Computer Vision II – Lecture 12

Multi-Object Tracking II

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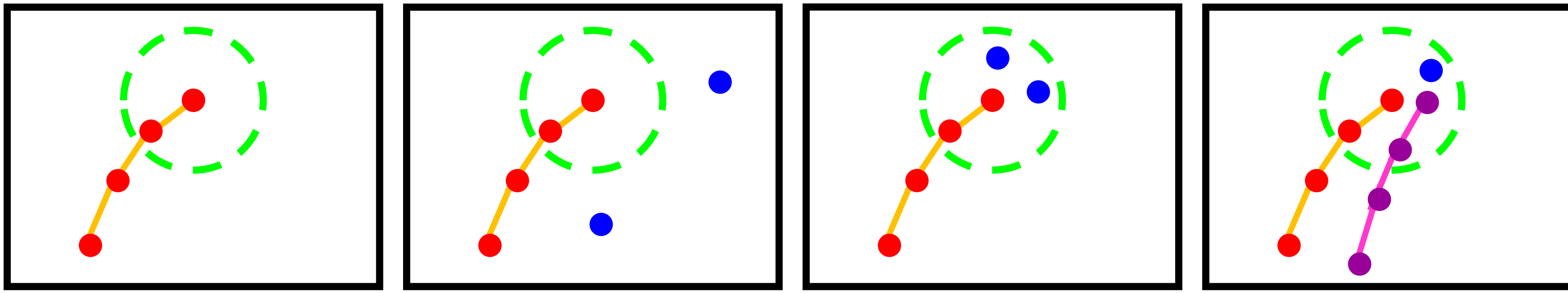
Course Outline

- Single-Object Tracking
- Bayesian Filtering
  - Kalman filters
  - Particle filters
  - Case studies
- Multi-Object Tracking
  - Introduction
  - MHT, JPDAF
  - Network Flow Optimization
- Articulated Tracking

Topics of This Lecture

- Recap: Track-Splitting Filter
  - Motivation
  - Ambiguities
- Multi-Hypothesis Tracking (MHT)
  - Basic idea
  - Hypothesis Generation
  - Assignment
  - Measurement Likelihood
  - Practical considerations

Recap: Motion Correspondence Ambiguities



1. **Predictions may not be supported by measurements**
   - Have the objects ceased to exist, or are they simply occluded?
2. **There may be unexpected measurements**
   - Newly visible objects, or just noise?
3. **More than one measurement may match a prediction**
   - Which measurement is the correct one (what about the others)?
4. **A measurement may match to multiple predictions**
   - Which object shall the measurement be assigned to?

Let's Formalize This

- **Multi-Object Tracking problem**
  - We represent a track by a state vector $\mathbf{x}$, e.g.,

$$\mathbf{x} = [x, y, v_x, v_y]^T$$

  - As the track evolves, we denote its state by the time index k:

$$\mathbf{x}^{(k)} = \left[x^{(k)}, y^{(k)}, v_x^{(k)}, v_y^{(k)}\right]^T$$

  - At each time step, we get a set of observations (measurements)

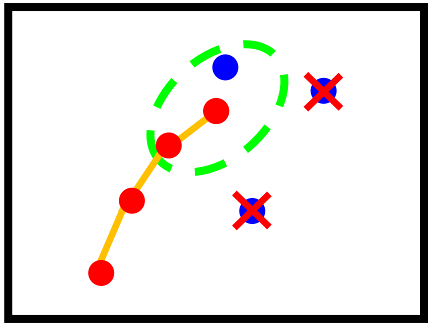
$$\mathbf{Y}^{(k)} = \left\{\mathbf{y}_1^{(k)}, \ldots, \mathbf{y}_{M_k}^{(k)}\right\}$$

