Computer Vision II - Lecture 11

Multi-Object Tracking I

17.06.2014

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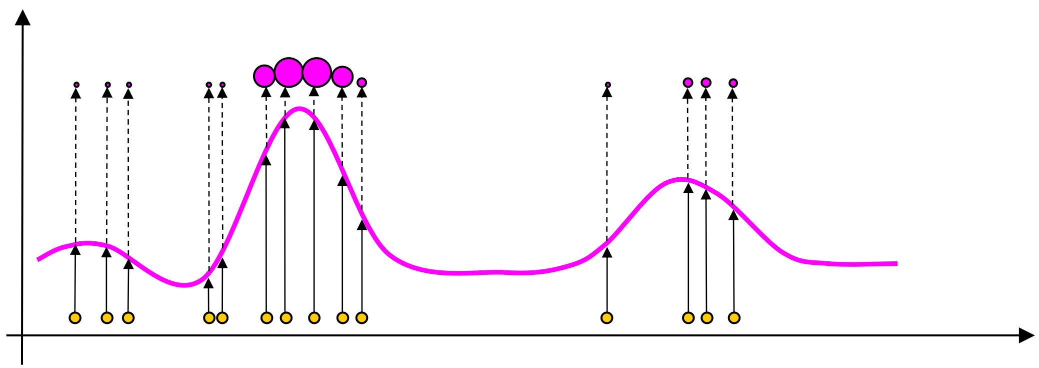
Course Outline

- Single-Object Tracking
- Bayesian Filtering
  - Kalman filters
  - Particle filters
  - Case studies
- Multi-Object Tracking
  - Introduction
  - MHT, JPDAF
  - Network Flow Optimization
- Articulated Tracking

Recap: Particle Filtering

- **Many variations, one general concept:**
  - ***Represent the posterior pdf by a set of randomly chosen weighted samples (particles)***



  - **Randomly Chosen = Monte Carlo (MC)**
  - **As the number of samples become very large - the characterization becomes an equivalent representation of the true pdf.**

Recap: Sequential Importance Sampling

$$\textbf{function} \; \left[\left\{\mathbf{x}_t^i, w_t^i\right\}_{i=1}^N\right] = SIS\left[\left\{\mathbf{x}_{t-1}^i, w_{t-1}^i\right\}_{i=1}^N, \mathbf{y}_t\right]$$

$$\eta = 0$$ **Initialize**

$$\textbf{for} \; i = 1{:}N$$

$$\mathbf{x}_t^i \sim q(\mathbf{x}_t | \mathbf{x}_{t-1}^i, \mathbf{y}_t)$$ **Sample from proposal pdf**

$$w_t^i = w_{t-1}^i \frac{p(\mathbf{y}_t | \mathbf{x}_t^i) p(\mathbf{x}_t^i | \mathbf{x}_{t-1}^i)}{q(\mathbf{x}_t | \mathbf{x}_{t-1}^i, \mathbf{y}_t)}$$ **Update weights**

$$\eta = \eta + w_t^i$$ **Update norm. factor**

$$\textbf{end}$$

$$\textbf{for} \; i = 1{:}N$$

$$w_t^i = w_t^i / \eta$$ **Normalize weights**

$$\textbf{end}$$

Recap: Sequential Importance Sampling

$$\text{function} \ \left[\{\mathbf{x}_t^i, w_t^i\}_{i=1}^N\right] = SIS\left[\{\mathbf{x}_{t-1}^i, w_{t-1}^i\}_{i=1}^N, \mathbf{y}_t\right]$$

$\eta = 0$ — **Initialize**

for $i = 1{:}N$

$$\mathbf{x}_t^i \sim q(\mathbf{x}_t|\mathbf{x}_{t-1}^i, \mathbf{y}_t)$$ — **Sample from proposal pdf**

$$w_t^i = w_{t-1}^i \frac{p(\mathbf{y}_t|\mathbf{x}_t^i)p(\mathbf{x}_t^i|\mathbf{x}_{t-1}^i)}{q(\mathbf{x}_t|\mathbf{x}_{t-1}^i, \mathbf{y}_t)}$$ — **Update weights**

$$\eta = \eta + w_t^i$$ — **Update norm. factor**

end

for $i = 1{:}N$

$$w_t^i = w_t^i / \eta$$ — **Normalize weights**

end

For a concrete algorithm, we need to define the importance density $q(.|.)$!

