Computer Vision II – Lecture 8

Tracking with Linear Dynamic Models

20.05.2014

Bastian Leibe
RWTH Aachen
http://www.vision.rwth-aachen.de

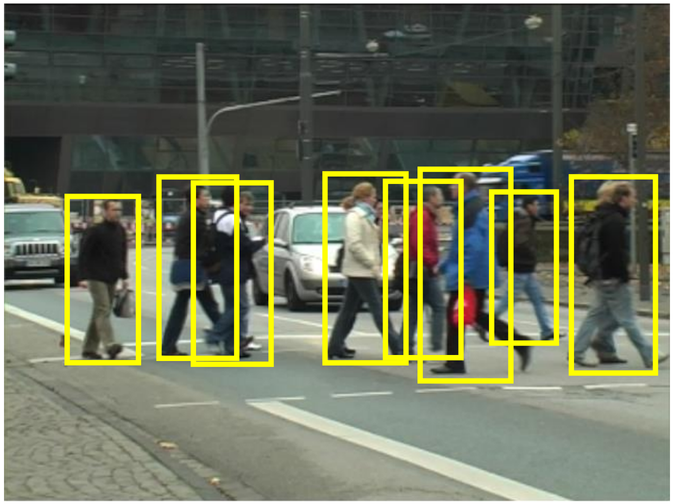
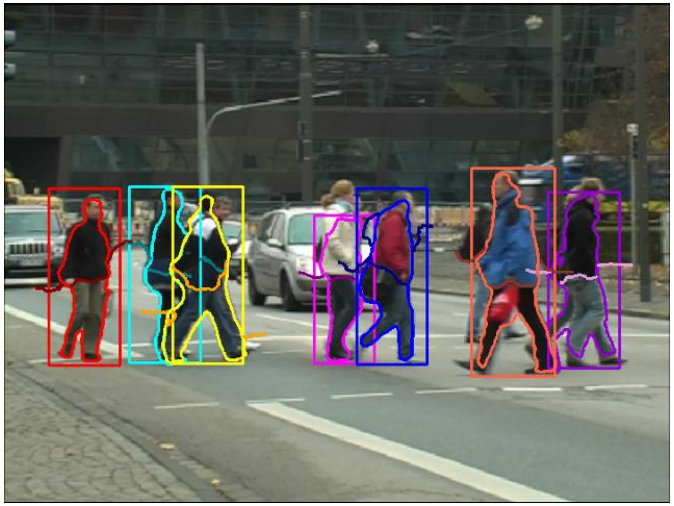
leibe@vision.rwth-aachen.de

Course Outline

- Single-Object Tracking
  - Background modeling
  - Template based tracking
  - Color based tracking
  - Contour based tracking
  - Tracking by online classification
  - Tracking-by-detection
- Bayesian Filtering
  - Kalman filter
  - Particle filter
- Multi-Object Tracking
- Articulated Tracking

Image source: Helmut Grabner, Disney/Pixar

Recap: Tracking-by-Detection

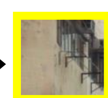
- **Main ideas**
  - Apply a generic object detector to find objects of a certain class
  - Based on the detections, extract object appearance models
  - Link detections into trajectories

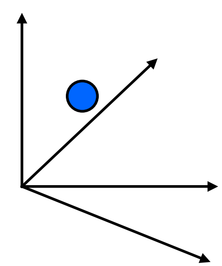
Recap: Sliding-Window Object Detection

Fleshing out this pipeline a bit more, we need to:

1. Obtain training data
2. Define features
3. Define classifier

Training examples





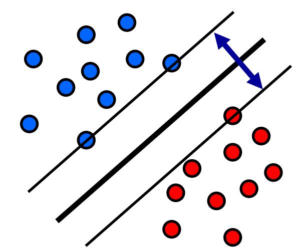
Feature extraction

Car/non-car Classifier

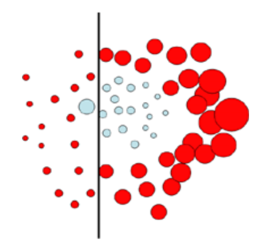
Recap: Object Detector Design

- **In practice, the classifier often determines the design.**
  - Types of features
  - Speedup strategies

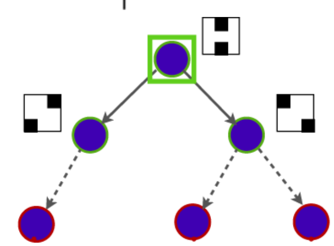
- **We'll look at 3 state-of-the-art detector designs**
  - Based on SVMs
    → Last lecture