  - We now need to make the data association between tracks $\left\{\mathbf{x}_1^{(k)}, \ldots, \mathbf{x}_{N_k}^{(k)}\right\}$ and observations $\left\{\mathbf{y}_1^{(k)}, \ldots, \mathbf{y}_{M_k}^{(k)}\right\}$:

$$z_l^{(k)} = j \text{ iff } \mathbf{y}_j^{(k)} \text{ is associated with } \mathbf{x}_l^{(k)}$$

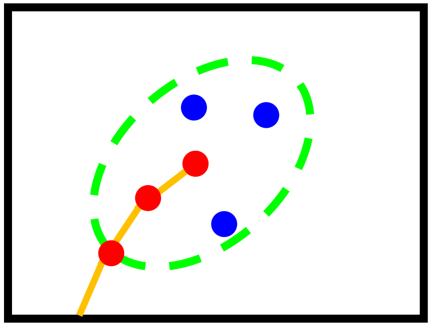
Recap: Reducing Ambiguities

- **Gating**
  - Only consider measurements within a certain area around the predicted location.
  - $\Rightarrow$ Large gain in efficiency, since only a small region needs to be searched



- **Nearest-Neighbor Filter**
  - Among the candidates in the gating region, only take the one closest to the prediction $\mathbf{x}_p$

$$z_l^{(k)} = \arg\min_j (\mathbf{x}_{p,l}^{(k)} - \mathbf{y}_j^{(k)})^T (\mathbf{x}_{p,l}^{(k)} - \mathbf{y}_j^{(k)})$$



  - Better: the one most likely under a Gaussian prediction model

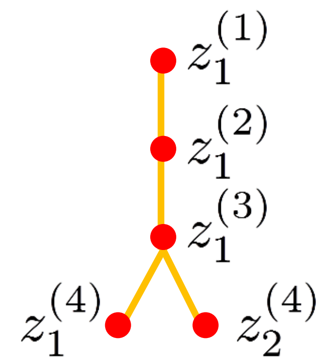
$$z_l^{(k)} = \arg\max_j \mathcal{N}(\mathbf{y}_j^{(k)}; \mathbf{x}_{p,l}^{(k)}, \mathbf{\Sigma}_{p,l}^{(k)})$$

which is equivalent to taking the Mahalanobis distance

$$z_l = \arg\min_j (\mathbf{x}_{p,l} - \mathbf{y}_j)^T \mathbf{\Sigma}_{p,l}^{-1} (\mathbf{x}_{p,l} - \mathbf{y}_j)$$

Recap: Track-Splitting Filter

- **Idea**
  - Instead of assigning the measurement that is currently closest, as in the NN algorithm, select the *sequence* of measurements that minimizes the *total* Mahalanobis distance over some interval!



  - Form a track tree for the different association decisions
  - Modified log-likelihood provides the merit of a particular node in the track tree.
  - Cost of calculating this is low, since most terms are needed anyway for the Kalman filter.
- **Problem**
  - The track tree grows exponentially, may generate a very large number of possible tracks that need to be maintained.

Recap: Pruning Strategies

- In order to keep this feasible, need to apply pruning
  - Deleting unlikely tracks
    - May be accomplished by comparing the modified log-likelihood $\lambda(k)$, which has a χ^2 distribution with kn_z degrees of freedom, with a threshold α (set according to χ^2 distribution tables).
    - Problem for long tracks: modified log-likelihood gets dominated by old terms and responds very slowly to new ones.

    $\Rightarrow$ Use sliding window or exponential decay term.
  - Merging track nodes
    - If the state estimates of two track nodes are similar, merge them.
    - E.g., if both tracks validate identical subsequent measurements.
  - Only keeping the most likely N tracks
    - Rank tracks based on their modified log-likelihood.

Summary: Track-Splitting Filter

- Properties
  - Very old algorithm
    - P. Smith, G. Buechler, A Branching Algorithm for Discriminating and Tracking Multiple Objects, IEEE Trans. Automatic Control, Vol. 20, pp. 101-104, 1975.
  - Improvement over NN assignment.
  - Assignment decisions are delayed until more information is available.
- Many problems remain
  - Exponential complexity, heuristic pruning needed.
  - Merging of track nodes is necessary, because tracks may share measurements, which is physically unrealistic.

