Computer Vision – Lecture 15

Epipolar Geometry & Stereo Basics

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Bastian Leibe
Visual Computing Institute
RWTH Aachen University
http://www.vision.rwth-aachen.de/

leibe@vision.rwth-aachen.de
Course Outline

• Image Processing Basics
• Segmentation & Grouping
• Object Recognition & Categorization
• Local Features & Matching
• Deep Learning
• 3D Reconstruction
  - Epipolar Geometry and Stereo Basics
  - Camera calibration & Uncalibrated Reconstruction
  - Multi-view Stereo
Topics of This Lecture

• **Geometric vision**
  - Visual cues
  - Stereo vision

• **Epipolar geometry**
  - Depth with stereo
  - Geometry for a simple stereo system
  - Case example with parallel optical axes
  - General case with calibrated cameras

• **Stereopsis & 3D Reconstruction**
  - Correspondence search
  - Additional correspondence constraints
  - Possible sources of error
  - Applications
Geometric vision

- Goal: Recovery of 3D structure
  - What cues in the image allow us to do this?
Visual Cues

• Shading

Merle Norman Cosmetics, Los Angeles

Slide credit: Steve Seitz
Visual Cues

- Shading

- Texture

*The Visual Cliff*, by William Vandivert, 1960
Visual Cues

- Shading
- Texture
- Focus

From *The Art of Photography*, Canon

Slide credit: Steve Seitz
Visual Cues

- Shading
- Texture
- Focus
- Perspective
Visual Cues

• Shading
• Texture
• Focus
• Perspective
• Motion

Figures from L. Zhang

http://www.brainconnection.com/teasers/?main=illusion/motion-shape
Our Goal: Recovery of 3D Structure

- We will focus on perspective and motion
- We need *multi-view geometry* because recovery of structure from one image is inherently ambiguous

Slide credit: Svetlana Lazebnik
To Illustrate This Point…

- Structure and depth are inherently ambiguous from single views.

Slide credit: Svetlana Lazebnik, Kristen Grauman
Stereo Vision

http://www.well.com/~jimg/stereo/stereo_list.html

Slide credit: Kristen Grauman
What Is Stereo Vision?

• Generic problem formulation: given several images of the same object or scene, compute a representation of its 3D shape
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What Is Stereo Vision?

• Narrower formulation: given a calibrated binocular stereo pair, fuse it to produce a depth image

![Image 1](image1.png) ![Image 2](image2.png)

Dense depth map

Slide credit: Svetlana Lazebnik, Steve Seitz
What Is Stereo Vision?

- Narrower formulation: given a calibrated binocular stereo pair, fuse it to produce a depth image.
  - Humans can do it

Stereograms: Invented by Sir Charles Wheatstone, 1838

Slide credit: Svetlana Lazebnik, Steve Seitz
What Is Stereo Vision?

- Narrower formulation: given a calibrated binocular stereo pair, fuse it to produce a depth image.
  - Humans can do it

Autostereograms: [http://www.magiceye.com](http://www.magiceye.com)
What Is Stereo Vision?

- Narrower formulation: given a calibrated binocular stereo pair, fuse it to produce a depth image.
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Autostereograms: [http://www.magiceye.com](http://www.magiceye.com)
Application of Stereo: Robotic Exploration

Nomad robot searches for meteorites in Antarctica

Real-time stereo on Mars

Slide credit: Steve Seitz
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Depth with Stereo: Basic Idea

- Basic Principle: Triangulation
  - Gives reconstruction as intersection of two rays
  - Requires
    - Camera pose (calibration)
    - Point correspondence

Slide credit: Steve Seitz
Camera Calibration

- **Parameters**
  - *Extrinsic*: rotation matrix and translation vector
  - *Intrinsic*: focal length, pixel sizes (mm), image center point, radial distortion parameters

We’ll assume for now that these parameters are given and fixed.
Geometry for a Simple Stereo System

