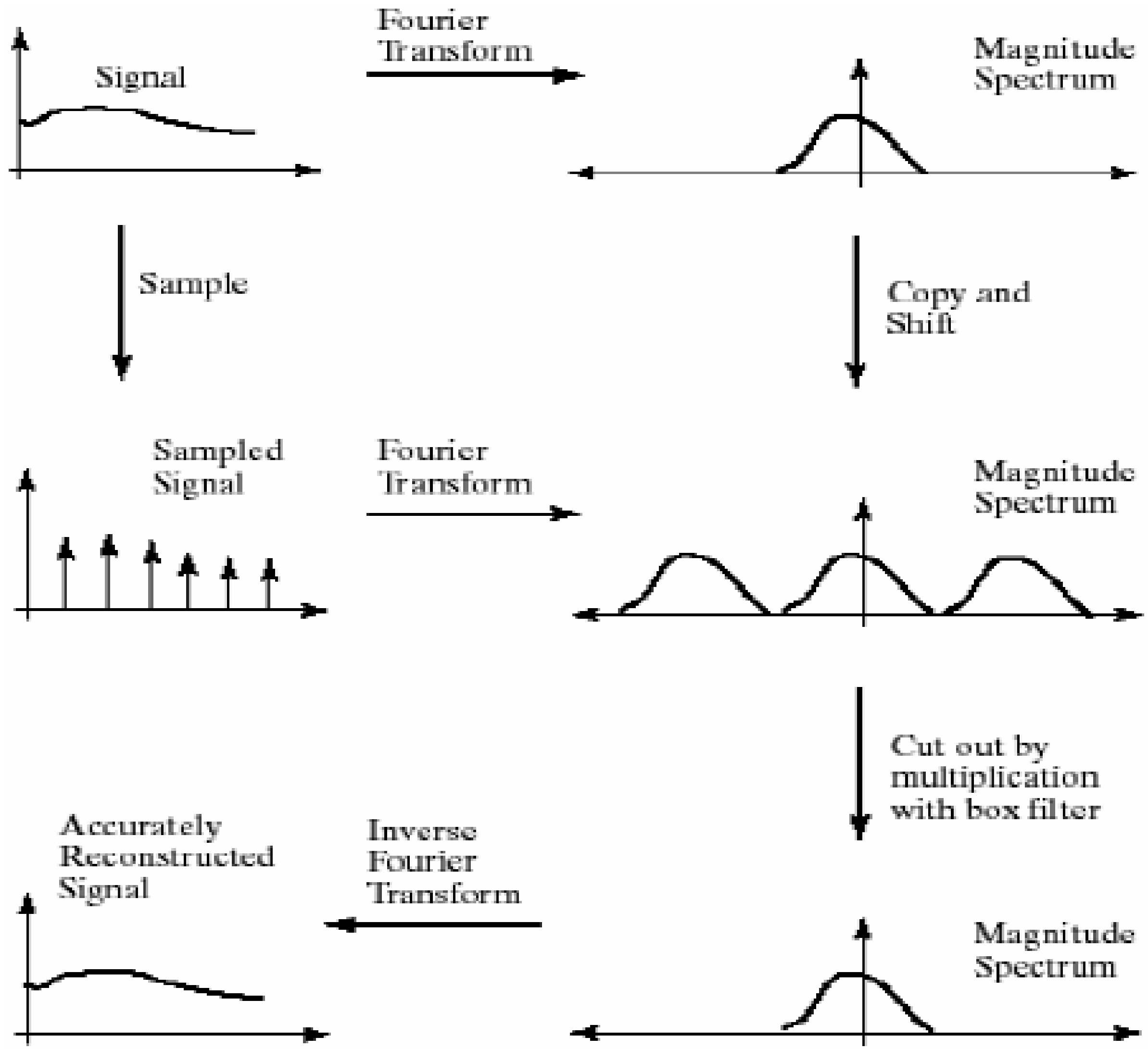


Image Source: Forsyth & Ponce

Recap: Sampling and Aliasing

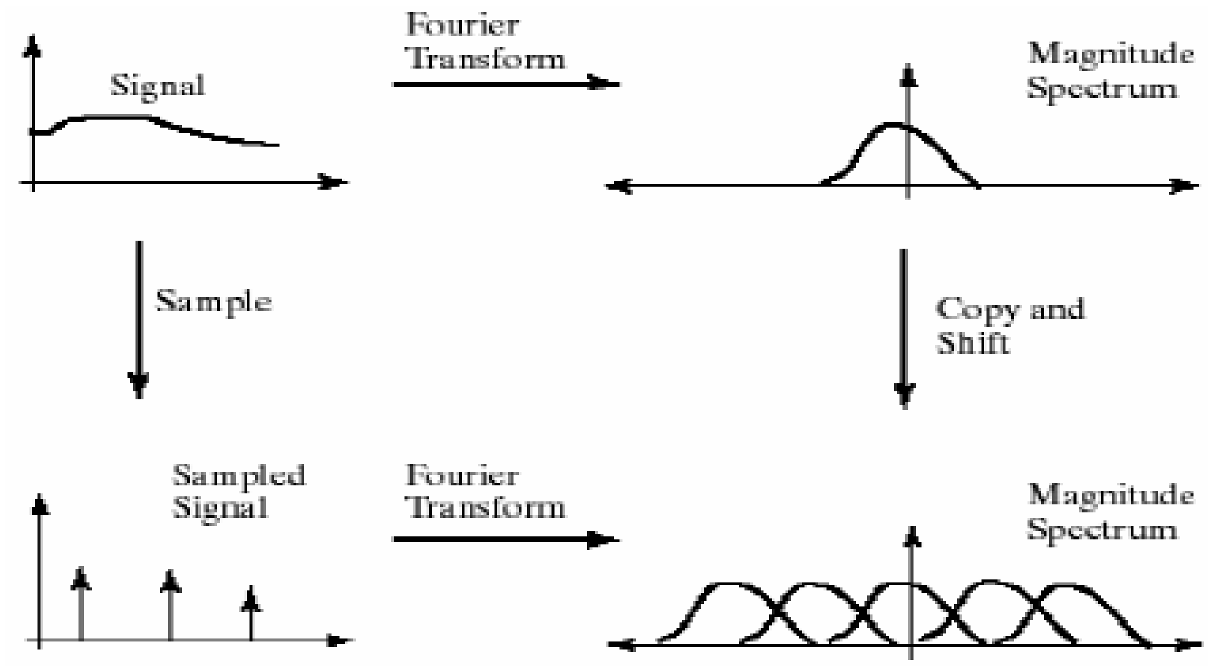


Image Source: Forsyth & Ponce

Recap: Sampling and Aliasing

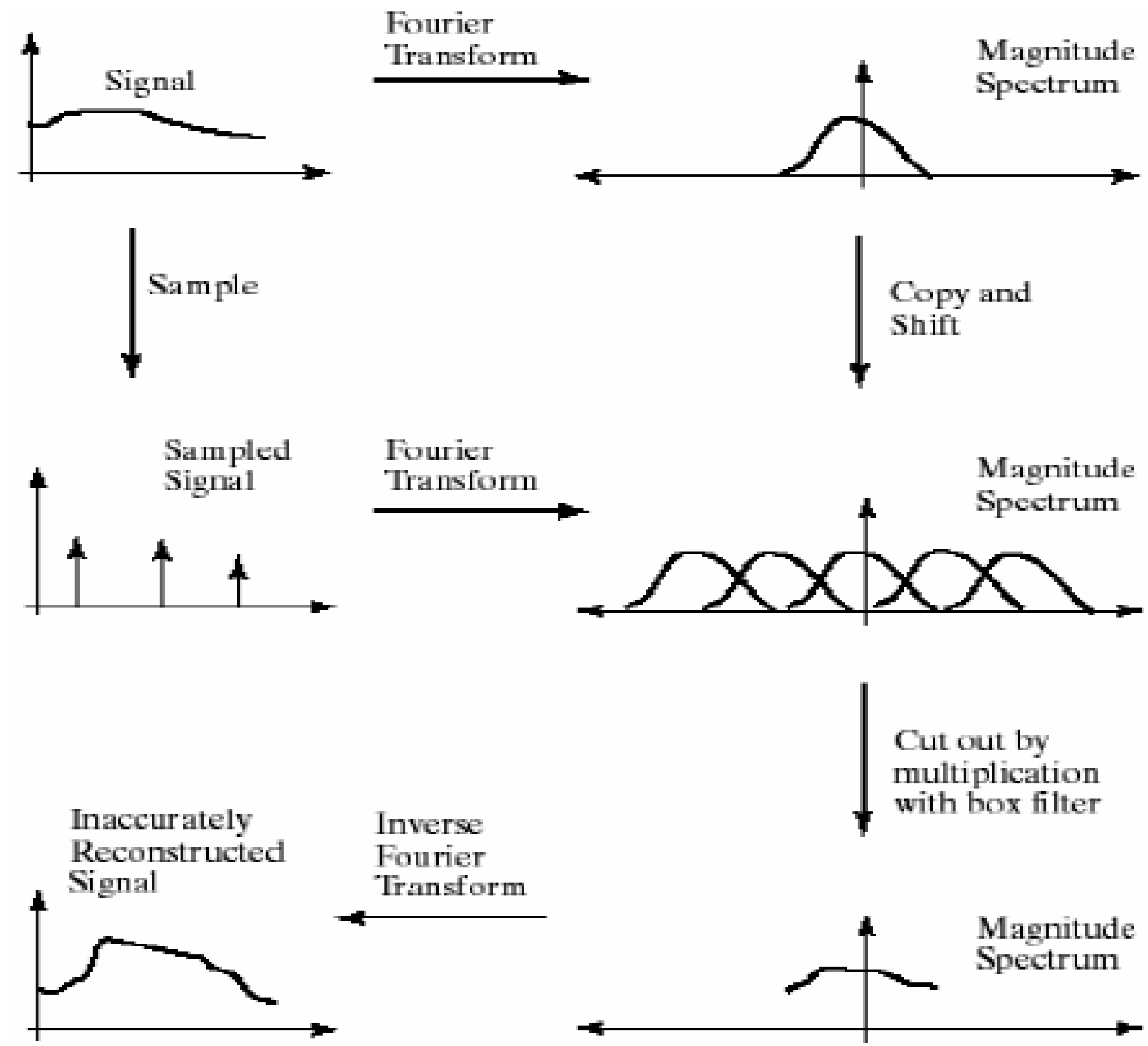
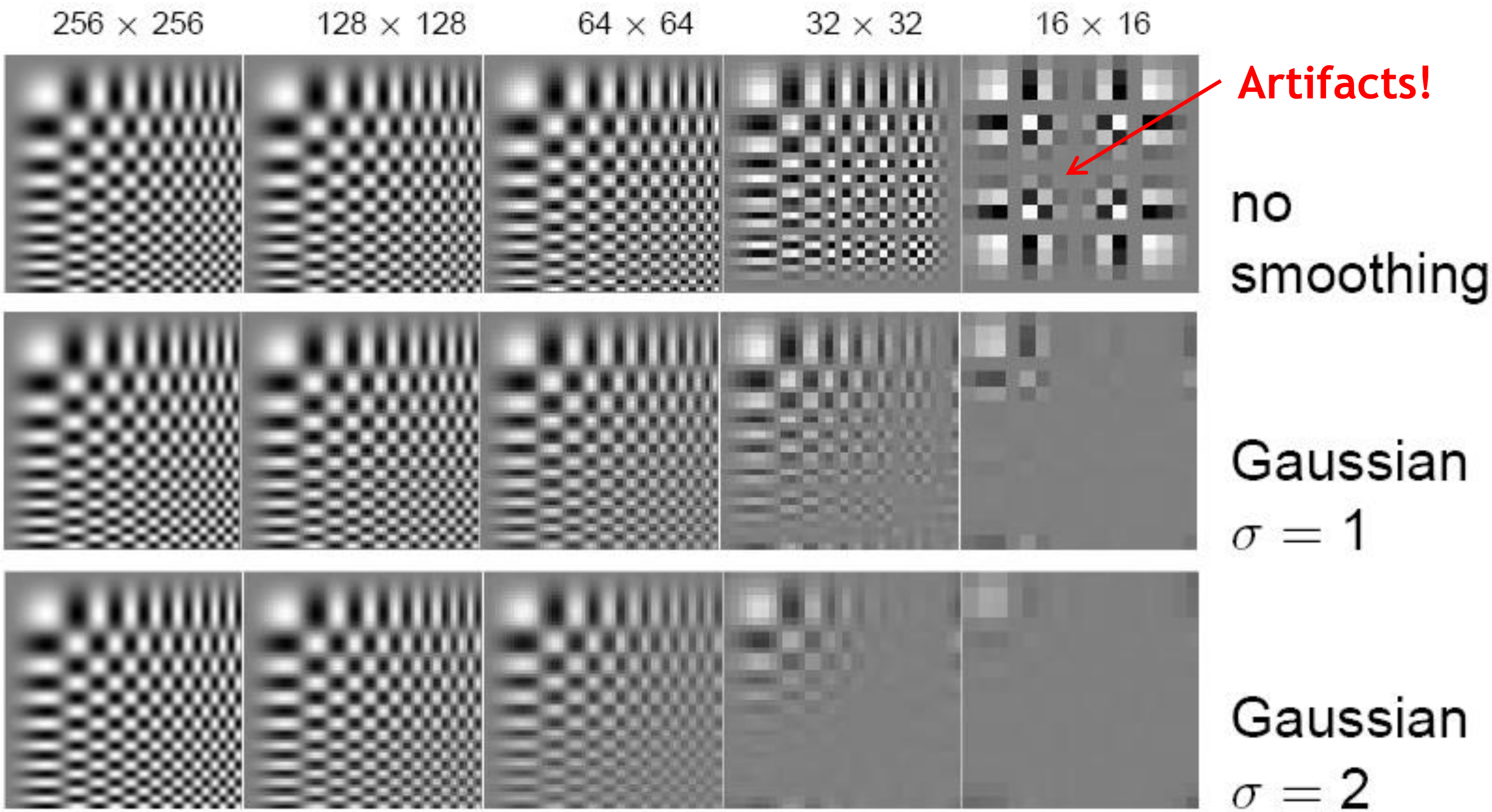