  ⇒ Would need to add exclusion constraints such that each measurement may only belong to a single track.

  ⇒ Impossible in this framework...

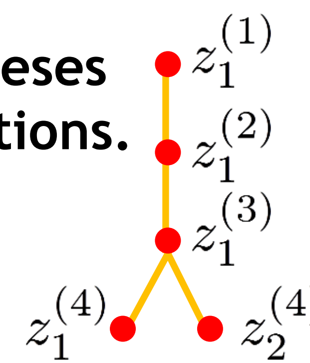
Topics of This Lecture

- Recap: Track-Splitting Filter
  - Motivation
  - Ambiguities
- Multi-Hypothesis Tracking (MHT)
  - Basic idea
  - Hypothesis Generation
  - Assignment
  - Measurement Likelihood
  - Practical considerations

Multi-Hypothesis Tracking (MHT)

- **Ideas**
  - Again associate sequences of measurements.
  - Evaluate the probabilities of all association hypotheses.
  - For each sequence of measurements (a hypothesized track), a standard KF yields the state estimate and covariance

- **Differences to Track-Splitting Filter**
  - Instead of forming a track tree, keep a set of hypotheses that generate child hypotheses based on the associations.
  - After each hypothesis generation step, merge and prune the current hypothesis set to keep the approach feasible.
  - Integrate track generation into the assignment process.



D. Reid, An Algorithm for Tracking Multiple Targets, IEEE Trans. Automatic Control, Vol. 24(6), pp. 843-854, 1979.

Target vs. Measurement Orientation

- Target-oriented approaches
  - Evaluate the probability that a measurement belongs to an established target.
- Measurement-oriented approaches
  - Evaluate the probability that an established target or a new target gave rise to a certain measurement sequence.
  - This makes it possible to include track initiation of new targets within the algorithmic framework.
- MHT
  - Measurement-oriented
  - Handles track initialization and termination

Challenge: Exponential Complexity

- Strategy
  - Generate all possible hypotheses and then depend on pruning these hypotheses to avoid the combinatorial explosion.
  - ⇒ Exhaustive search
  - Tree data structures are used to keep this search efficient

- Commonly used pruning techniques
  - Clustering to reduce the combinatorial complexity
  - Pruning of low-probability hypotheses
  - N-scan pruning
  - Merging of similar hypotheses