  - Based on Boosting
    → Last lecture

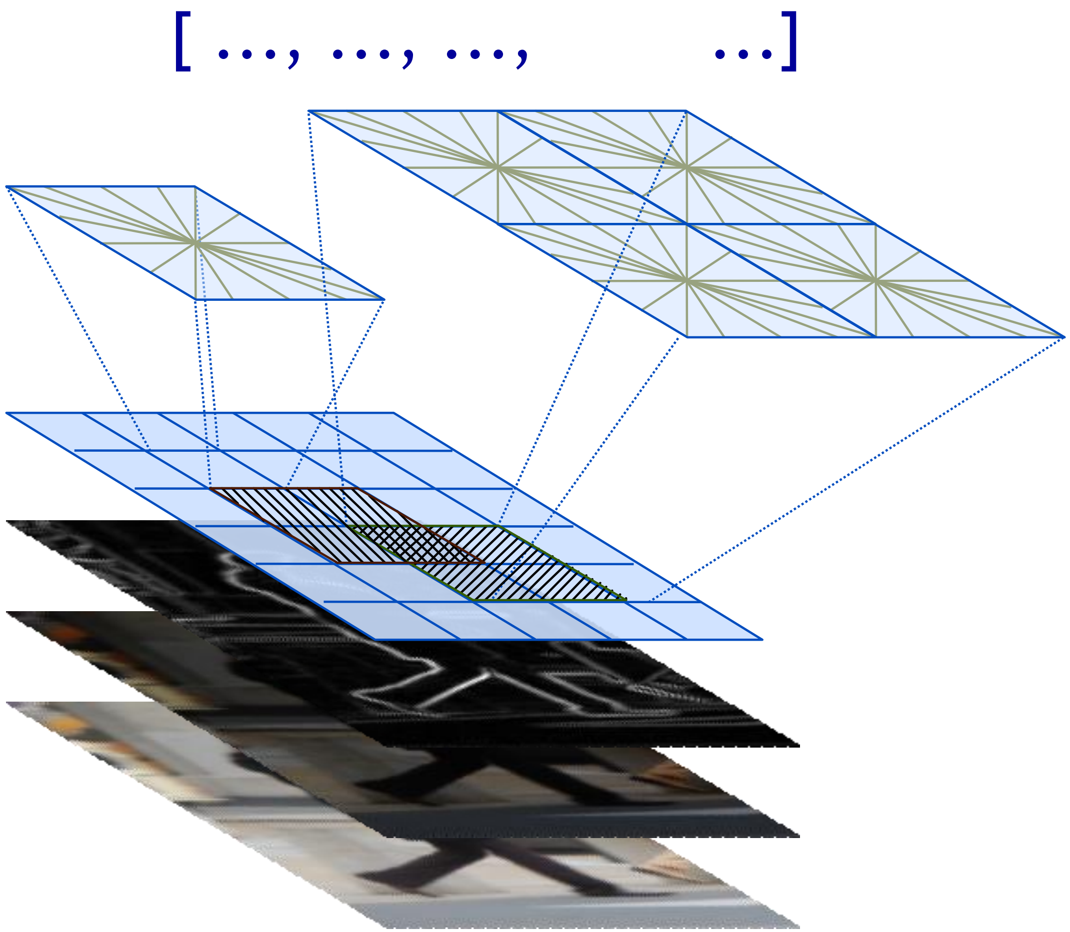


  - Based on Random Forests
    → Postponed to a later slot...



Recap: Histograms of Oriented Gradients (HOG)

- **Holistic object representation**
  - **Localized gradient orientations**



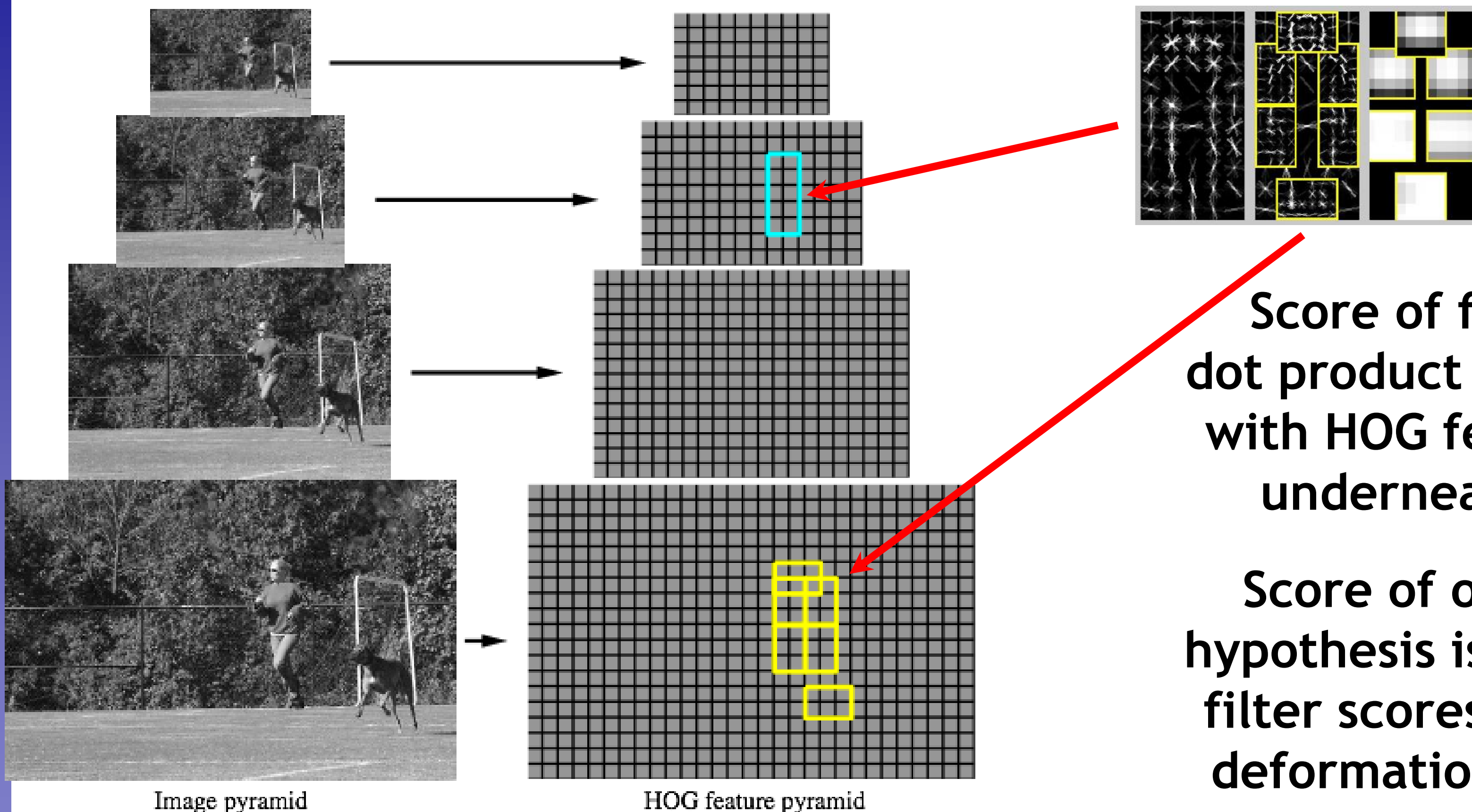
Object/Non-object

- Linear SVM
- Collect HOGs over detection window
- Contrast normalize over overlapping spatial cells
- Weighted vote in spatial & orientation cells
- Compute gradients
- Gamma compression

Image Window

Slide adapted from Navneet Dalal

Recap: Deformable Part-based Model (DPM)