Image Source: Forsyth & Ponce

Recap: Resampling with Prior Smoothing

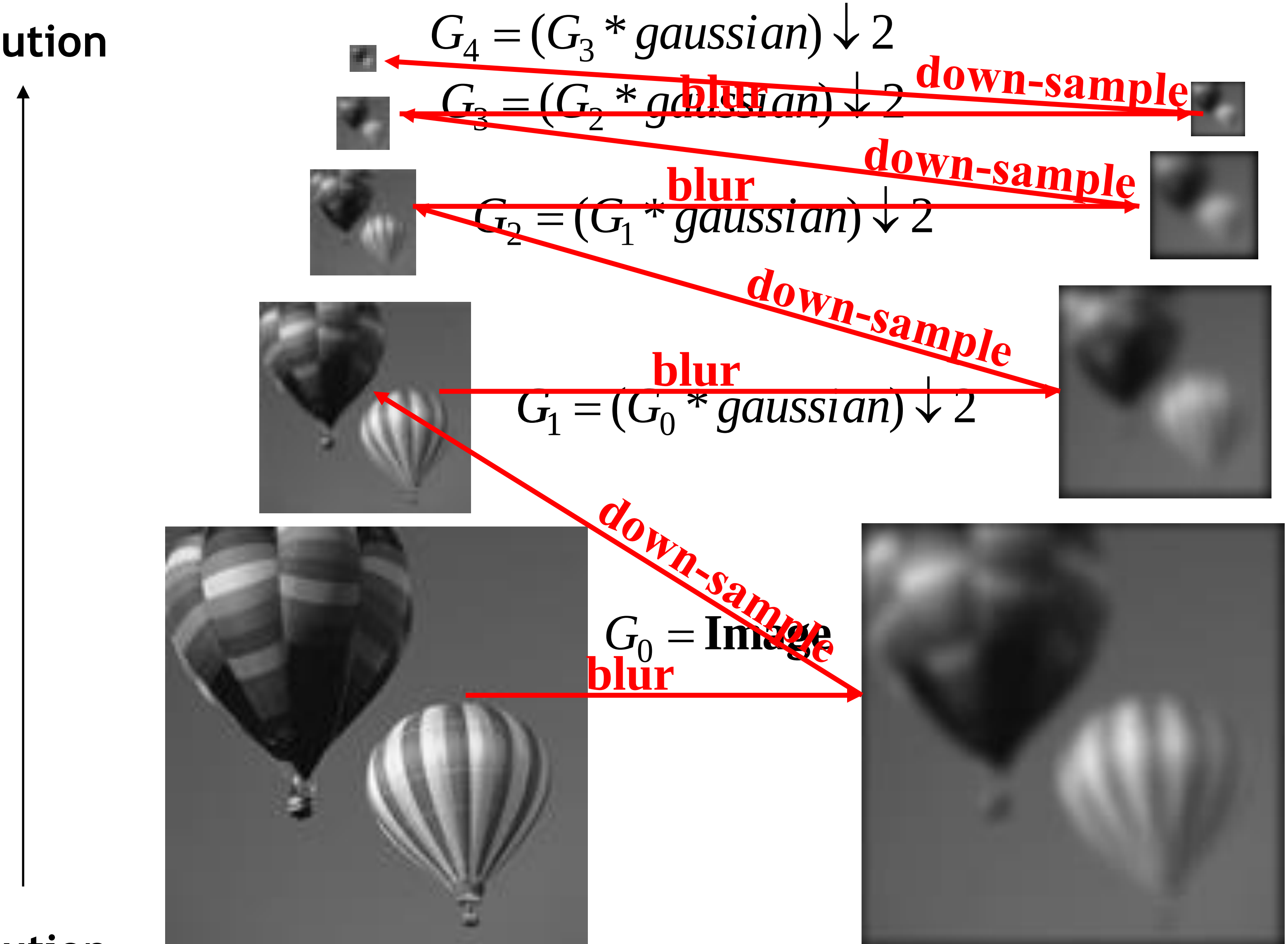


- **Note: We cannot recover the high frequencies, but we can avoid artifacts by smoothing before resampling.**

Image Source: Forsyth & Ponce

The Gaussian Pyramid

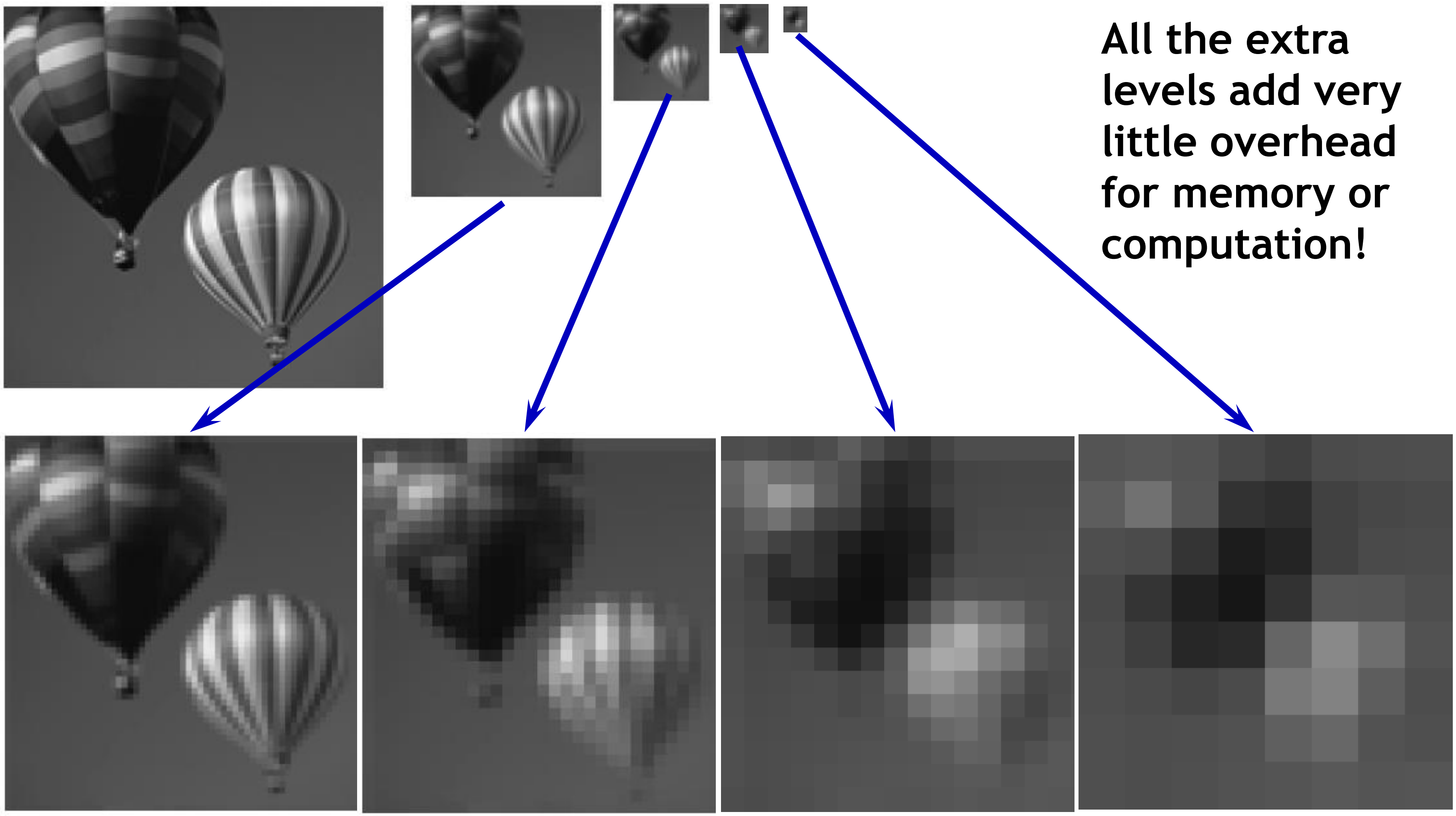
Low resolution



High resolution

Source: Irani & Basri

Gaussian Pyramid – Stored Information



All the extra levels add very little overhead for memory or computation!