Recap: SIS Algorithm with Transitional Prior

$$\text{function}\ \left[\left\{\mathbf{x}_t^i, w_t^i\right\}_{i=1}^N\right] = SIS\left[\left\{\mathbf{x}_{t-1}^i, w_{t-1}^i\right\}_{i=1}^N, \mathbf{y}_t\right]$$

$\eta = 0$ — **Initialize**

$\text{for}\ \ i = 1{:}N$

$\quad \mathbf{x}_t^i \sim p(\mathbf{x}_t|\mathbf{x}_{t-1}^i)$ — **Sample from proposal pdf**

$\quad w_t^i = w_{t-1}^i p(\mathbf{y}_t|\mathbf{x}_t^i)$ — **Update weights**

$\quad \eta = \eta + w_t^i$ — **Update norm. factor**

end

Transitional prior

$$q(\mathbf{x}_t|\mathbf{x}_{t-1}^i, \mathbf{y}_t) = p(\mathbf{x}_t|\mathbf{x}_{t-1}^i)$$

$\text{for}\ \ i = 1{:}N$

$\quad w_t^i = w_t^i/\eta$ — **Normalize weights**

end

Slide adapted from Michael Rubinstein

Recap: Resampling

- Degeneracy problem with SIS
  - After a few iterations, most particles have negligible weights.
  - Large computational effort for updating particles with very small contribution to $p(\mathbf{x}_t \mid \mathbf{y}_{1:t})$.
- Idea: Resampling
  - Eliminate particles with low importance weights and increase the number of particles with high importance weight.

$$\{\mathbf{x}_t^i, w_t^i\}_{i=1}^N \rightarrow \left\{\mathbf{x}_t^{i*}, \frac{1}{N}\right\}_{i=1}^N$$

  - The new set is generated by sampling with replacement from the discrete representation of $p(\mathbf{x}_t \mid \mathbf{y}_{1:t})$ such that

$$Pr\left\{\mathbf{x}_t^{i*} = \mathbf{x}_t^j\right\} = w_t^j$$

Recap: Efficient Resampling Approach

- **From Arulampalam paper:**

Algorithm 2: Resampling Algorithm
$[\{\mathbf{x}_k^{j*}, w_k^j, i^j\}_{j=1}^{N_s}] = $ RESAMPLE $[\{\mathbf{x}_k^i, w_k^i\}_{i=1}^{N_s}]$

- Initialize the CDF: $c_1 = 0$
- FOR $i = 2{:}\ N_s$
  - Construct CDF: $c_i = c_{i-1} + w_k^i$
- END FOR
- Start at the bottom of the CDF: $i = 1$
- Draw a starting point: $u_1 \sim \mathbb{U}[0, N_s^{-1}]$
- FOR $j = 1{:}\ N_s$
  - Move along the CDF: $u_j = u_1 + N_s^{-1}(j-1)$
  - WHILE $u_j > c_i$
    - $i = i + 1$
  - END WHILE
  - Assign sample: $\mathbf{x}_k^{j*} = \mathbf{x}_k^i$
  - Assign weight: $w_k^j = N_s^{-1}$
  - Assign parent: $i^j = i$
- END FOR

Basic idea: choose one initial small random number; deter-ministically sample the rest by "crawling" up the cdf. This is $\mathcal{O}(N)$!

Recap: Generic Particle Filter

$$\textbf{function} \quad \left[\left\{\mathbf{x}_t^i, w_t^i\right\}_{i=1}^{N}\right] = PF\left[\left\{\mathbf{x}_{t-1}^i, w_{t-1}^i\right\}_{i=1}^{N}, \mathbf{y}_t\right]$$

$$\textit{Apply SIS filtering} \quad \left[\left\{\mathbf{x}_t^i, w_t^i\right\}_{i=1}^{N}\right] = SIS\left[\left\{\mathbf{x}_{t-1}^i, w_{t-1}^i\right\}_{i=1}^{N}, \mathbf{y}_t\right]$$

$$\textit{Calculate } N_{eff} = \frac{1}{\sum_{i=1}^{N} (w_t^i)^2}$$

$$\textbf{if} \quad N_{eff} < N_{thr}$$

$$\left[\left\{\mathbf{x}_t^i, w_t^i\right\}_{i=1}^{N}\right] = RESAMPLE\left[\left\{\mathbf{x}_t^i, w_t^i\right\}_{i=1}^{N}\right]$$

$$\textbf{end}$$

- **We can also apply resampling selectively**
  - **Only resample when it is needed, i.e., N_{eff} is too low.**
  - **⇒ Avoids drift when there the tracked state is stationary.**