Score of filter: dot product of filter with HOG features underneath it

Score of object hypothesis is sum of filter scores minus deformation costs

- **Multiscale model captures features at two resolutions**

Recap: DPM Hypothesis Score

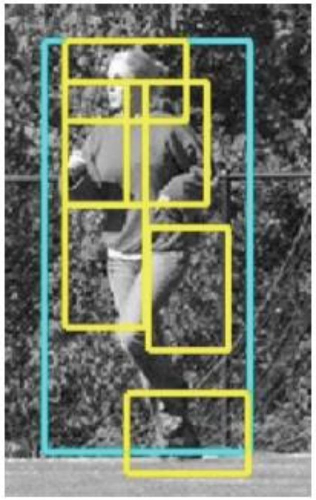
"data term" "spatial prior"

$$\text{score}(p_0, \ldots, p_n) = \sum_{i=0}^{n} F_i \cdot \phi(H, p_i) - \sum_{i=1}^{n} d_i \cdot (dx_i^2, dy_i^2)$$

filters

displacements

deformation parameters



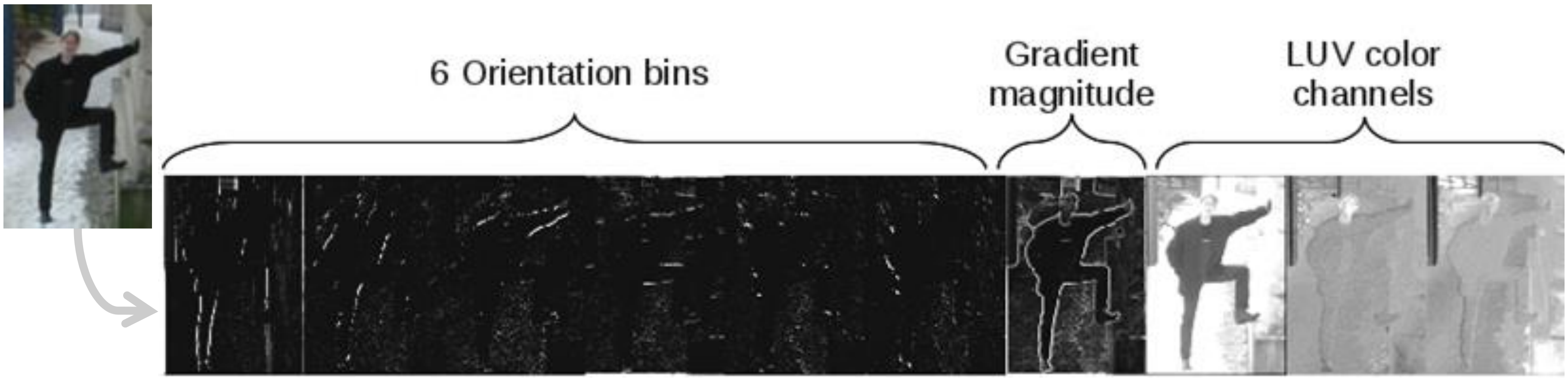
$$\text{score}(z) = \beta \cdot \Psi(H, z)$$

concatenation filters and deformation parameters

concatenation of HOG features and part displacement features

[Felzenszwalb, McAllister, Ramanan, CVPR'08]

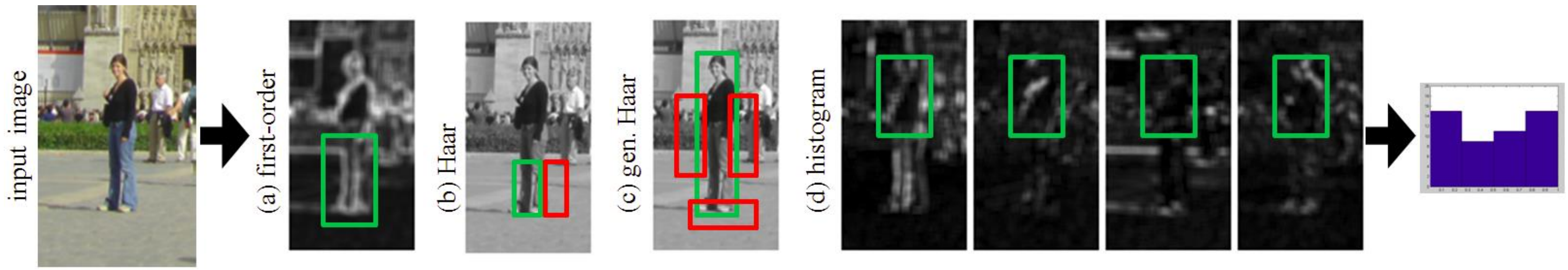
Recap: Integral Channel Features



- **Generalization of Haar Wavelet idea from Viola-Jones**
  - Instead of only considering intensities, also take into account other feature channels (gradient orientations, color, texture).
  - Still efficiently represented as integral images.