MHT Outline

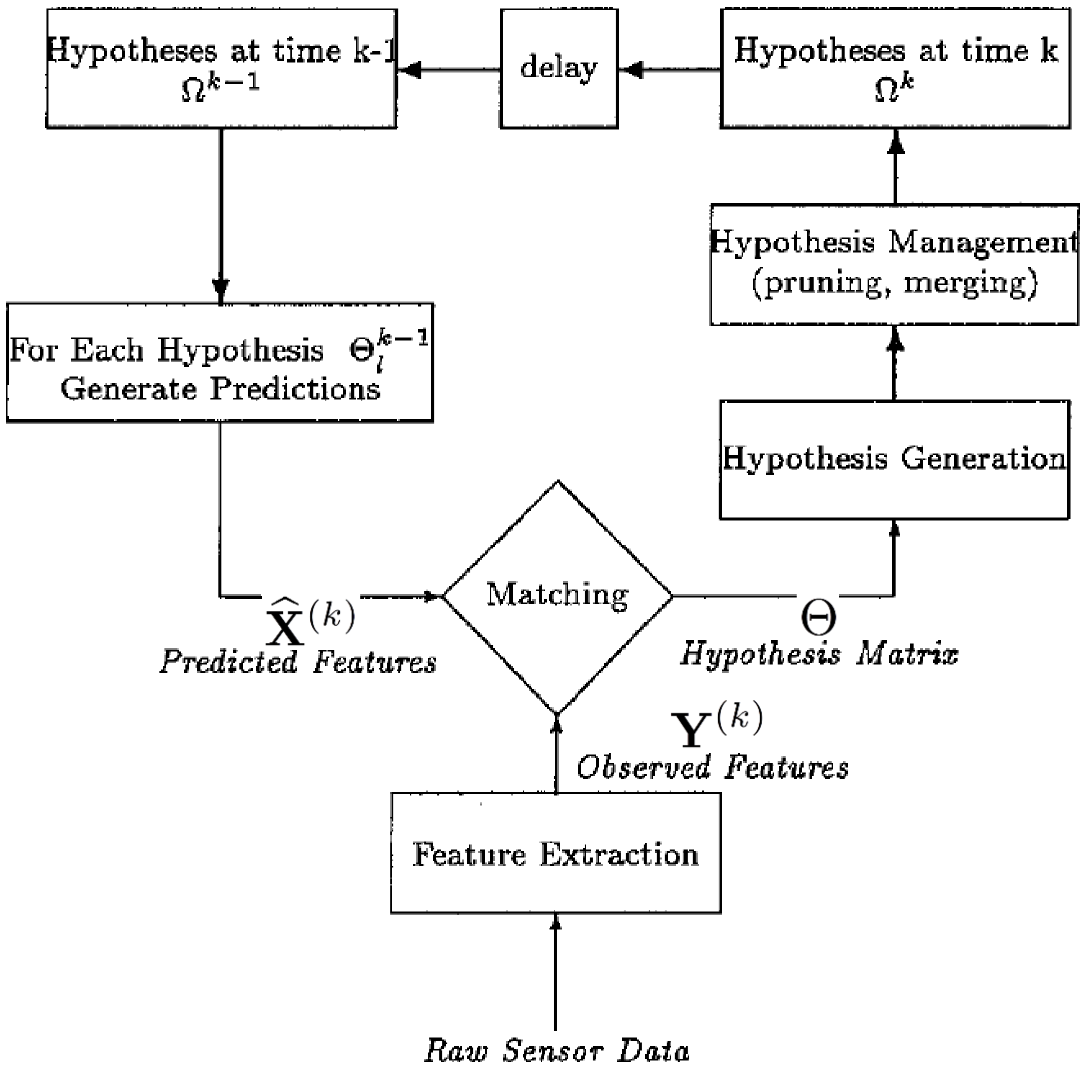


Image source: [Cox, IJCV'93]

Hypothesis Generation

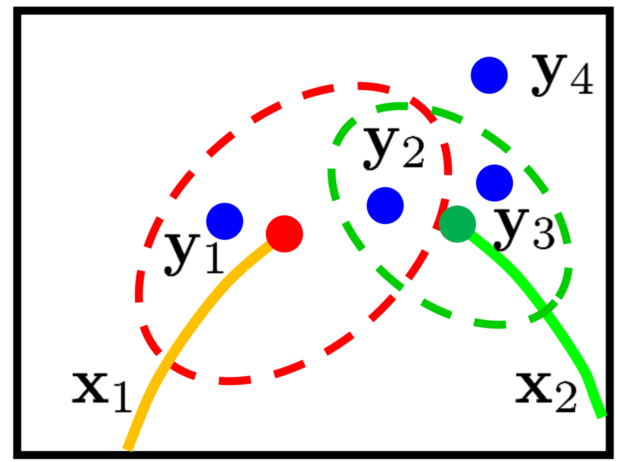
- Formalization
  - Set of hypotheses at time k: $\boldsymbol{\Omega}^{(\mathbf{k})} = \left\{ \Omega_j^{(k)} \right\}$
  - This set is obtained from $\boldsymbol{\Omega}^{(k\text{-}1)}$ and the latest set of measurements
$$\mathbf{Y}^{(k)} = \left\{ \mathbf{y}_1^{(k)}, \ldots, \mathbf{y}_{M_k}^{(k)} \right\}$$
  - The set $\boldsymbol{\Omega}^{(k)}$ is generated from $\boldsymbol{\Omega}^{(k\text{-}1)}$ by performing all feasible associations between the old hypotheses and the new measurements $\mathbf{Y}^{(k)}$.