Sampling-Importance-Resampling Algorithm

function $[\mathcal{X}_t] = SIR\,[\mathcal{X}_{t-1}, \mathbf{y}_t]$

$\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$ — **Initialize**

for $i = 1{:}N$

$\quad Sample\ \ \mathbf{x}_t^i \sim p(\mathbf{x}_t | \mathbf{x}_{t-1}^i)$ — **Generate new samples**

$\quad w_t^i = p(\mathbf{y}_t | \mathbf{x}_t^i)$ — **Update weights**

end

for $i = 1{:}N$

$\quad Draw\ i\ with\ probability \propto w_t^i$

$\quad Add\ \mathbf{x}_t^i\ \ to\ \mathcal{X}_t$

Resample

end

Sampling-Importance-Resampling Algorithm

function $[\mathcal{X}_t] = SIR\,[\mathcal{X}_{t-1}, \mathbf{y}_t]$

$\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$

for $i = 1{:}N$

$\quad Sample\;\; \mathbf{x}_t^i \sim p(\mathbf{x}_t | \mathbf{x}_{t-1}^i)$

$\quad w_t^i = p(\mathbf{y}_t | \mathbf{x}_t^i)$

end

for $i = 1{:}N$

$\quad Draw\ i\ with\ probability \propto w_t^i$

$\quad Add\ \mathbf{x}_t^i\ \ to\ \mathcal{X}_t$

end

Important property:

Particles are distributed according to pdf from previous time step.

Particles are distributed according to posterior from this time step.

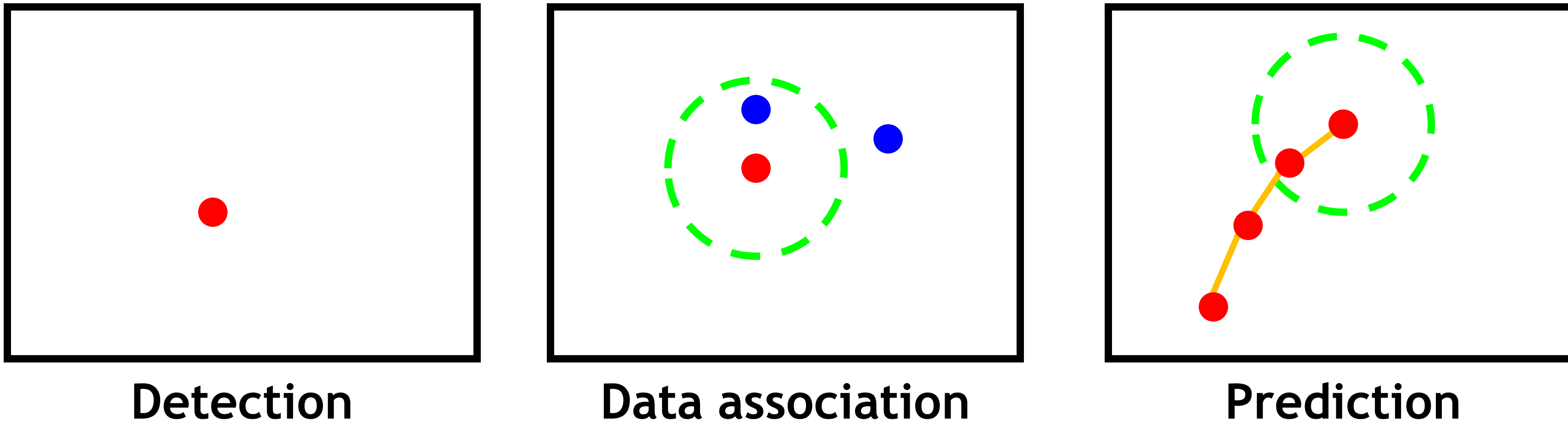
Today: Multi-Object Tracking

[Ess, Leibe, Schindler, Van Gool, CVPR'08; ICRA'09; PAMI'09]

Topics of This Lecture

- **Multi-Object Tracking**
  - Motivation
  - Ambiguities
- **Simple Approaches**
  - Gating
  - Mahalanobis distance
  - Nearest-Neighbor Filter
- **Track-Splitting Filter**
  - Derivation
  - Properties
- **Outlook**