Source: Irani & Basri

Summary: Gaussian Pyramid

- Construction: create each level from previous one
  - Smooth and sample
- Smooth with Gaussians, in part because
  - a Gaussian*Gaussian = another Gaussian
  - $G(\sigma_1) * G(\sigma_2) = G(\mathrm{sqrt}(\sigma_1^{\,2} + \sigma_2^{\,2}))$
- Gaussians are low-pass filters, so the representation is redundant once smoothing has been performed.

  $\Rightarrow$ There is no need to store smoothed images at the full original resolution.

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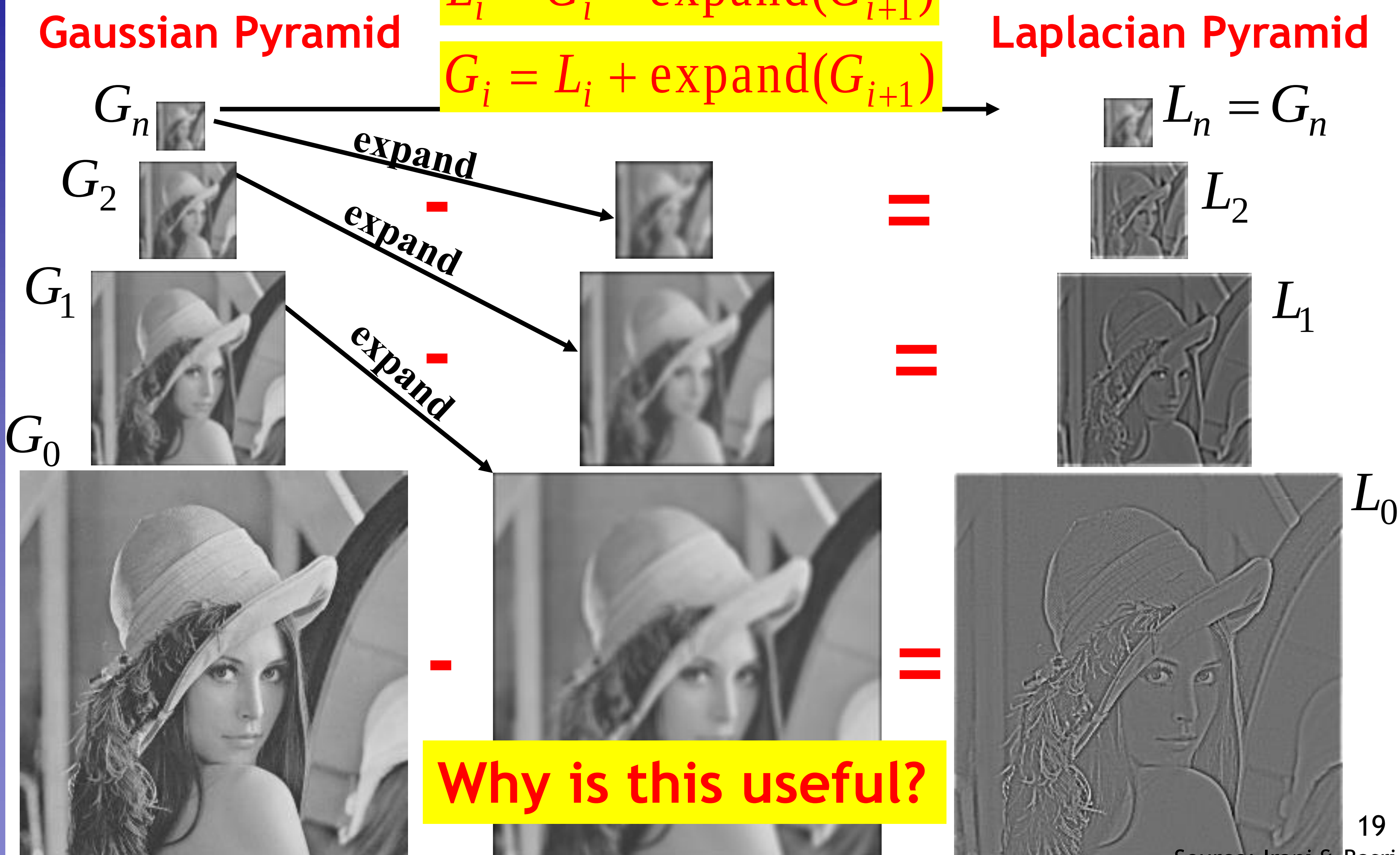
The Laplacian Pyramid

$$L_i = G_i - \mathrm{expand}(G_{i+1})$$

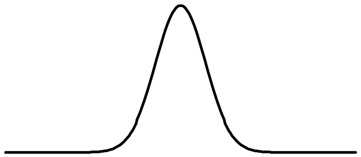
$$G_i = L_i + \mathrm{expand}(G_{i+1})$$



Why is this useful?

Source: Irani & Basri

Laplacian ~ Difference of Gaussian



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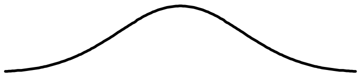

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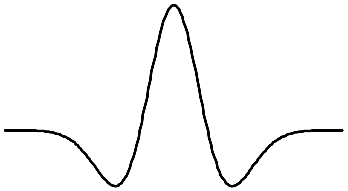

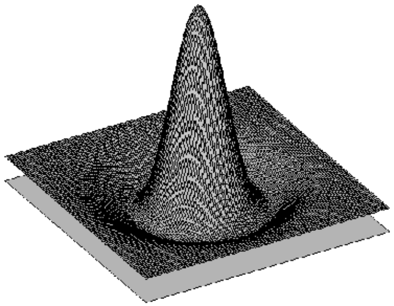

DoG = Difference of Gaussians

Cheap approximation - no derivatives needed.

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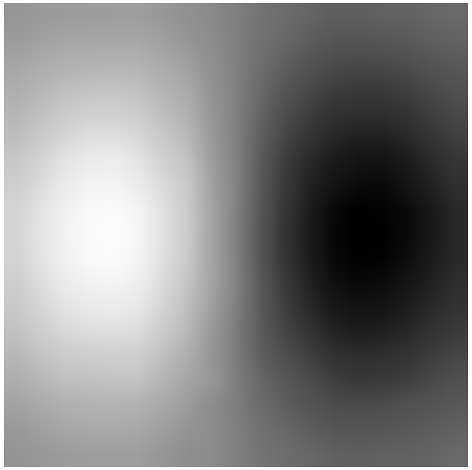
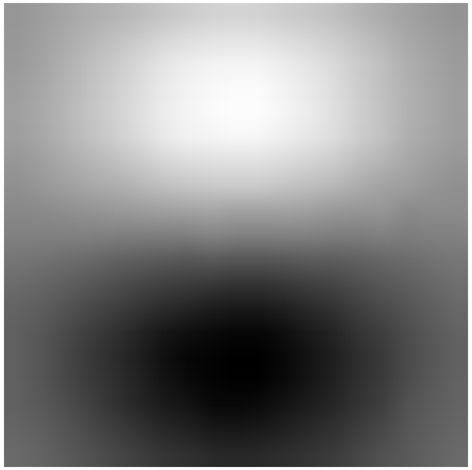
- Filters as templates
  - Correlation as template matching
- Image gradients
  - Derivatives of Gaussian
- Edge detection
  - Canny edge detector

Note: Filters are Templates

- Applying a filter at some point can be seen as taking a dot-product between the image and some vector.
- Filtering the image is a set of dot products.

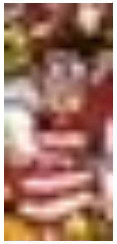
- Insight
  - Filters look like the effects they are intended to find.
  - Filters find effects they look like.





Where's Waldo?

Scene

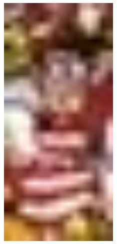


Template

Slide credit: Kristen Grauman

Where's Waldo?

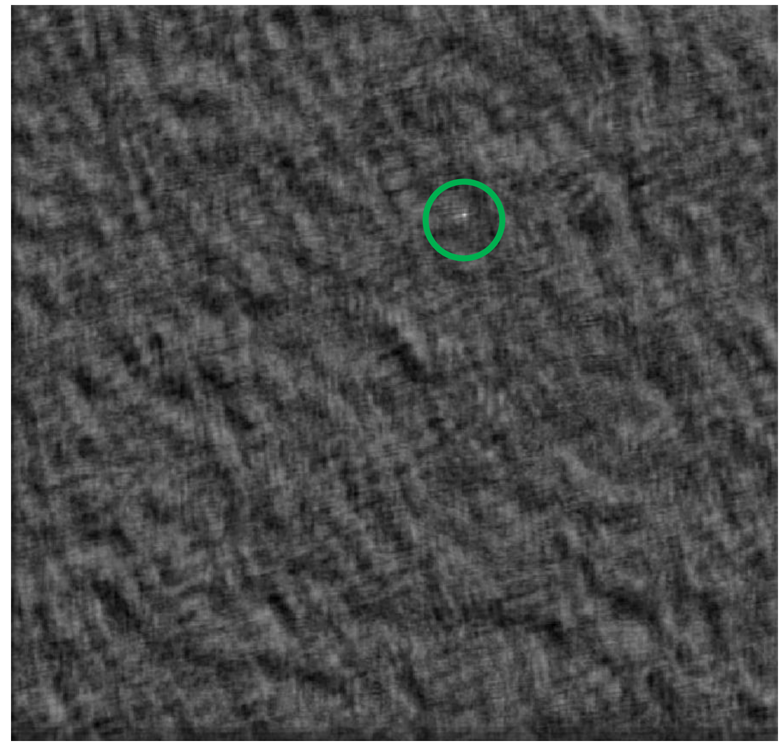
Detected template



Template

Where's Waldo?