- Feasible associations can be
  - A continuation of a previous track
  - A false alarm
  - A new target

Hypothesis Matrix

- Visualize feasible associations by a hypothesis matrix

$$
\Theta = \begin{array}{c c}
\begin{array}{cccc} \mathbf{x}_1 & \mathbf{x}_2 & \mathbf{x}_{fa} & \mathbf{x}_{nt} \end{array} & \\
\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} & \begin{array}{c} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \\ \mathbf{y}_4 \end{array}
\end{array}
$$



- Interpretation
  - Columns represent tracked objects
  - Rows represent measurements
  - A non-zero element at matrix position (i,j) denotes that measurement $\mathbf{y}_i$ is contained in the validation region of track $\mathbf{x}_j$.
  - Extra column $\mathbf{x}_{fa}$ for association as *false alarm*.
  - Extra column $\mathbf{x}_{nt}$ for association as *new track*.

Assignments

- Turning feasible associations into assignments
  - For each feasible association, we generate a new hypothesis.
  - Let $\Omega_j^{(k)}$ be the j-th hypothesis at time k and $\Omega_{p(j)}^{(k-1)}$ be the parent hypothesis from which $\Omega_j^{(k)}$ was derived.
  - Let $Z_j^{(k)}$ denote the set of assignments that gives rise to $\Omega_j^{(k)}$.
  - Assignments are again best visualized in matrix form

Z_j	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_{fa}$	$\mathbf{x}_{nt}$
$\mathbf{y}_1$	0	0	1	0
$\mathbf{y}_2$	1	0	0	0
$\mathbf{y}_3$	0	1	0	0
$\mathbf{y}_4$	0	0	0	1

Assignments

Z_j	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_{fa}$	$\mathbf{x}_{nt}$
$\mathbf{y}_1$	0	0	1	0
$\mathbf{y}_2$	1	0	0	0
$\mathbf{y}_3$	0	1	0	0
$\mathbf{y}_4$	0	0	0	1

- Impose constraints
  - A measurement can originate from only one object.

  $\Rightarrow$ Any row has only a single non-zero value.
  - An object can have at most one associated measurement per time step.

  $\Rightarrow$ Any column has only a single non-zero value, except for $\mathbf{x}_{fa}$, $\mathbf{x}_{nt}$

Calculating Hypothesis Probabilities

- **Probabilistic formulation**
  - It is straightforward to enumerate all possible assignments.
  - However, we also need to calculate the probability of each child hypothesis.
  - This is done recursively:

$$
\begin{aligned}
p(\Omega_j^{(k)}|\mathbf{Y}^{(k)}) &= p(Z_j^{(k)}, \Omega_{p(j)}^{(k-1)}|\mathbf{Y}^{(k)}) \\
&\overset{Bayes}{=} \eta p(\mathbf{Y}^{(k)}|Z_j^{(k)}, \Omega_{p(j)}^{(k-1)}) p(Z_j^{(k)}, \Omega_{p(j)}^{(k-1)}) \\
&= \underset{\text{Normalization factor}}{\eta} \underbrace{p(\mathbf{Y}^{(k)}|Z_j^{(k)}, \Omega_{p(j)}^{(k-1)})}_{\text{Measurement likelihood}} \underbrace{p(Z_j^{(k)}|\Omega_{p(j)}^{(k-1)})}_{\text{Prob. of assignment set}} \underbrace{p(\Omega_{p(j)}^{(k-1)})}_{\text{Prob. of parent}}
\end{aligned}
$$