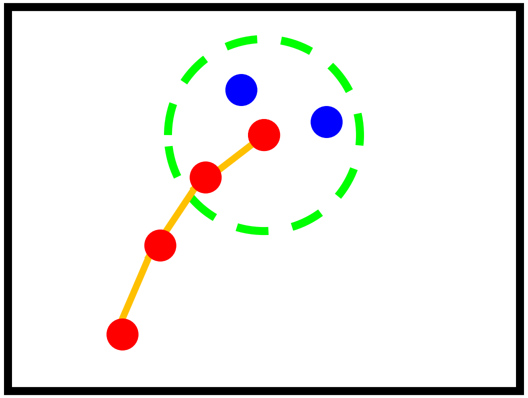
Elements of Tracking



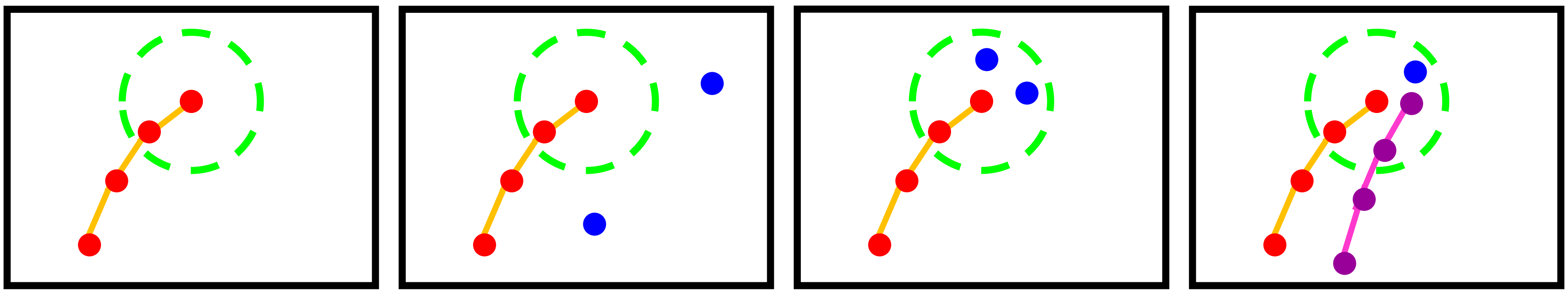
- **Detection** — Lecture 7
  - *Where are candidate objects?*
- **Data association** — Today's topic
  - *Which detection corresponds to which object?*
- **Prediction** — Lectures 8-10
  - *Where will the tracked object be in the next time step?*

Motion Correspondence

- Motion correspondence problem
  - Do two measurements at different times originate from the same object?
- Why is it hard?
  - First make predictions for the expected locations of the current set of objects
  - Match predictions to actual measurements
  - This is where ambiguities may arise...



Motion Correspondence Ambiguities



1. Predictions may not be supported by measurements
   - Have the objects ceased to exist, or are they simply occluded?
2. There may be unexpected measurements
   - Newly visible objects, or just noise?
3. More than one measurement may match a prediction
   - Which measurement is the correct one (what about the others)?
4. A measurement may match to multiple predictions
   - Which object shall the measurement be assigned to?

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Let's Formalize This

- **Multi-Object Tracking problem**
  - We represent a track by a state vector $\mathbf{x}$, e.g.,

$$\mathbf{x} = [x, y, v_x, v_y]^T$$

  - As the track evolves, we denote its state by the time index k:

$$\mathbf{x}^{(k)} = \left[x^{(k)}, y^{(k)}, v_x^{(k)}, v_y^{(k)}\right]^T$$

  - At each time step, we get a set of observations (measurements)

$$\mathbf{Y}^{(k)} = \left\{\mathbf{y}_1^{(k)}, \ldots, \mathbf{y}_{M_k}^{(k)}\right\}$$

  - We now need to make the data association between tracks

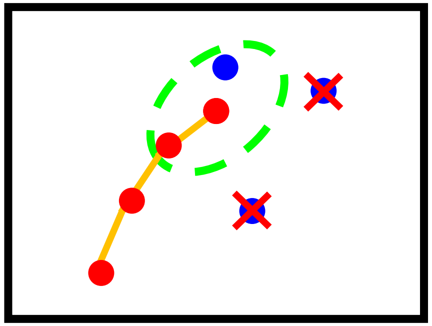
$\left\{\mathbf{x}_1^{(k)}, \ldots, \mathbf{x}_{N_k}^{(k)}\right\}$ and observations $\left\{\mathbf{y}_1^{(k)}, \ldots, \mathbf{y}_{M_k}^{(k)}\right\}$:

$z_l^{(k)} = j \text{ iff } \mathbf{y}_j^{(k)}$ is associated with $\mathbf{x}_l^{(k)}$

Reducing Ambiguities: Simple Approaches

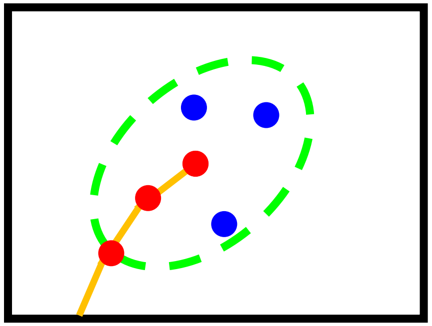
- **Gating**
  - Only consider measurements within a certain area around the predicted location.

  ⇒ Large gain in efficiency, since only a small region needs to be searched



- **Nearest-Neighbor Filter**
  - Among the candidates in the gating region, only take the one closest to the prediction $\mathbf{x}_p$

$$z_l^{(k)} = \arg\min_j (\mathbf{x}_{p,l}^{(k)} - \mathbf{y}_j^{(k)})^T (\mathbf{x}_{p,l}^{(k)} - \mathbf{y}_j^{(k)})$$



  - Better: the one most likely under a Gaussian prediction model

$$z_l^{(k)} = \arg\max_j \mathcal{N}(\mathbf{y}_j^{(k)}; \mathbf{x}_{p,l}^{(k)}, \mathbf{\Sigma}_{p,l}^{(k)})$$

which is equivalent to taking the Mahalanobis distance

$$z_l = \arg\min_j (\mathbf{x}_{p,l} - \mathbf{y}_j)^T \mathbf{\Sigma}_{p,l}^{-1} (\mathbf{x}_{p,l} - \mathbf{y}_j)$$

Gating with Mahalanobis Distance

- Recall: Kalman filter
  - Provides exactly the quantities necessary to perform this
  - Predicted mean location $\mathbf{x}_p$
  - Prediction covariance $\boldsymbol{\Sigma}_p$

  - The Kalman filter prediction covariance also defines a useful gating area.

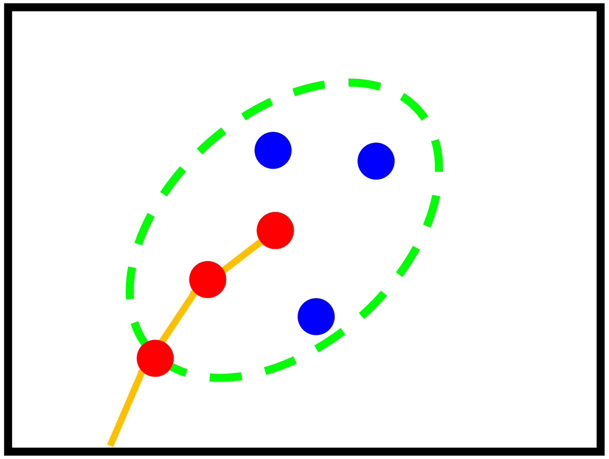
  ⇒ E.g., choose the gating area size such that 95% of the probability mass is covered.

- Side note
  - The Mahalanobis distance is χ^2 distributed with the number of degrees of freedom n_z equal to the dimension of $\mathbf{x}$.
  - For a given probability bound, the corresponding threshold on the Mahalanobis distance can be got from χ^2 distribution tables.

Mahalanobis Distance

- Additional notation
  - Our KF state of track $\mathbf{x}_l$ is given by the prediction $\mathbf{x}_{p,l}^{(k)}$ and covariance $\mathbf{\Sigma}_{p,l}^{(k)}$.
  - We define the innovation that measurement $\mathbf{y}_j$ brings to track $\mathbf{x}_l$ at time k as

$$\mathbf{v}_{j,l}^{(k)} = (\mathbf{y}_j^{(k)} - \mathbf{x}_{p,l}^{(k)})$$



  - With this, we can write the observation likelihood shortly as

$$p(\mathbf{y}_j^{(k)}|\mathbf{x}_l^{(k)}) \sim \exp\left\{-\frac{1}{2}\mathbf{v}_{j,l}^{(k)^T}\mathbf{\Sigma}_{p,l}^{(k)^{-1}}\mathbf{v}_{j,l}^{(k)}\right\}$$