• First, assuming parallel optical axes, known camera parameters (i.e., calibrated cameras):
Focal length

Baseline

World point

Depth of p

Image point (left)

Optical center (left)

Optical center (right)

Image point (right)
Geometry for a Simple Stereo System

- Assume parallel optical axes, known camera parameters (i.e., calibrated cameras). We can triangulate via:

  Similar triangles \((p_l, P, p_r)\) and \((O_l, P, O_r)\):

  \[
  \frac{T - (x_r - x_l)}{Z - f} = \frac{T}{Z}
  \]

  \[
  Z = f \frac{T}{x_r - x_l}
  \]

  “disparity”
Depth From Disparity

Image $I(x, y)$

Disparity map $D(x, y)$

Image $I'(x', y')$

$$(x', y') = (x + D(x, y), y)$$
General Case With Calibrated Cameras

- The two cameras need not have parallel optical axes.
Stereo Correspondence Constraints

- Given $p$ in the left image, where can the corresponding point $p'$ in the right image be?
Stereo Correspondence Constraints

• Given $p$ in the left image, where can the corresponding point $p'$ in the right image be?
Stereo Correspondence Constraints
Stereo Correspondence Constraints

• Geometry of two views allows us to constrain where the corresponding pixel for some image point in the first view must occur in the second view.

• Epipolar constraint: Why is this useful?
  - Reduces correspondence problem to 1D search along conjugate epipolar lines.
Epipolar Geometry

- Epipolar Plane
- Epipoles
- Baseline
- Epipolar Lines

Slide adapted from Marc Pollefeys
Epipolar Geometry: Terms

- **Baseline**
  - Line joining the camera centers

- **Epipole**
  - Point of intersection of baseline with the image plane

- **Epipolar plane**
  - Plane containing baseline and world point

- **Epipolar line**
  - Intersection of epipolar plane with the image plane

- **Properties**
  - All epipolar lines intersect at the epipole.
  - An epipolar plane intersects the left and right image planes in epipolar lines.
Epipolar Constraint

- Potential matches for $p$ have to lie on the corresponding epipolar line $l'$.
- Potential matches for $p'$ have to lie on the corresponding epipolar line $l$.

http://www.ai.sri.com/~luong/research/Meta3DViewer/EpipolarGeo.html

Slide credit: Marc Pollefeys
Example
Example: Converging Cameras

As position of 3D point varies, epipolar lines “rotate” about the baseline.

Figure from Hartley & Zisserman
Example: Motion Parallel With Image Plane

\[ e \text{ at } \infty \quad \rightarrow \quad e' \text{ at } \infty \]

Figure from Hartley & Zisserman
Example: Forward Motion

- Epipole has same coordinates in both images.
- Points move along lines radiating from $e$: "Focus of expansion"
Let’s Formalize This!

• For a given stereo rig, how do we express the epipolar constraints algebraically?

• For this, we will need some linear algebra.

• But don’t worry! We’ll go through it step by step…
Stereo Geometry With Calibrated Cameras

- If the rig is calibrated, we know:
  - How to rotate and translate camera reference frame 1 to get to camera reference frame 2.
    - Rotation: 3 x 3 matrix; translation: 3 vector.
Rotation Matrix

Express 3D rotation as series of rotations around coordinate axes by angles $\alpha$, $\beta$, $\gamma$

Overall rotation is product of these elementary rotations:

$$
\mathbf{R} = \mathbf{R}_x(\alpha) \mathbf{R}_y(\beta) \mathbf{R}_z(\gamma)
$$
3D Rigid Transformation

\[
\begin{bmatrix}
X' \\
Y' \\
Z'
\end{bmatrix} =
\begin{bmatrix}
r_{11} & r_{12} & r_{13} \\
r_{21} & r_{22} & r_{23} \\
r_{31} & r_{32} & r_{33}
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} +
\begin{bmatrix}
T_x \\
T_y \\
T_z
\end{bmatrix}
\]

\(X' = RX + T\)
Stereo Geometry With Calibrated Cameras

- Camera-centered coordinate systems are related by known rotation $R$ and translation $T$:
  
  $$X' = RX + T$$

Slide credit: Kristen Grauman
Excursion: Cross Product

\[ \vec{a} \times \vec{b} = \vec{c} \]
\[ \vec{a} \cdot \vec{c} = 0 \]
\[ \vec{b} \cdot \vec{c} = 0 \]

- Vector cross product takes two vectors and returns a third vector that’s perpendicular to both inputs.