Detected template



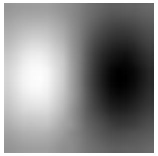
Correlation map

Correlation as Template Matching

- **Think of filters as a dot product of the filter vector with the image region**
  - **Now measure the angle between the vectors**

$$a \cdot b = |a||b|\cos\theta \qquad \cos\theta = \frac{a \cdot b}{|a||b|}$$

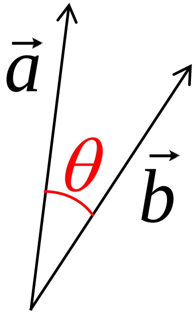
  - **Angle (similarity) between vectors can be measured by normalizing the length of each vector to 1 and taking the dot product.**



Template

$\cdot$

Image region

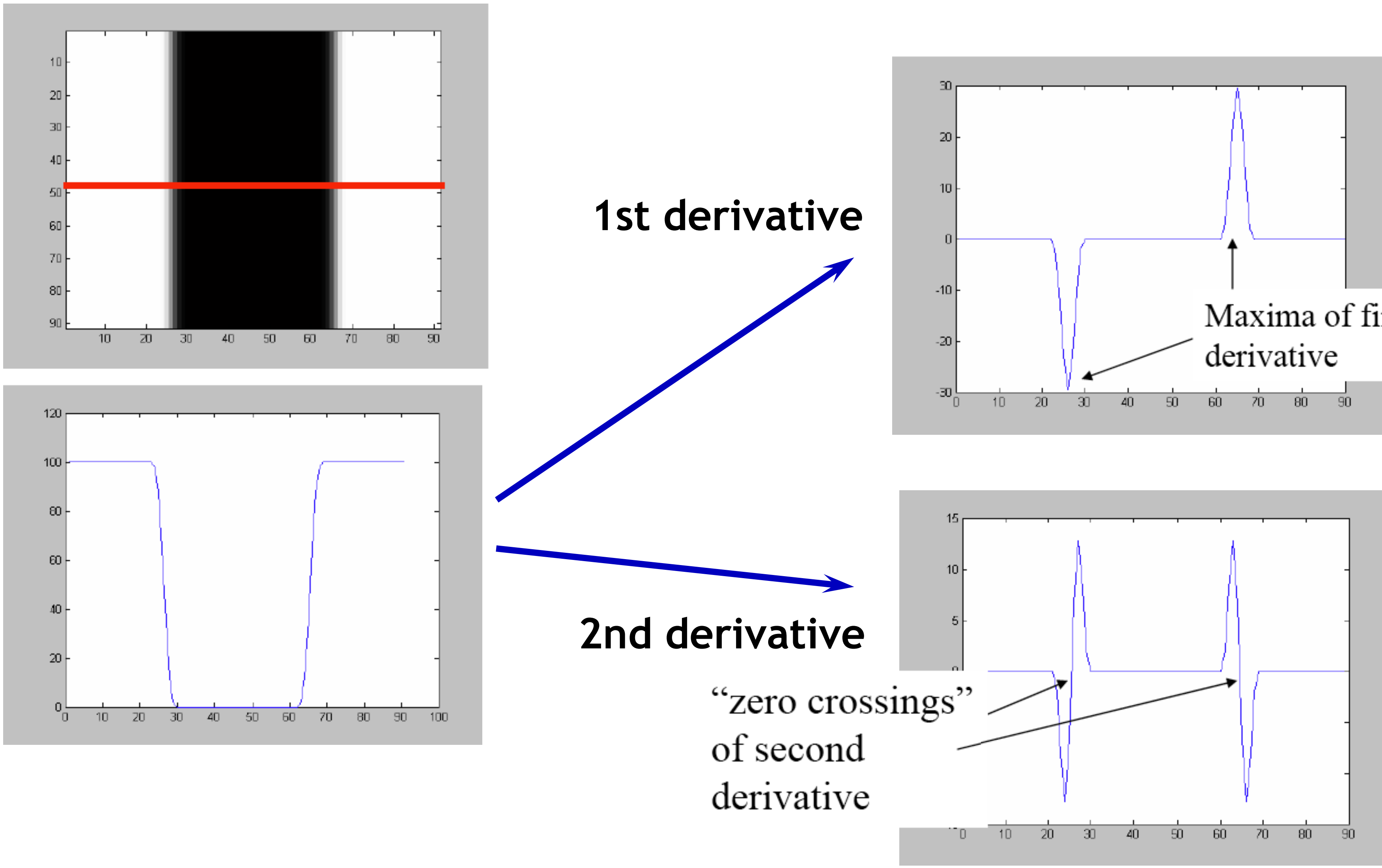


Vector interpretation

Topics of This Lecture

- Recap: Linear Filters
- Multi-Scale representations
  - How to properly rescale an image?
- Filters as templates
  - Correlation as template matching
- **Image gradients**
  - **Derivatives of Gaussian**
- Edge detection
  - Canny edge detector

Derivatives and Edges…



Differentiation and Convolution

- For the 2D function $f(x,y)$, the partial derivative is:

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon, y) - f(x,y)}{\varepsilon}$$

- For discrete data, we can approximate this using finite differences:

$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1, y) - f(x,y)}{1}$$

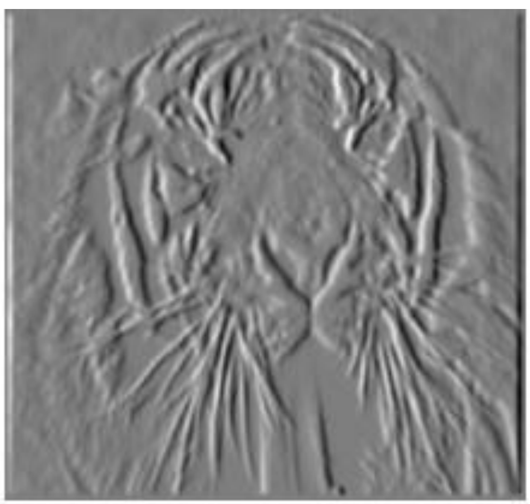
- To implement the above as convolution, what would be the associated filter?

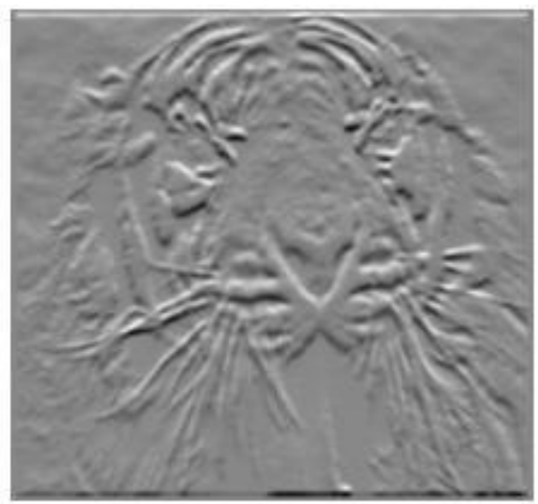
1	-1

Partial Derivatives of an Image

$$\frac{\partial f(x,y)}{\partial x}$$

$$\begin{bmatrix} -1 & 1 \end{bmatrix}$$





$$\frac{\partial f(x,y)}{\partial y}$$

$$\begin{bmatrix} -1 \\ 1 \end{bmatrix} \text{ ? or } \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Which shows changes with respect to x?

Assorted Finite Difference Filters

Prewitt: $M_x =$

-1	0	1
-1	0	1
-1	0	1

; $M_y =$

1	1	1
0	0	0
-1	-1	-1

Sobel: $M_x =$

-1	0	1
-2	0	2
-1	0	1

; $M_y =$

1	2	1
0	0	0
-1	-2	-1

Roberts: $M_x =$

0	1
-1	0

; $M_y =$

1	0
0	-1

```
>> My = fspecial('sobel');
>> outim = imfilter(double(im), My);
>> imagesc(outim);
>> colormap gray;
```

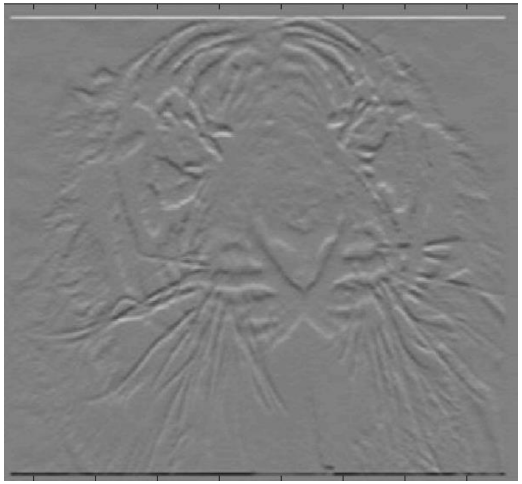
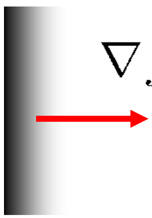


Image Gradient

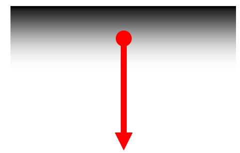
- The gradient of an image:

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

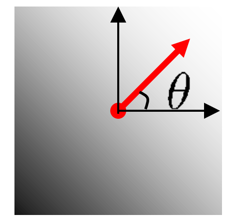
- The gradient points in the direction of most rapid intensity change



$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$



$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

- The gradient direction (orientation of edge normal) is given by:

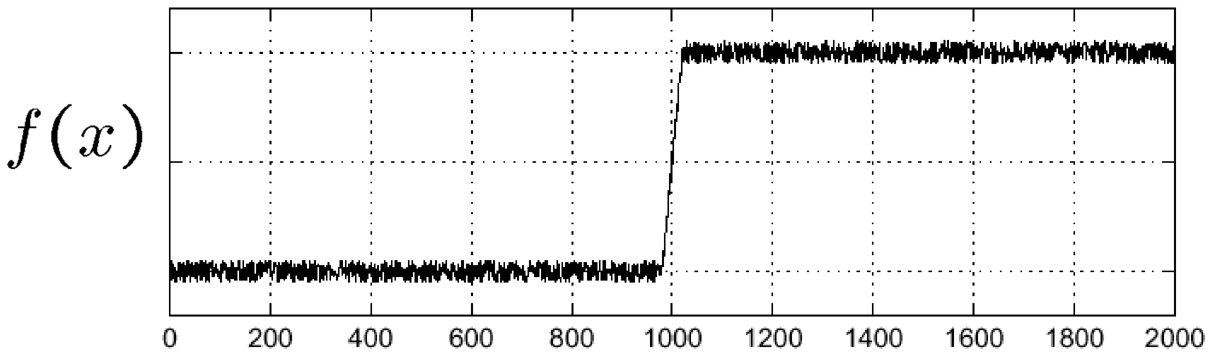
$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

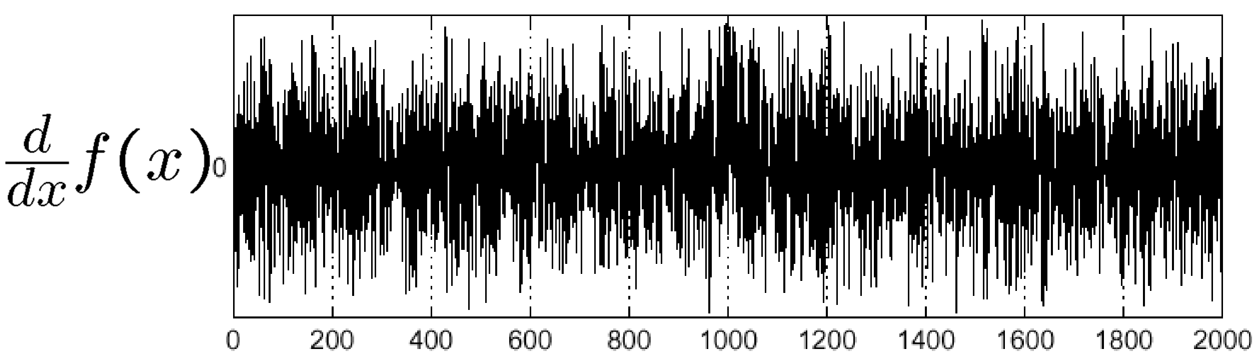
- The edge strength is given by the gradient magnitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Effect of Noise

- Consider a single row or column of the image
  - Plotting intensity as a function of position gives a signal





Where is the edge?