  - We define the ellipsoidal gating or validation volume as

$$V^{(k)}(\gamma) = \left\{\mathbf{y}|(\mathbf{y} - \mathbf{x}_{p,l}^{(k)})^T\mathbf{\Sigma}_{p,l}^{(k)^{-1}}(\mathbf{y} - \mathbf{x}_{p,l}^{(k)}) \leq \gamma\right\}$$

Problems with NN Assignment

- **Limitations**
  - For NN assignments, there is always a finite chance that the association is incorrect, which can lead to serious effects.
  - ⇒ If a Kalman filter is used, a misassigned measurement may lead the filter to lose track of its target.
  - The NN filter makes assignment decisions only based on the current frame.
  - More information is available by examining subsequent images.
  - ⇒ Let's make use of this information by postponing the decision process until a future frame will resolve the ambiguity...

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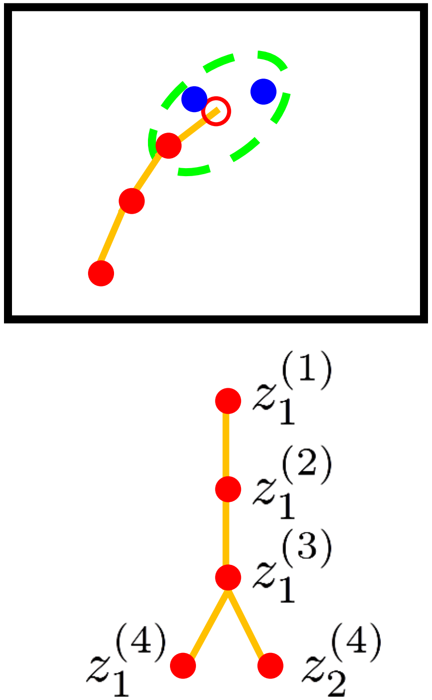
- Multi-Object Tracking
  - Motivation
  - Ambiguities
- Simple Approaches
  - Gating
  - Mahalanobis distance
  - Nearest-Neighbor Filter
- **Track-Splitting Filter**
  - **Derivation**
  - **Properties**
- Outlook

Track-Splitting Filter

- Idea
  - Problem with NN filter was hard assignment.
  - Rather than arbitrarily assigning the closest measurement, form a tree.
  - Branches denote alternate assignments.
  - No assignment decision is made at this stage!
  - ⇒ Decisions are postponed until additional measurements have been gathered...



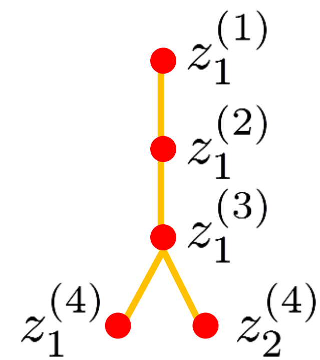
- Potential problems?
  - Track trees can quickly become very large due to combinatorial explosion.
  - ⇒ We need some measure of the likelihood of a track, so that we can prune the tree!

Track Likelihoods

- **Expressing track likelihoods**
  - Given a track l, denote by $\theta_{k,l}$ the event that the sequence of assignments

$$Z_{k,l} = \left\{ z_{i_1,l}^{(1)}, \ldots, z_{i_k,l}^{(k)} \right\}$$

from time 1 to k originate from the same object.



  - The likelihood of $\theta_{k,l}$ is the joint probability over all observations in the track

$$L(\theta_{k,l}) = \prod_{j=1}^{k} p(z_{i_j,l}^{(j)} | Z_{(j-1),l}, \theta_{k,l})$$

  - If we assume Gaussian observation likelihoods, this becomes

$$L(\theta_{k,l}) = \prod_{j=1}^{k} \frac{1}{(2\pi)^{\frac{d}{2}} |\mathbf{\Sigma}_l^{(j)}|^{\frac{1}{2}}} \exp\left[-\frac{1}{2} \sum_{j=1}^{k} \mathbf{v}_{i_j,l}^{(j)^T} \mathbf{\Sigma}_l^{(j)^{-1}} \mathbf{v}_{i_j,l}^{(j)} \right]$$

Track Likelihoods (2)