Measurement Likelihood

- Use KF prediction
  - Assume that a measurement $\mathbf{y}_i^{(k)}$ associated to a track $\mathbf{x}_j$ has a Gaussian pdf centered around the measurement prediction $\hat{\mathbf{x}}_j^{(k)}$ with innovation covariance $\widehat{\mathbf{\Sigma}}_j^{(k)}$.
  - Further assume that the pdf of a measurement belonging to a new track or false alarm is uniform in the observation volume W (the sensor's field-of-view) with probability W^{-1}.
  - Thus, the measurement likelihood can be expressed as

$$p\left(\mathbf{Y}^{(k)} | Z_j^{(k)}, \Omega_{p(j)}^{(k-1)}\right) = \prod_{i=1}^{M_k} \mathcal{N}\left(\mathbf{y}_i^{(k)}; \hat{\mathbf{x}}_j, \widehat{\mathbf{\Sigma}}_j^{(k)}\right)^{\delta_i} W^{-(1-\delta_i)}$$

$$= W^{-(N_{fal}+N_{new})} \prod_{i=1}^{M_k} \mathcal{N}\left(\mathbf{y}_i^{(k)}; \hat{\mathbf{x}}_j, \widehat{\mathbf{\Sigma}}_j^{(k)}\right)^{\delta_i}$$

Probability of an Assignment Set

$$p(Z_j^{(k)}|\Omega_{p(j)}^{(k-1)})$$

- Composed of three terms
  1. Probability of the **number of tracks** N_{det}, N_{fal}, N_{new}
     - Assumption 1: N_{det} follows a binomial distribution

$$p(N_{det}|\Omega_{p(j)}^{(k-1)}) \;=\; \binom{N}{N_{det}} p_{det}^{N_{det}}(1-p_{det})^{(N-N_{det})}$$

       where N is the number of tracks in the parent hypothesis

     - Assumption 2: N_{fal} and N_{new} both follow a Poisson distribution with expected number of events $\lambda_{fal}W$ and $\lambda_{new}W$

$$p(N_{det},N_{fal},N_{new}|\Omega_{p(j)}^{(k-1)}) \;=\; \binom{N}{N_{det}} p_{det}^{N_{det}}(1-p_{det})^{(N-N_{det})} \cdot\mu(N_{fal};\lambda_{fal}W)\cdot\mu(N_{new};\lambda_{new}W)$$

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Probability of an Assignment Set

2. Probability of a specific assignment of measurements
   - Such that $M_k = N_{det} + N_{fal} + N_{new}$ holds.
   - This is determined as 1 over the number of combinations

$$\binom{M_k}{N_{det}} \binom{M_k - N_{det}}{N_{fal}} \binom{M_k - N_{det} - N_{fal}}{N_{new}}$$

3. Probability of a specific assignment of tracks
   - Given that a track can be either *detected* or not *detected*.
   - This is determined as 1 over the number of assignments

$$\frac{N!}{(N - N_{det})!} \binom{N - N_{det}}{N_{det}}$$

Measurement Likelihood

- Combining all the different parts
  - Nice property: many terms cancel out!
  - (Derivation left as exercise)

$\Rightarrow$ The final probability $p\left(\Omega_j^{(k)} | \mathbf{Y}^{(k)}\right)$ can be computed in a very simple form.

  - This was the main contribution by Reid and it is one of the reasons why the approach is still popular.

- Practical issues
  - Exponential complexity remains
  - Heuristic pruning strategies must be applied to contain the growth of the hypothesis set.
  - E.g., dividing hypotheses into spatially disjoint clusters.

References and Further Reading

- A good tutorial on Data Association
  - I.J. Cox. A Review of Statistical Data Association Techniques for Motion Correspondence. In *International Journal of Computer Vision*, Vol. 10(1), pp. 53-66, 1993.
- Reid's original MHT paper
  - D. Reid, An Algorithm for Tracking Multiple Targets, IEEE Trans. Automatic Control, Vol. 24(6), pp. 843-854, 1979.