Slide credit: Steve Seitz

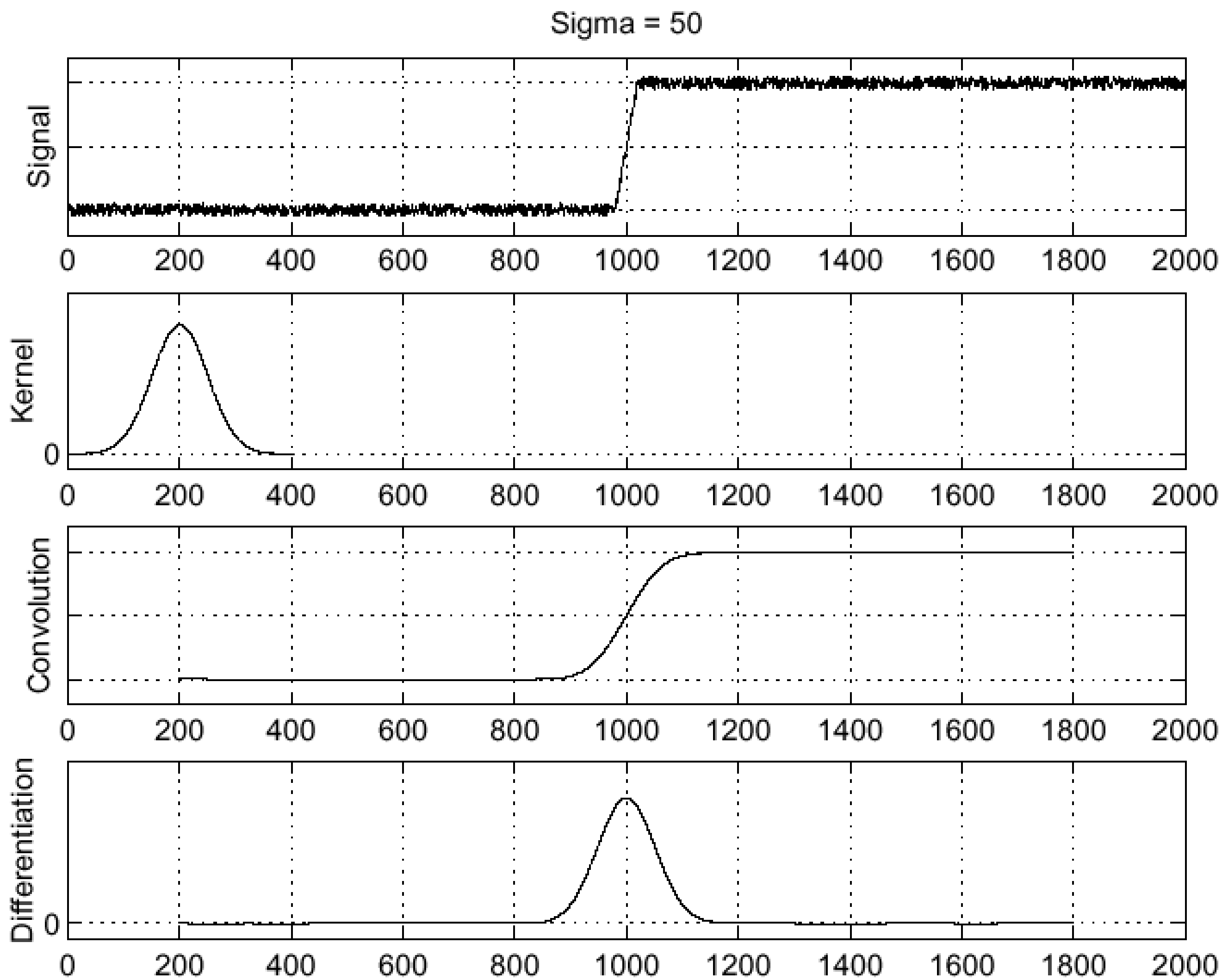
Solution: Smooth First

f

h

$h \star f$

$\frac{\partial}{\partial x}(h \star f)$



Where is the edge? **Look for peaks in** $\frac{\partial}{\partial x}(h \star f)$

Derivative Theorem of Convolution

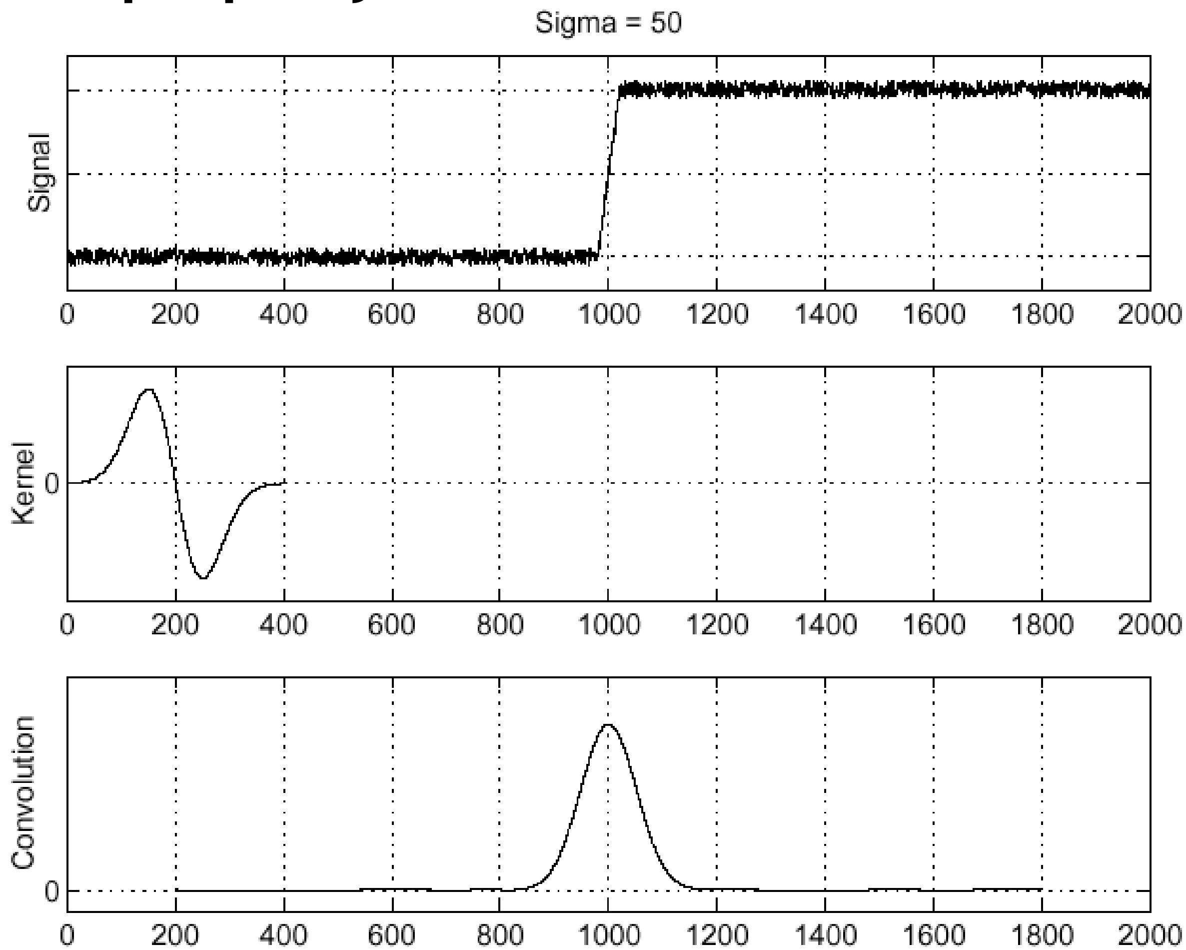
$$\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$$

- **Differentiation property of convolution.**

f

$\frac{\partial}{\partial x}h$

$(\frac{\partial}{\partial x}h) \star f$



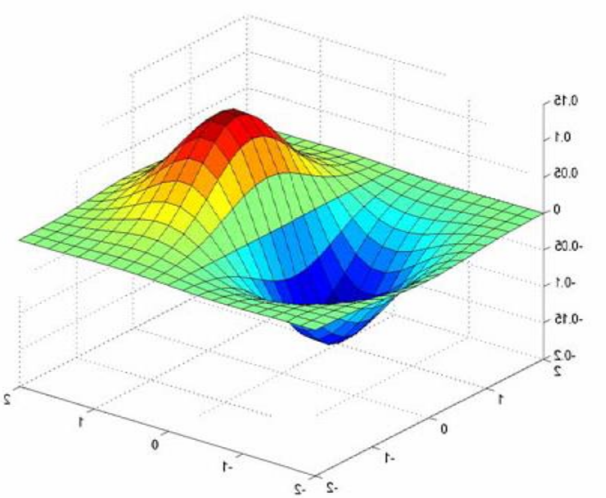
Slide credit: Steve Seitz

Derivative of Gaussian Filter

$$g * (h * I) = (g * h) * I$$

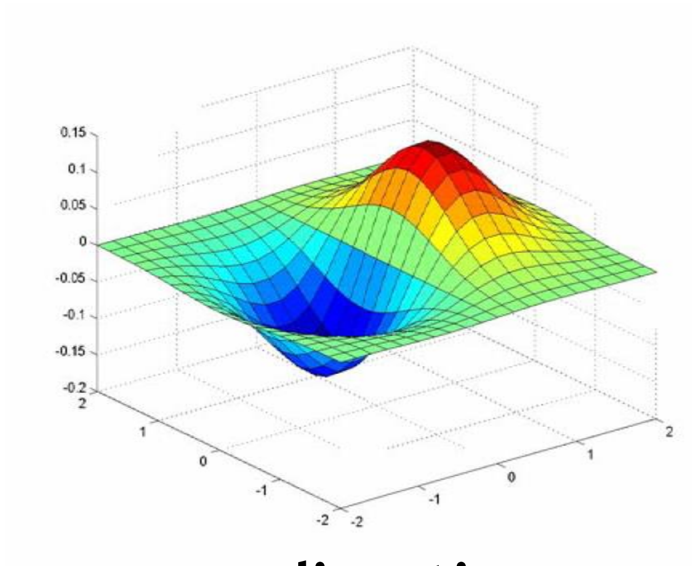
$$\begin{bmatrix} 1 & -1 \end{bmatrix} * \begin{bmatrix} 0.0030 & 0.0133 & 0.0219 & 0.0133 & 0.0030 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.0219 & 0.0983 & 0.1621 & 0.0983 & 0.0219 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.0030 & 0.0133 & 0.0219 & 0.0133 & 0.0030 \end{bmatrix}$$

Why is this preferable?