- **Starting from the likelihood**

$$L(\theta_{k,l}) = \prod_{j=1}^{k} \frac{1}{(2\pi)^{\frac{d}{2}} |\mathbf{\Sigma}_l^{(j)}|^{\frac{1}{2}}} \exp\left[-\frac{1}{2} \sum_{j=1}^{k} \mathbf{v}_{i_j,l}^{(j)^T} \mathbf{\Sigma}_l^{(j)^{-1}} \mathbf{v}_{i_j,l}^{(j)} \right]$$

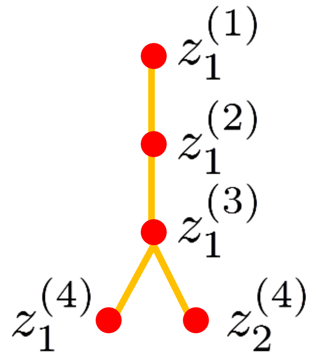
  - **Define the modified log-likelihood λ_l for track l as**

$$\begin{aligned} \lambda_l(k) &= -2\log\left[\frac{L(\theta_{k,l})}{\prod_{j=1}^{k} (2\pi)^{-\frac{d}{2}} |\mathbf{\Sigma}_l^{(j)}|^{-\frac{1}{2}}} \right] \\ &= \sum_{j=1}^{k} \mathbf{v}_{i_j,l}^{(j)^T} \mathbf{\Sigma}_l^{(j)^{-1}} \mathbf{v}_{i_j,l}^{(j)} \\ &= \lambda_l(k-1) + \mathbf{v}_{i_k,l}^{(k)^T} \mathbf{\Sigma}_l^{(k)^{-1}} \mathbf{v}_{i_k,l}^{(k)} \end{aligned}$$

$\Rightarrow$ **Recursive calculation, sum of Mahalanobis distances of all the measurements assigned to track l.**

Track-Splitting Filter

- Effect
  - Instead of assigning the measurement that is currently closest, as in the NN algorithm, we can select the *sequence* of measurements that minimizes the *total* Mahalanobis distance over some interval!



  - Modified log-likelihood provides the merit of a particular node in the track tree.
  - Cost of calculating this is low, since most terms are needed anyway for the Kalman filter.

- Problem
  - The track tree grows exponentially, may generate a very large number of possible tracks that need to be maintained.

Pruning Strategies

- In order to keep this feasible, need to apply pruning
  - Deleting unlikely tracks
    - May be accomplished by comparing the modified log-likelihood $\lambda(k)$, which has a χ^2 distribution with kn_z degrees of freedom, with a threshold α (set according to χ^2 distribution tables).
    - Problem for long tracks: modified log-likelihood gets dominated by old terms and responds very slowly to new ones.

    $\Rightarrow$ Use sliding window or exponential decay term.
  - Merging track nodes
    - If the state estimates of two track nodes are similar, merge them.
    - E.g., if both tracks validate identical subsequent measurements.
  - Only keeping the most likely N tracks
    - Rank tracks based on their modified log-likelihood.

Summary: Track-Splitting Filter

- Properties
  - Very old algorithm
    - P. Smith, G. Buechler, A Branching Algorithm for Discriminating and Tracking Multiple Objects, IEEE Trans. Automatic Control, Vol. 20, pp. 101-104, 1975.
  - Improvement over NN assignment.
  - Assignment decisions are delayed until more information is available.
- Many problems remain
  - Exponential complexity, heuristic pruning needed.
  - Merging of track nodes is necessary, because tracks may share measurements, which is physically unrealistic.

  ⇒ Would need to add exclusion constraints such that each measurement may only belong to a single track.

  ⇒ Impossible in this framework...

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Outlook for the Next Lectures

- More powerful approaches
  - Multi-Hypothesis Tracking (MHT)
    - Well-suited for KF, EKF approaches [Reid, 1979]
  - Joint Probabilistic Data Association Filters (JPDAF)
    - Well-suited for PF approaches [Fortmann, 1983]

- Data association as convex optimization problem
  - Bipartite Graph Matching (Hungarian algorithm)
  - Network Flow Optimization

  $\Rightarrow$ Efficient, globally optimal solutions for subclass of problems.

References and Further Reading

- A good tutorial on Data Association
  - I.J. Cox. A Review of Statistical Data Association Techniques for Motion Correspondence. In *International Journal of Computer Vision*, Vol. 10(1), pp. 53-66, 1993.