P. Dollar, Z. Tu, P. Perona, S. Belongie. Integral Channel Features, BMVC'09.

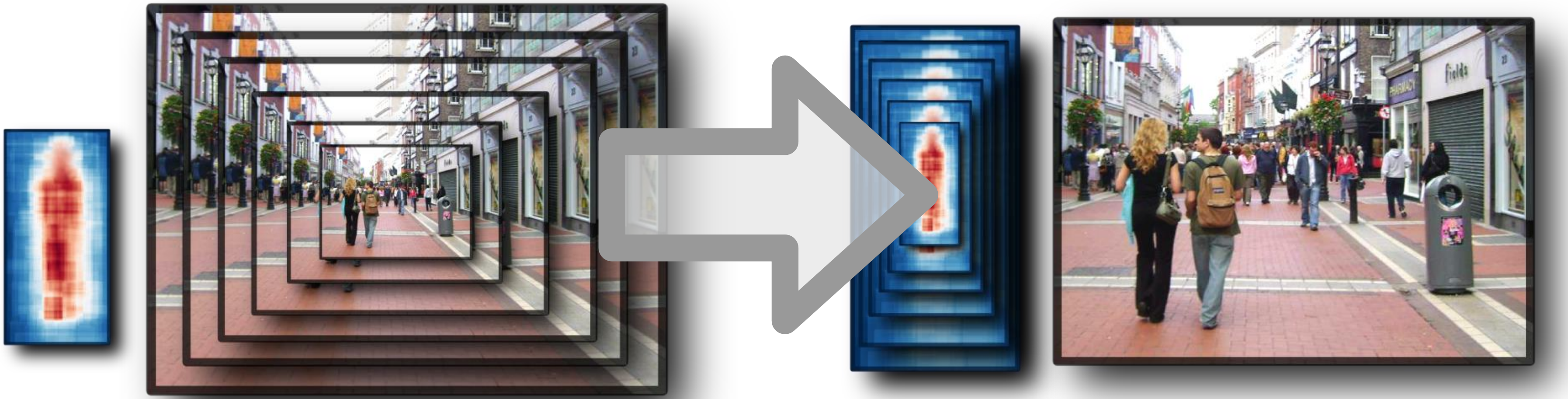
Recap: Integral Channel Features



- Generalize also block computation
  - 1st order features:
    - Sum of pixels in rectangular region.
  - 2nd-order features:
    - Haar-like difference of sum-over-blocks
  - Generalized Haar:
    - More complex combinations of weighted rectangles
  - Histograms
    - Computed by evaluating local sums on quantized images.