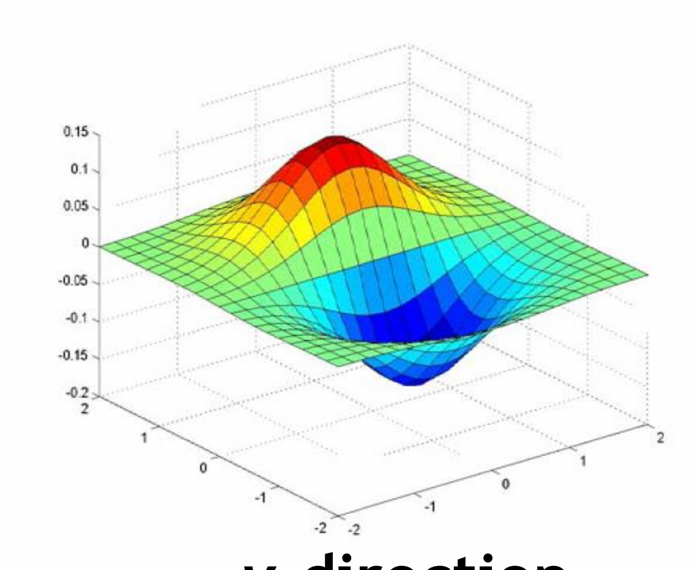


Slide adapted from Kristen Grauman

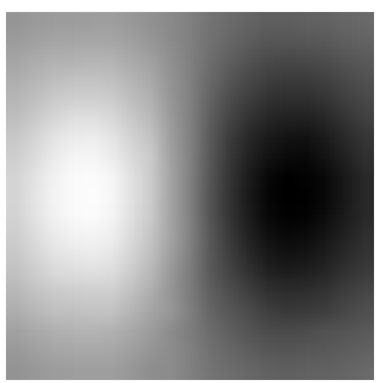
Derivative of Gaussian Filters

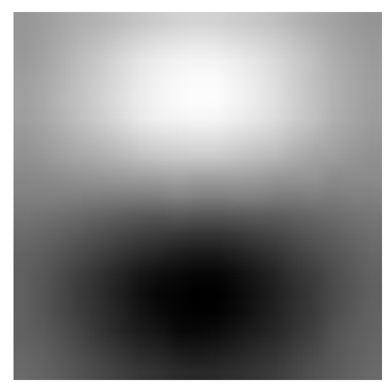


x-direction



y-direction





Source: Svetlana Lazebnik

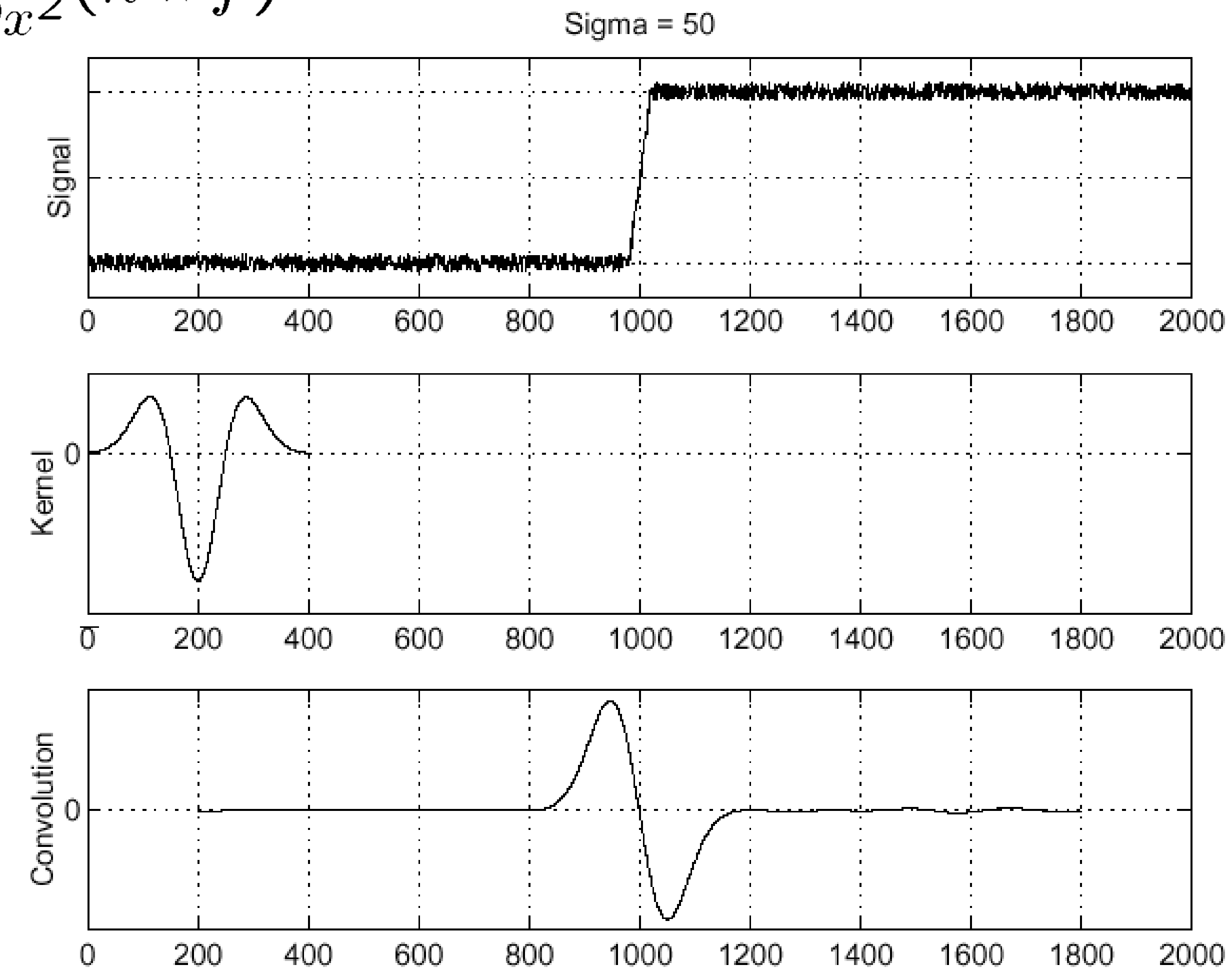
Laplacian of Gaussian (LoG)

- **Consider** $\frac{\partial^2}{\partial x^2}(h \star f)$

f

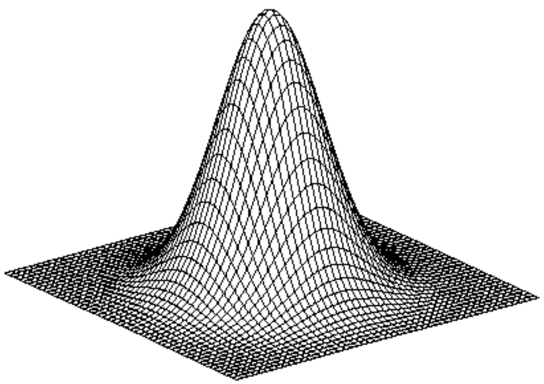
$\frac{\partial^2}{\partial x^2}h$

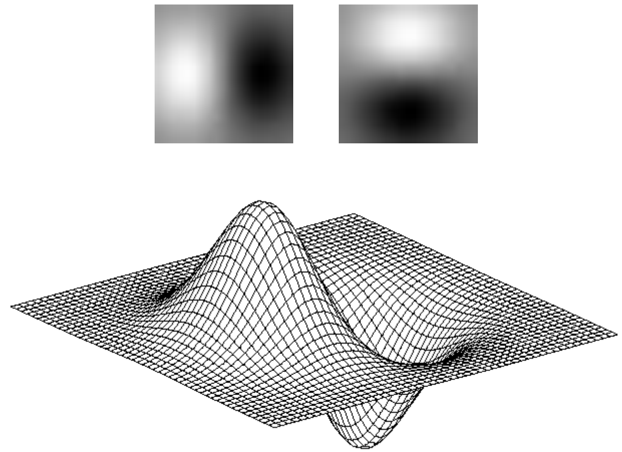
$(\frac{\partial^2}{\partial x^2}h) \star f$



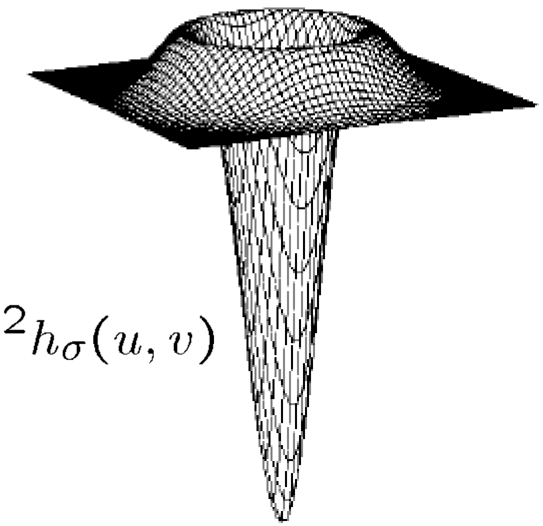
Where is the edge? **Zero-crossings of bottom graph**

Summary: 2D Edge Detection Filters





Laplacian of Gaussian



Gaussian

$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$

Derivative of Gaussian

$$\frac{\partial}{\partial x} h_\sigma(u,v)$$

$$\nabla^2 h_\sigma(u,v)$$

- ∇^2 **is the Laplacian operator:**

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Topics of This Lecture

- Recap: Linear Filters
- Multi-Scale representations
  - How to properly rescale an image?
- Filters as templates
  - Correlation as template matching
- Image gradients
  - Derivatives of Gaussian
- **Edge detection**
  - **Canny edge detector**

Edge Detection

- Goal: map image from 2D array of pixels to a set of curves or line segments or contours.
- Why?