- So here, \( \vec{c} \) is perpendicular to both \( \vec{a} \) and \( \vec{b} \), which means the dot product is 0.
From Geometry to Algebra

\[ X' = RX + T \]

\[ T \times X' = T \times RX + T \times T \]

Normal to the plane

\[ = T \times RX \]

\[ X' \cdot (T \times X') = X' \cdot (T \times RX) \]

\[ 0 = X' \cdot (T \times RX) \]

Slide credit: Kristen Grauman
Matrix Form of Cross Product

\[ \vec{a} \times \vec{b} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \]

“skew symmetric” matrix

\[ \vec{a} \times \vec{b} = [a_x] \vec{b} \]

\[ \vec{a} \cdot \vec{c} = 0 \]
\[ \vec{b} \cdot \vec{c} = 0 \]

Slide credit: Kristen Grauman
From Geometry to Algebra

\[ X' = RX + T \]

\[ T \times X' = T \times RX + T \times T \]

Normal to the plane

\[ T \times RX \]

\[ X' \cdot (T \times X') = X' \cdot (T \times RX) \]

\[ 0 = X' \cdot (T \times RX) \]
Essential Matrix

\[ X' \cdot (T \times RX) = 0 \]
\[ X' \cdot (T_x RX) = 0 \]

Let \( E = T_x R \)

\[ X'^T EX = 0 \]

- This holds for the rays \( p \) and \( p' \) that are parallel to the camera-centered position vectors \( X \) and \( X' \), so we have:

\[ p'^T Ep = 0 \]

- \( E \) is called the **essential matrix**, which relates corresponding image points [Longuet-Higgins 1981]
Essential Matrix and Epipolar Lines

\[ \mathbf{p'}^T \mathbf{E} \mathbf{p} = 0 \]

Epipolar constraint: if we observe point \( p \) in one image, then its position \( p' \) in second image must satisfy this equation.

\[ \mathbf{l}' = \mathbf{E} \mathbf{p} \]

is the coordinate vector representing the epipolar line for point \( p \)

(i.e., the line is given by: \( \mathbf{l'}^T \mathbf{x} = 0 \))

\[ \mathbf{l} = \mathbf{E}^T \mathbf{p'} \]

is the coordinate vector representing the epipolar line for point \( p' \)

Slide credit: Kristen Grauman
Essential Matrix: Properties

- Relates image of corresponding points in both cameras, given rotation and translation.
- Assuming intrinsic parameters are known

\[ E = T_x R \]
Essential Matrix Example: Parallel Cameras

For the parallel cameras, image of any point must lie on same horizontal line in each image plane.

\[ R = I \]

\[ T = [-d, 0, 0]^T \]

\[ E = [T_p]R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & d \\ 0 & -d & 0 \end{bmatrix} \]

\[ p'^T E p = 0 \]

\[ \begin{bmatrix} x' & y' & f \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & d \\ 0 & -d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ f \end{bmatrix} = 0 \]

\[ \Leftrightarrow \begin{bmatrix} x' & y' & f \end{bmatrix} \begin{bmatrix} 0 \\ df \\ -dy \end{bmatrix} = 0 \]

\[ \Leftrightarrow y = y' \]
More General Case

Image $I(x, y)$  Disparity map $D(x, y)$  Image $I'(x', y')$

$(x', y') = (x + D(x, y), y)$

What about when cameras’ optical axes are not parallel?