Recap: VeryFast Detector

- **Idea 1: Invert the relation**



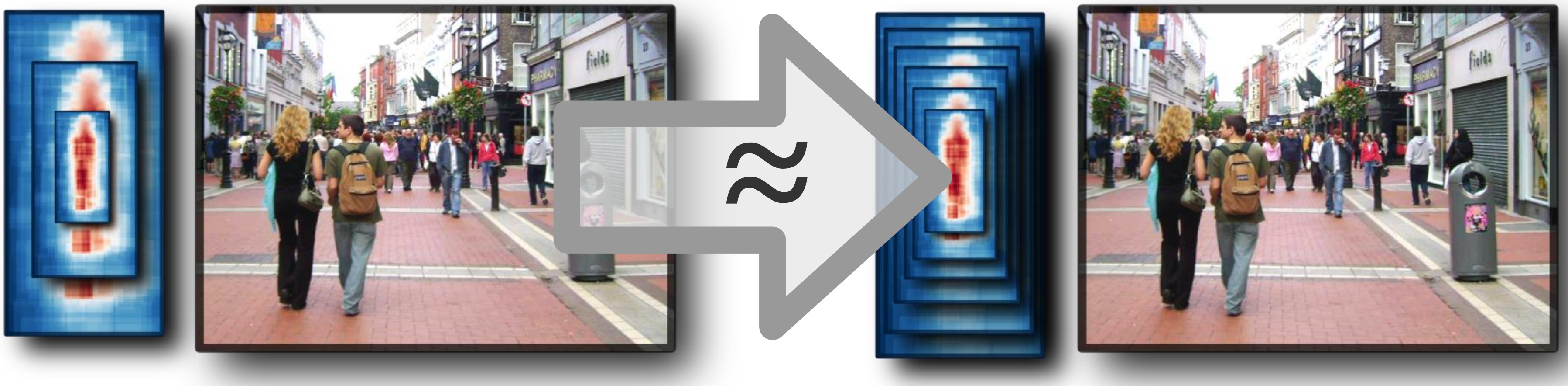
1 model,
50 image scales

50 models,
1 image scale

R. Benenson, M. Mathias, R. Timofte, L. Van Gool. Pedestrian Detection at 100 Frames per Second, CVPR'12.

Recap: VeryFast Detector

- Idea 2: Reduce training time by feature interpolation

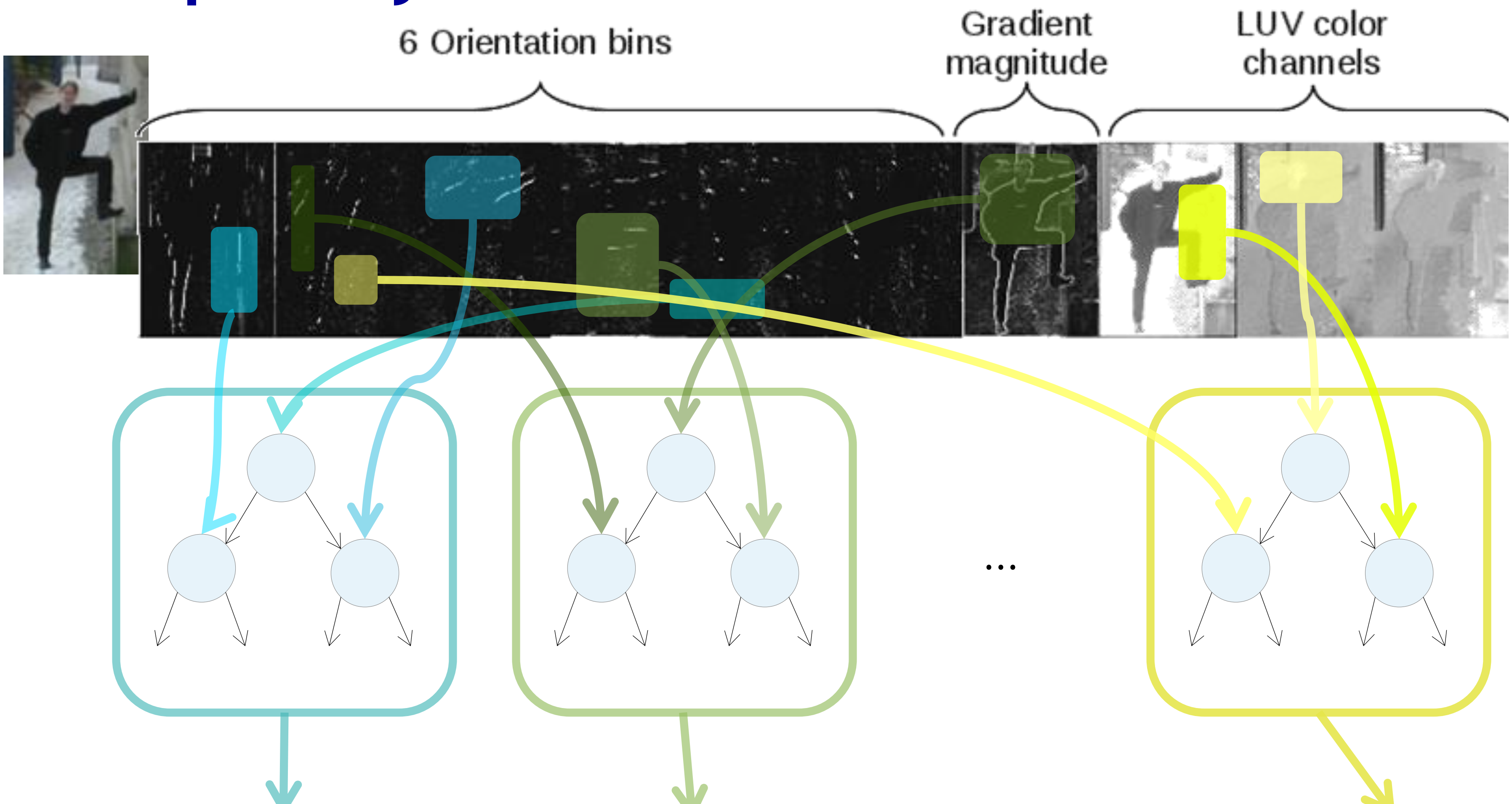


5 models,
1 image scale

50 models,
1 image scale

- Shown to be possible for Integral Channel features
  - P. Dollár, S. Belongie, Perona. The Fastest Pedestrian Detector in the West, BMVC 2010.

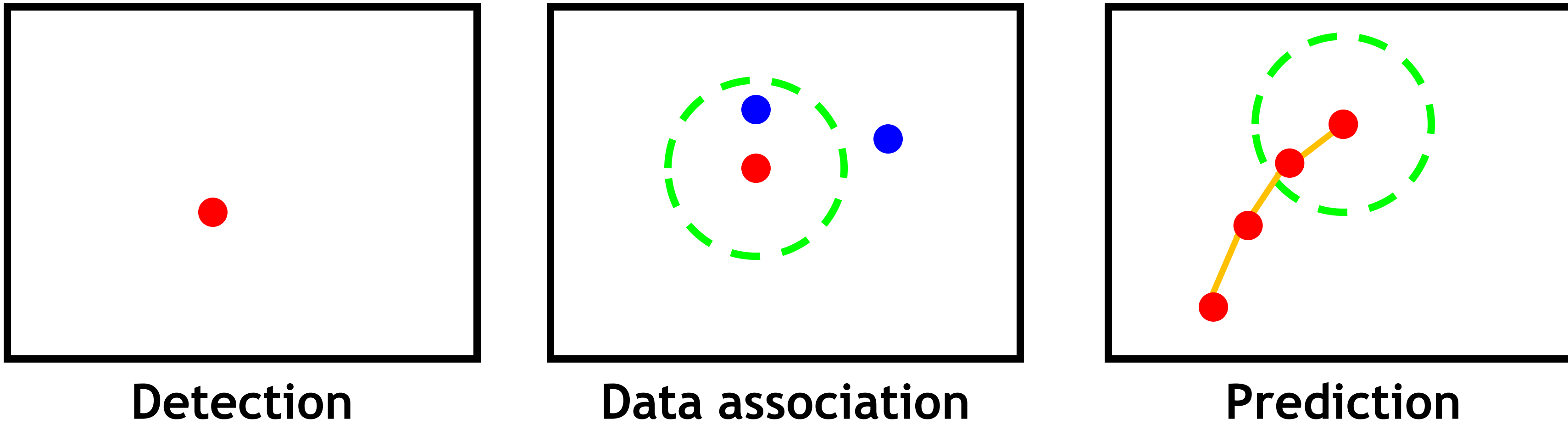
Recap: VeryFast Classifier Construction



$$score = w_1 \cdot h_1 + \quad w_2 \cdot h_2 + \quad \ldots \quad + w_N \cdot h_N$$

- **Ensemble of short trees, learned by AdaBoost**

Elements of Tracking



- **Detection**
  - *Where are candidate objects?*

Last lecture

- **Data association**
  - *Which detection corresponds to which object?*
- **Prediction**
  - *Where will the tracked object be in the next time step?*