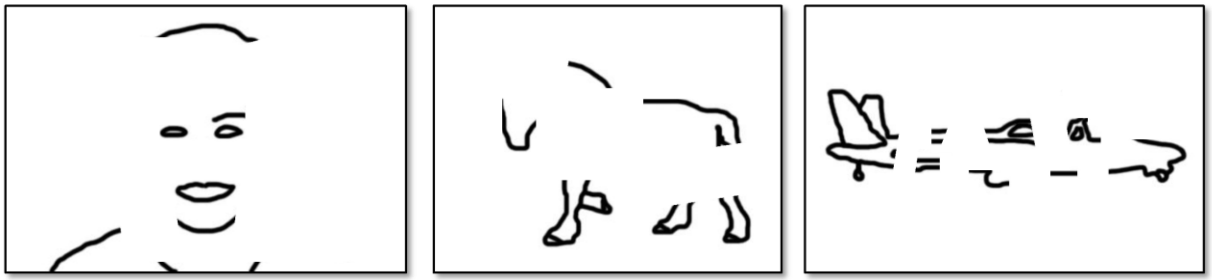
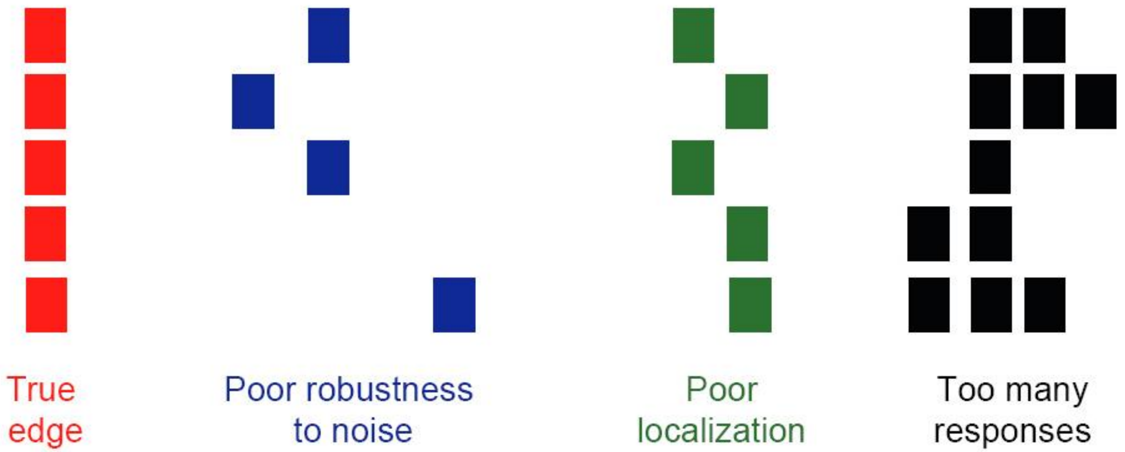


Figure from J. Shotton et al., PAMI 2007

- Main idea: look for strong gradients, post-process

Designing an Edge Detector

- Criteria for an "optimal" edge detector:
  - **Good detection**: the optimal detector should minimize the probability of false positives (detecting spurious edges caused by noise), as well as that of false negatives (missing real edges).
  - **Good localization**: the edges detected should be as close as possible to the true edges.
  - **Single response**: the detector should return one point only for each true edge point; that is, minimize the number of local maxima around the true edge.



Source: Li Fei-Fei

Gradients → Edges

Primary edge detection steps

1. Smoothing: suppress noise
2. Edge enhancement: filter for contrast
3. Edge localization
   - Determine which local maxima from filter output are actually edges vs. noise
   - Thresholding, thinning

- Two issues
  - At what scale do we want to extract structures?
  - How sensitive should the edge extractor be?

Scale: Effect of σ on Derivatives

σ = 1 pixel   σ = 3 pixels

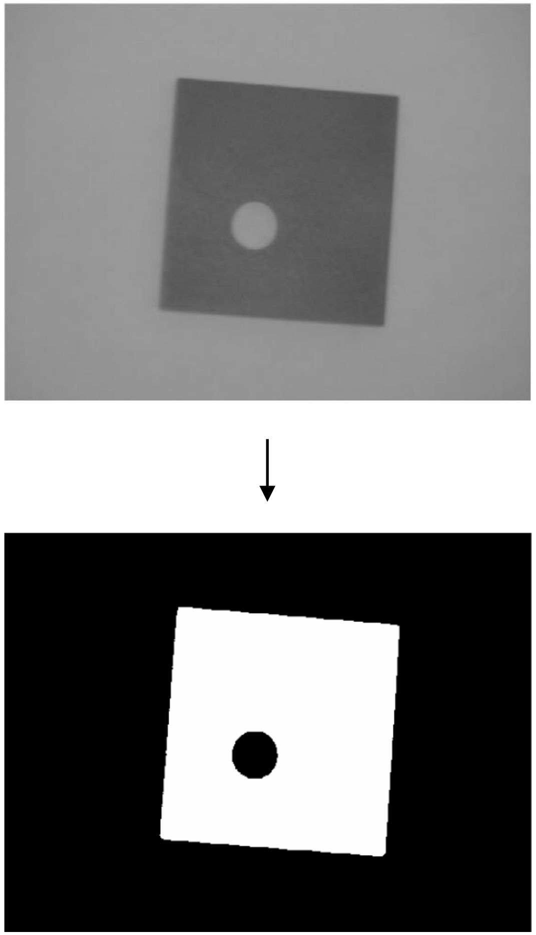
- The apparent structures differ depending on Gaussian's scale parameter.

⇒ Larger values: larger-scale edges detected
⇒ Smaller values: finer features detected

Sensitivity: Recall Thresholding

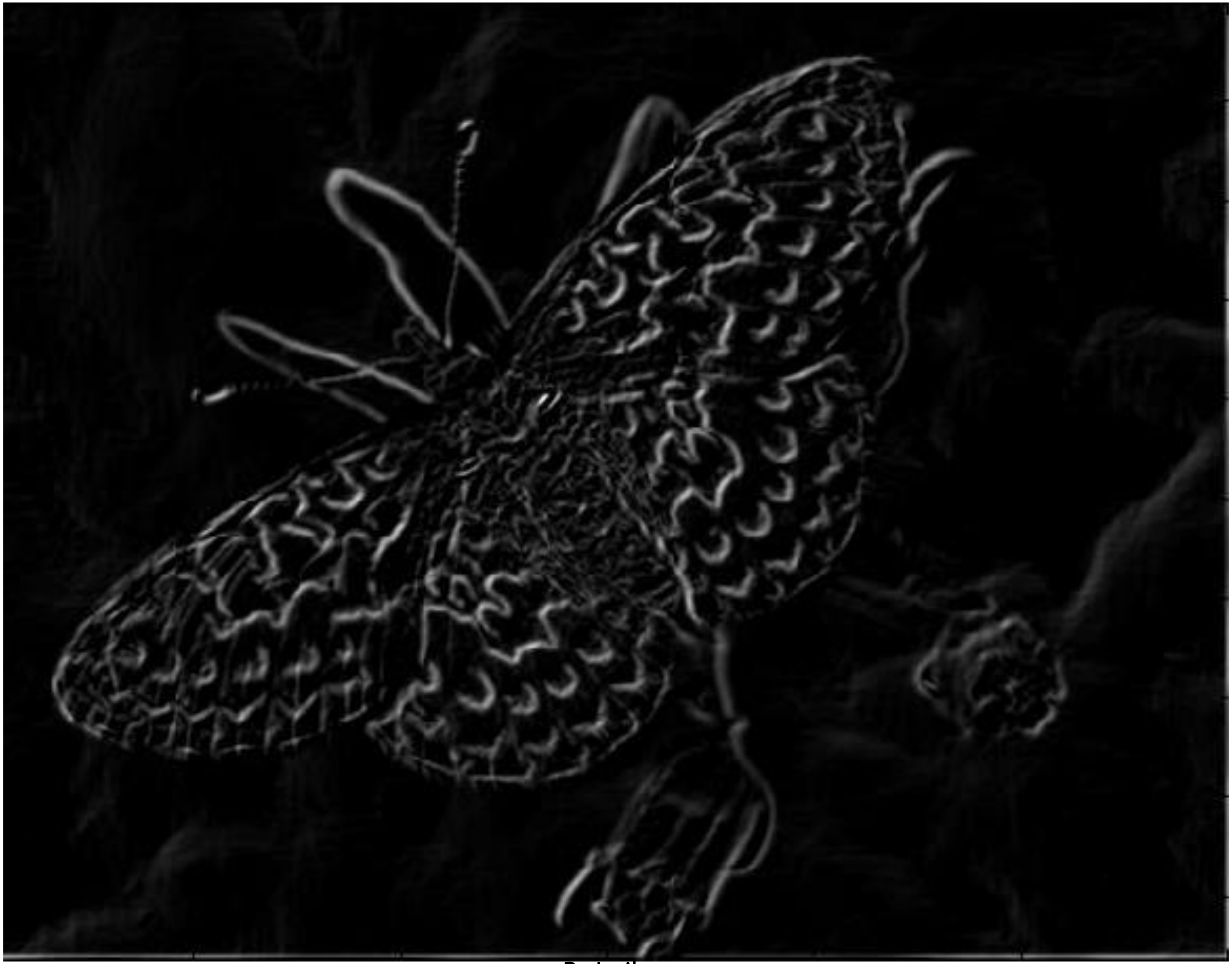
- Choose a threshold t
- Set any pixels less than t to zero (off).
- Set any pixels greater than or equal t to one (on).

$$F_T[i,j] = \begin{cases} 1, & \text{if } F[i,j] \geq t \\ 0, & \text{otherwise} \end{cases}$$



Original Image

Gradient Magnitude Image



Slide credit: Kristen Grauman

Thresholding with a Lower Threshold

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Thresholding with a Higher Threshold

Canny Edge Detector

- Probably the most widely used edge detector in computer vision
- Theoretical model: step-edges corrupted by additive Gaussian noise
- Canny has shown that the first derivative of the Gaussian closely approximates the operator that optimizes the product of *signal-to-noise ratio* and localization.

J. Canny, A Computational Approach To Edge Detection, *IEEE Trans. Pattern Analysis and Machine Intelligence*, 8:679-714, 1986.