Slide credit: Kristen Grauman
Stereo Image Rectification

• In practice, it is convenient if image scanlines are the epipolar lines.

• Algorithm
  - Reproject image planes onto a common plane parallel to the line between optical centers
  - Pixel motion is horizontal after this transformation
  - Two homographies (3×3 transforms), one for each input image reprojection

C. Loop & Z. Zhang, Computing Rectifying Homographies for Stereo Vision, CVPR'99
Stereo Image Rectification: Example
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  ➢ Possible sources of error
  ➢ Applications
Stereo Reconstruction

• Main Steps
  - Calibrate cameras
  - Rectify images
  - Compute disparity
  - Estimate depth
Correspondence Problem

Multiple match hypotheses satisfy epipolar constraint, but which is correct?

Figure from Gee & Cipolla 1999

Oc

Oc'

Left image

Right image

Hypothesis 1
Hypothesis 2
Hypothesis 3

Slide credit: Kristen Grauman

B. Leibe
Dense Correspondence Search

- For each pixel in the first image
  - Find corresponding epipolar line in the right image
  - Examine all pixels on the epipolar line and pick the best match (e.g. SSD, correlation)
  - Triangulate the matches to get depth information

- This is easiest when epipolar lines are scanlines
  \[\Rightarrow\] Rectify images first

adapted from Svetlana Lazebnik, Li Zhang
Example: Window Search

- Data from University of Tsukuba
Example: Window Search

- Data from University of Tsukuba

Window-based matching (best window size)  Ground truth
Effect of Window Size

Want window large enough to have sufficient intensity variation, yet small enough to contain only pixels with about the same disparity.

$W = 3$  
$W = 20$

Slide credit: Kristen Grauman  
Figures from Li Zhang
Alternative: Sparse Correspondence Search

• Idea: Restrict search to sparse set of detected features
• Rather than pixel values (or lists of pixel values) use *feature descriptor* and an associated *feature distance*
• Still narrow search further by epipolar geometry

*What would make good features?*
Dense vs. Sparse

• Sparse
  - Efficiency
  - Can have more reliable feature matches, less sensitive to illumination than raw pixels
  - But...
    - Have to know enough to pick good features
    - Sparse information

• Dense
  - Simple process
  - More depth estimates, can be useful for surface reconstruction
  - But...
    - Breaks down in textureless regions anyway
    - Raw pixel distances can be brittle
    - Not good with very different viewpoints

Slide credit: Kristen Grauman
Difficulties in Similarity Constraint

Untextured surfaces

Occlusions

Slide credit: Kristen Grauman
Possible Sources of Error?

- Low-contrast / textureless image regions
- Occlusions
- Camera calibration errors
- Violations of *brightness constancy* (e.g., specular reflections)
- Large motions
Application: View Interpolation

Right Image

Slide credit: Svetlana Lazebnik
Application: View Interpolation

Left Image

Slide credit: Svetlana Lazebnik
Application: View Interpolation

Disparity

Slide credit: Svetlana Lazebnik
Application: View Interpolation
Application: Free-Viewpoint Video

http://www.liberovision.com
Summary: Stereo Reconstruction

• Main Steps
  - Calibrate cameras
  - Rectify images
  - Compute disparity
  - Estimate depth

• So far, we have only considered calibrated cameras...

• Next lecture
  - Uncalibrated cameras
  - Camera parameters
  - Revisiting epipolar geometry
  - Robust fitting
References and Further Reading

• Background information on epipolar geometry and stereopsis can be found in Chapters 10.1-10.2 and 11.1-11.3 of

D. Forsyth, J. Ponce, 
Computer Vision – A Modern Approach. 
Prentice Hall, 2003

• More detailed information (if you really want to implement 3D reconstruction algorithms) can be found in Chapters 9 and 10 of

R. Hartley, A. Zisserman 
Multiple View Geometry in Computer Vision 
2nd Ed., Cambridge Univ. Press, 2004