Today's topic

Today: Tracking with Linear Dynamic Models

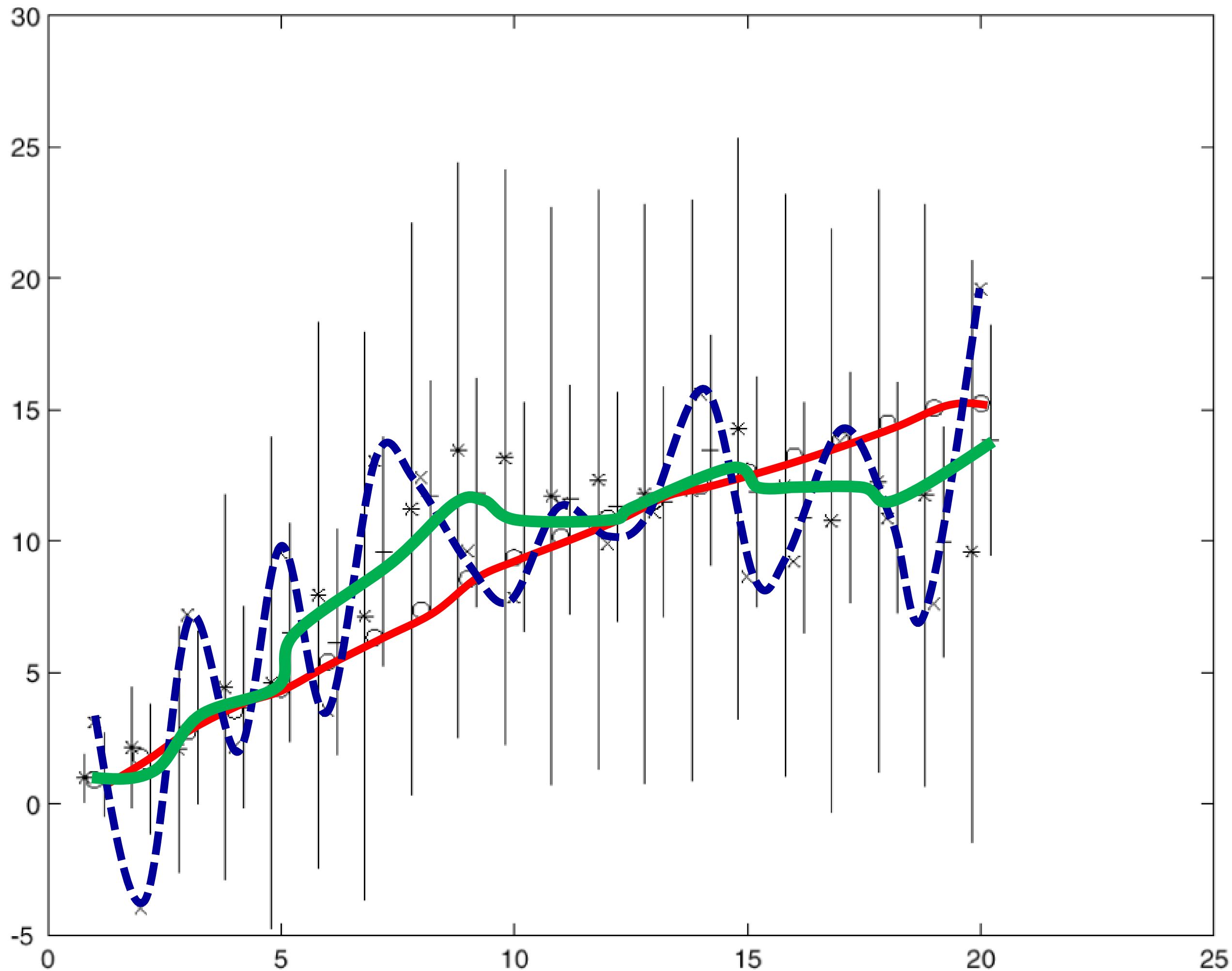


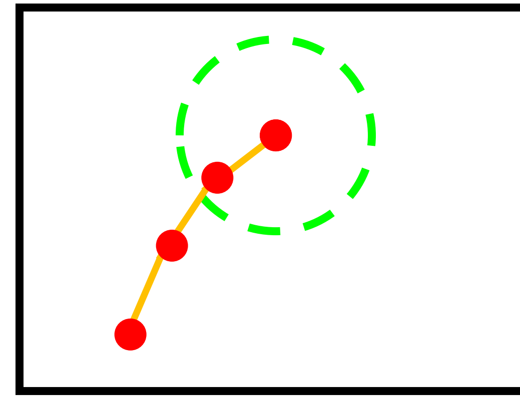
Figure from Forsyth & Ponce

Topics of This Lecture

- Tracking with Dynamics
  - Detection vs. Tracking
  - Tracking as probabilistic inference
  - Prediction and Correction
- Linear Dynamic Models
  - Zero velocity model
  - Constant velocity model
  - Constant acceleration model
- The Kalman Filter
  - Kalman filter for 1D state
  - General Kalman filter
  - Limitations

Tracking with Dynamics

- Key idea
  - Given a model of expected motion, predict where objects will occur in next frame, even before seeing the image.

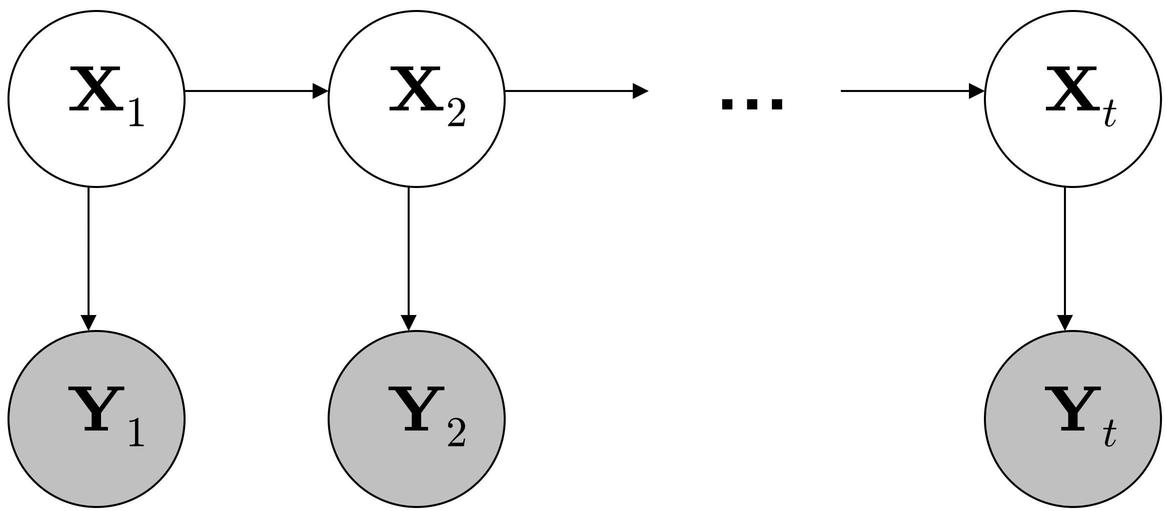


- Goals
  - Restrict search for the object
  - Improved estimates since measurement noise is reduced by trajectory smoothness.
- Assumption: continuous motion patterns
  - Camera is not moving instantly to new viewpoint.
  - Objects do not disappear and reappear in different places.
  - Gradual change in pose between camera and scene.