Source: Li Fei-Fei

Canny Edge Detector

1. Filter image with derivative of Gaussian
2. Find magnitude and orientation of gradient
3. Non-maximum suppression:
   - Thin multi-pixel wide "ridges" down to single pixel width
4. Linking and thresholding (hysteresis):
   - Define two thresholds: low and high
   - Use the high threshold to start edge curves and the low threshold to continue them

- MATLAB:

```
>> edge(image, 'canny');
>> help edge
```


Source: D. Lowe, L. Fei-Fei

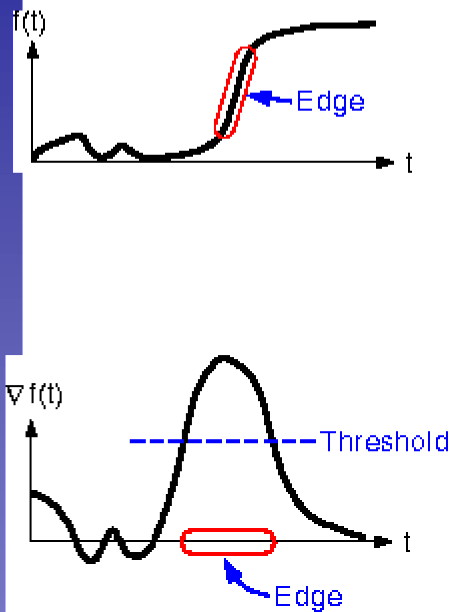
The Canny Edge Detector

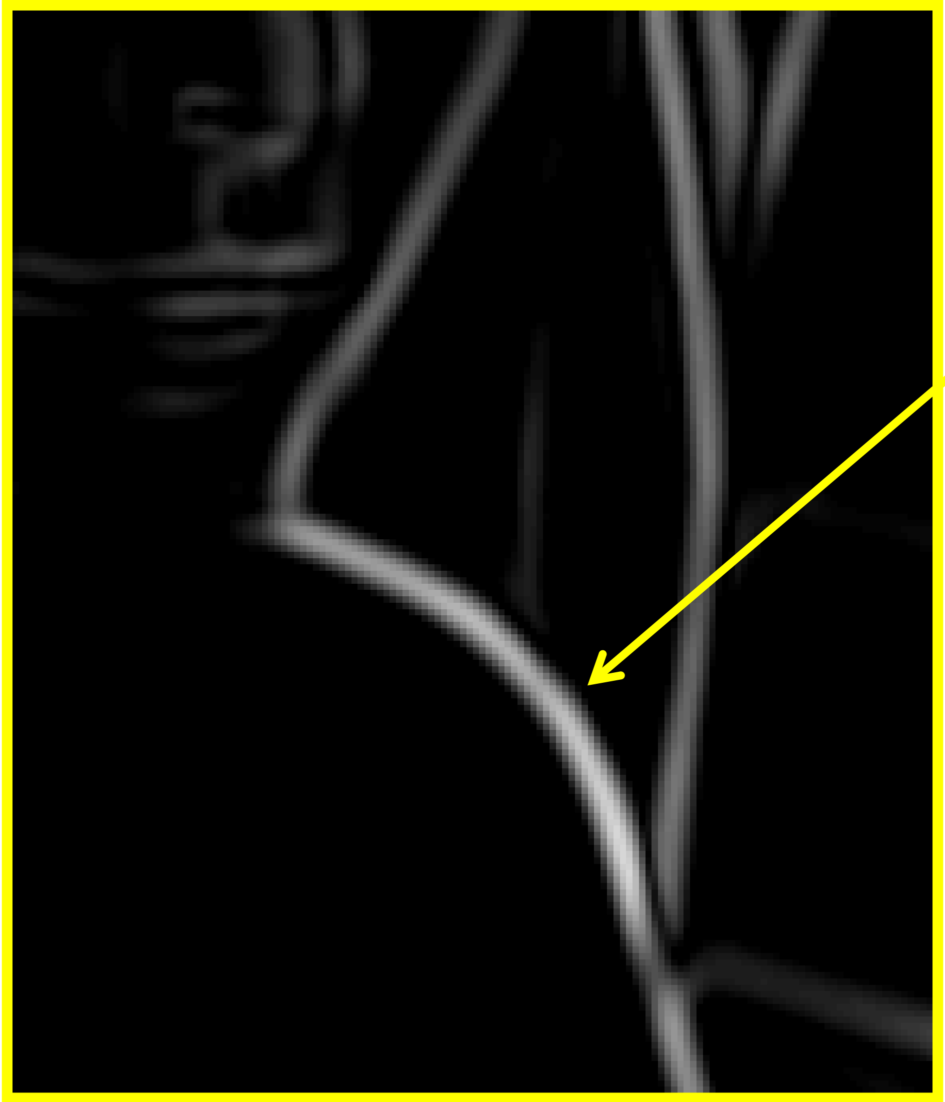
Original image (Lena)

The Canny Edge Detector

Gradient magnitude

The Canny Edge Detector

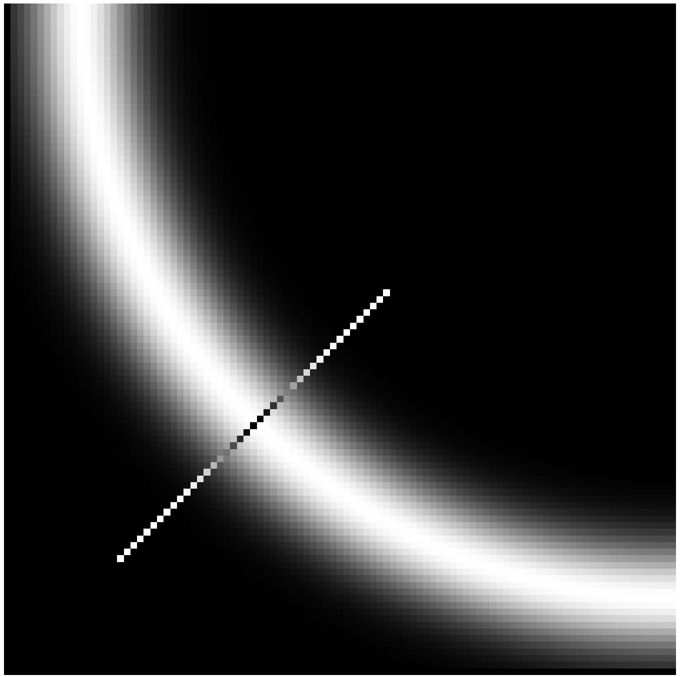


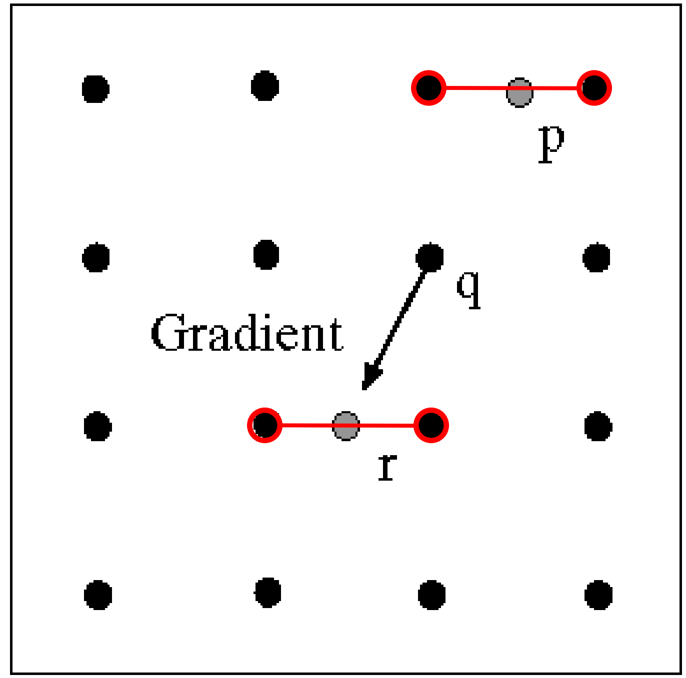


How to turn these thick regions of the gradient into curves?

Slide credit: Kristen Grauman

Non-Maximum Suppression





- **Check if pixel is local maximum along gradient direction, select single max across width of the edge**
  - **Requires checking interpolated pixels p and r**
  - **⇒ Linear interpolation based on gradient direction**

Source: Forsyth & Ponce

The Canny Edge Detector

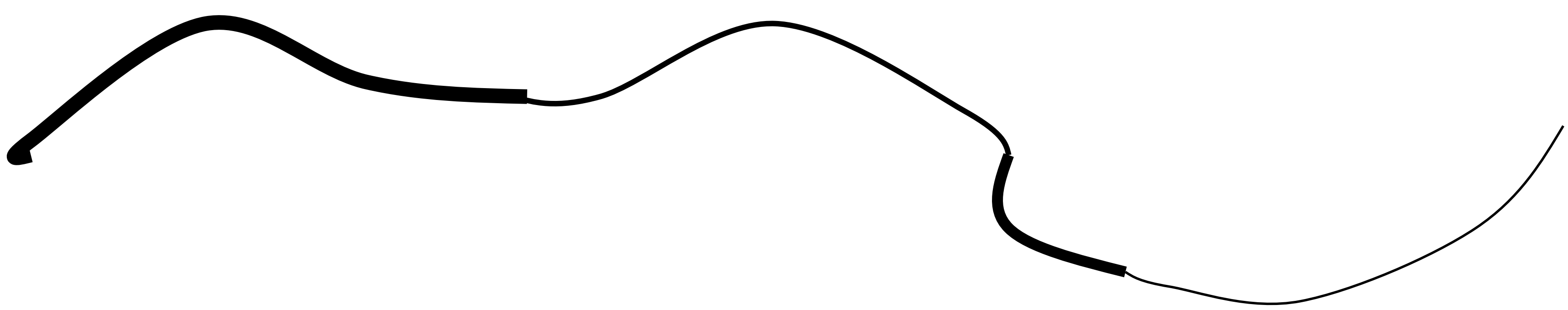
Problem: pixels along this edge didn’t survive the thresholding.

Thinning (non-maximum suppression)

Solution: Hysteresis Thresholding

- Hysteresis: A lag or momentum factor
- Idea: Maintain two thresholds k_{high} and k_{low}
  - Use k_{high} to find strong edges to start edge chain
  - Use k_{low} to find weak edges which continue edge chain
- Typical ratio of thresholds is roughly

$$k_{high} \;/\; k_{low} = 2$$



Source: D. Lowe, S. Seitz

Hysteresis Thresholding

Original image

High threshold
(strong edges)

Low threshold
(weak edges)

courtesy of G. Loy

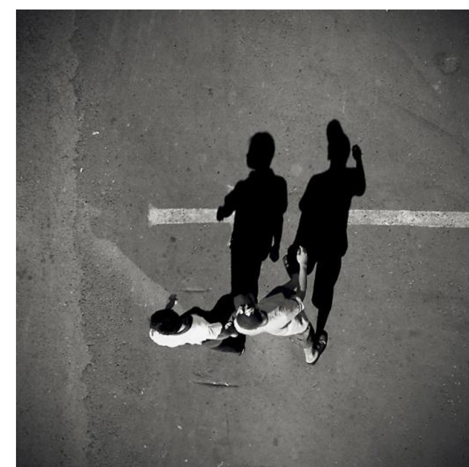
Hysteresis threshold

Source: L. Fei-Fei

Object Boundaries vs. Edges

Background

Texture



Shadows

Edge Detection is Just the Beginning…

Image | Human segmentation | Gradient magnitude

- **Berkeley segmentation database:**
  http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/segbench/

Source: L. Lazebnik

References and Further Reading

- **Background information on linear filters and their connection with the Fourier transform can be found in Chapter 7 of F&P. Additional information on edge detection is available in Chapter 8.**
  - **D. Forsyth, J. Ponce,**
    ***Computer Vision - A Modern Approach*.**
    Prentice Hall, 2003