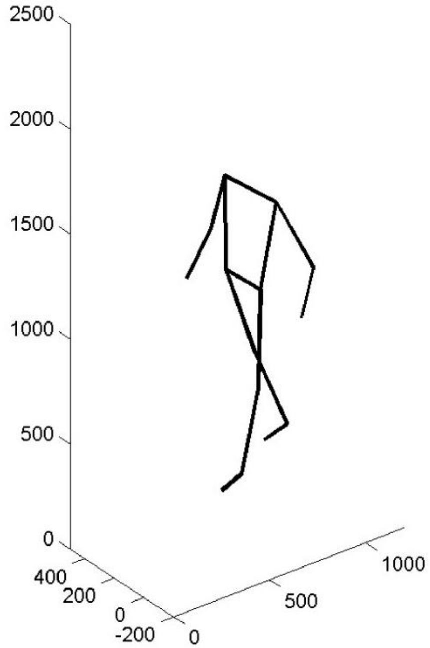
Slide adapted from S. Lazebnik, K. Grauman

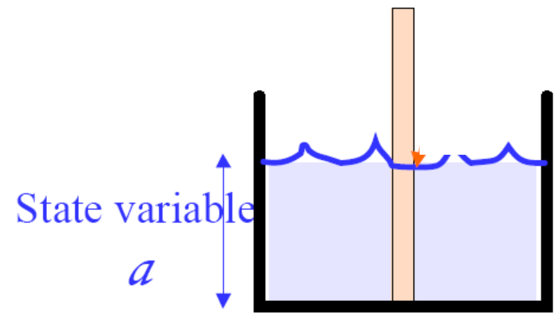
General Model for Tracking

- Representation
  - The moving object of interest is characterized by an underlying *state* $\mathbf{X}$.
  - State $\mathbf{X}$ gives rise to *measurements* or *observations* $\mathbf{Y}$.
  - At each time t, the state changes to $\mathbf{X}_t$ and we get a new observation $\mathbf{Y}_t$.



State vs. Observation





- **Hidden state : parameters of interest**
- **Measurement: what we get to directly observe**

Tracking as Inference

- Inference problem
  - The hidden state consists of the true parameters we care about, denoted $\mathbf{X}$.
  - The measurement is our noisy observation that results from the underlying state, denoted $\mathbf{Y}$.
  - At each time step, state changes (from $\mathbf{X}_{t-1}$ to $\mathbf{X}_t$) and we get a new observation $\mathbf{Y}_t$.
- Our goal: recover most likely state $\mathbf{X}_t$ given
  - All observations seen so far.
  - Knowledge about dynamics of state transitions.

Steps of Tracking

- **Prediction**: What is the next state of the object given past measurements?

$$P\left(X_t \middle| Y_0 = y_0, \ldots, Y_{t-1} = y_{t-1}\right)$$

- **Correction**: Compute an updated estimate of the state from prediction and measurements.

$$P\left(X_t \middle| Y_0 = y_0, \ldots, Y_{t-1} = y_{t-1}, Y_t = y_t\right)$$

- Tracking can be seen as the process of propagating the posterior distribution of state given measurements across time.

Simplifying Assumptions

- **Only the immediate past matters**

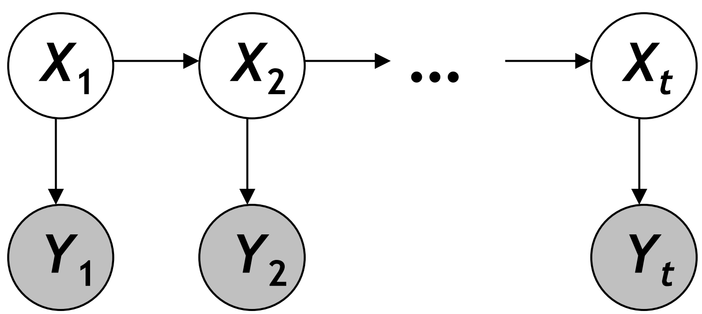
$$P(X_t | X_0, \ldots, X_{t-1}) = \boxed{P(X_t | X_{t-1})}$$

Dynamics model

- **Measurements depend only on the current state**

$$P(Y_t | X_0, Y_0 \ldots, X_{t-1}, Y_{t-1}, X_t) = \boxed{P(Y_t | X_t)}$$

Observation model



Tracking as Induction

- **Base case:**
  - Assume we have initial prior that predicts state in absence of any evidence: $P(\mathbf{X}_0)$
  - At the first frame, *correct* this given the value of $\mathbf{Y}_0 = \mathbf{y}_o$

$$P(X_0 \mid Y_0 = y_0) = \frac{P(y_0 \mid X_0) P(X_0)}{P(y_0)} \propto P(y_0 \mid X_0) P(X_0)$$

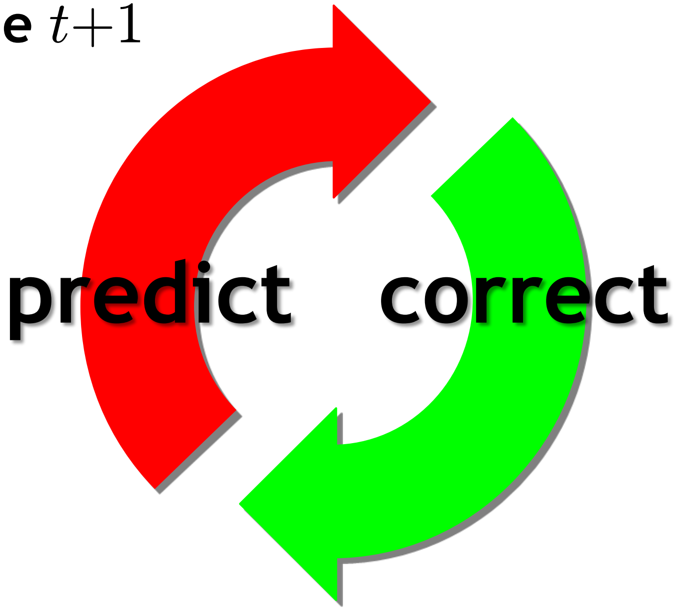
Posterior prob. of state given measurement

Likelihood of measurement

Prior of the state

Tracking as Induction

- Base case:
  - Assume we have initial prior that predicts state in absence of any evidence: $P(\mathbf{X}_0)$
  - At the first frame, *correct* this given the value of $\mathbf{Y}_0 = \mathbf{y}_0$
- Given corrected estimate for frame t:
  - Predict for frame $t+1$
  - Correct for frame $t+1$



Slide credit: Svetlana Lazebnik

Induction Step: Prediction

- **Prediction involves representing $P\left(X_t \middle| y_0,\ldots,y_{t-1}\right)$ given $P\left(X_{t-1} \middle| y_0,\ldots,y_{t-1}\right)$**

$$P\left(X_t \middle| y_0,\ldots,y_{t-1}\right)$$

$$= \int P\left(X_t, X_{t-1} \middle| y_0,\ldots,y_{t-1}\right) dX_{t-1}$$

Law of total probability

$$P(A) = \int P(A,B)\,dB$$

Induction Step: Prediction

- **Prediction involves representing** $P(X_t | y_0, \ldots, y_{t-1})$ **given** $P(X_{t-1} | y_0, \ldots, y_{t-1})$

$$
\begin{aligned}
P(X_t | y_0, \ldots, y_{t-1}) \\
&= \int P(X_t, X_{t-1} | y_0, \ldots, y_{t-1}) dX_{t-1} \\
&= \int P(X_t \mid X_{t-1}, y_0, \ldots, y_{t-1}) P(X_{t-1} \mid y_0, \ldots, y_{t-1}) dX_{t-1}
\end{aligned}
$$

Conditioning on X_{t-1}

$$P(A, B) = P(A \mid B) P(B)$$

RWTH AACHEN UNIVERSITY

Induction Step: Prediction

- **Prediction involves representing** $P\left(X_t \middle| y_0, \ldots, y_{t-1}\right)$ **given** $P\left(X_{t-1} \middle| y_0, \ldots, y_{t-1}\right)$

$$
\begin{aligned}
& P\left(X_t \middle| y_0, \ldots, y_{t-1}\right) \\
& = \int P\left(X_t, X_{t-1} \middle| y_0, \ldots, y_{t-1}\right) dX_{t-1} \\
& = \int P\left(X_t \mid X_{t-1}, y_0, \ldots, y_{t-1}\right) P\left(X_{t-1} \mid y_0, \ldots, y_{t-1}\right) dX_{t-1} \\
& = \int P\left(X_t \mid X_{t-1}\right) P\left(X_{t-1} \mid y_0, \ldots, y_{t-1}\right) dX_{t-1}
\end{aligned}
$$

Independence assumption

Induction Step: Correction

- **Correction involves computing** $P\left(X_t \middle| y_0, \ldots, y_t\right)$ **given predicted value** $P\left(X_t \middle| y_0, \ldots, y_{t-1}\right)$

$$P\left(X_t \middle| y_0, \ldots, y_t\right)$$

$$= \frac{P\left(y_t \mid X_t, y_0, \ldots, y_{t-1}\right) P\left(X_t \mid y_0, \ldots, y_{t-1}\right)}{P\left(y_t \mid y_0, \ldots, y_{t-1}\right)}$$

Bayes rule

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Induction Step: Correction

- **Correction involves computing $P\left(X_t \middle| y_0, \ldots, y_t\right)$ given predicted value $P\left(X_t \middle| y_0, \ldots, y_{t-1}\right)$**

$$P\left(X_t \middle| y_0, \ldots, y_t\right)$$

$$= \frac{P\left(y_t \mid X_t, y_0, \ldots, y_{t-1}\right) P\left(X_t \mid y_0, \ldots, y_{t-1}\right)}{P\left(y_t \mid y_0, \ldots, y_{t-1}\right)}$$

$$= \frac{P\left(y_t \mid X_t\right) P\left(X_t \mid y_0, \ldots, y_{t-1}\right)}{P\left(y_t \mid y_0, \ldots, y_{t-1}\right)}$$

Independence assumption
(observation y_t depends only on state X_t)

Induction Step: Correction

- **Correction involves computing** $P(X_t | y_0, \ldots, y_t)$ **given predicted value** $P(X_t | y_0, \ldots, y_{t-1})$

$$P(X_t | y_0, \ldots, y_t)$$
$$= \frac{P(y_t \mid X_t, y_0, \ldots, y_{t-1}) P(X_t \mid y_0, \ldots, y_{t-1})}{P(y_t \mid y_0, \ldots, y_{t-1})}$$
$$= \frac{P(y_t \mid X_t) P(X_t \mid y_0, \ldots, y_{t-1})}{P(y_t \mid y_0, \ldots, y_{t-1})}$$
$$= \frac{P(y_t \mid X_t) P(X_t \mid y_0, \ldots, y_{t-1})}{\int P(y_t \mid X_t) P(X_t \mid y_0, \ldots, y_{t-1}) dX_t}$$

Conditioning on X_t

Slide credit: Svetlana Lazebnik

Summary: Prediction and Correction

- **Prediction:**

$$P\left(X_t \mid y_0,\ldots,y_{t-1}\right) = \int \underbrace{P\left(X_t \mid X_{t-1}\right)}_{\text{Dynamics model}} \underbrace{P\left(X_{t-1} \mid y_0,\ldots,y_{t-1}\right)}_{\text{Corrected estimate from previous step}} dX_{t-1}$$

Slide credit: Svetlana Lazebnik

Summary: Prediction and Correction

- **Prediction:**

$$P(X_t \mid y_0,\ldots,y_{t-1}) = \int \underbrace{P(X_t \mid X_{t-1})}_{\text{Dynamics model}} \underbrace{P(X_{t-1} \mid y_0,\ldots,y_{t-1})}_{\text{Corrected estimate from previous step}} dX_{t-1}$$

- **Correction:**

$$P(X_t \mid y_0,\ldots,y_t) = \frac{\overbrace{P(y_t \mid X_t)}^{\text{Observation model}} \overbrace{P(X_t \mid y_0,\ldots,y_{t-1})}^{\text{Predicted estimate}}}{\int P(y_t \mid X_t) P(X_t \mid y_0,\ldots,y_{t-1}) dX_t}$$

Slide credit: Svetlana Lazebnik

Topics of This Lecture

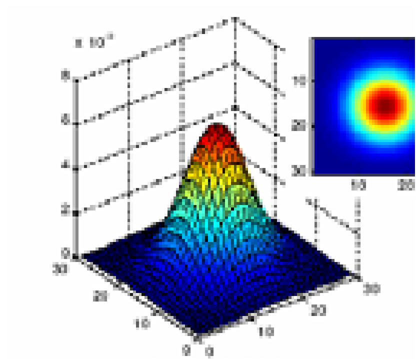
- Tracking with Dynamics
  - Detection vs. Tracking
  - Tracking as probabilistic inference
  - Prediction and Correction
- Linear Dynamic Models
  - Zero velocity model
  - Constant velocity model
  - Constant acceleration model
- The Kalman Filter
  - Kalman filter for 1D state
  - General Kalman filter
  - Limitations

Notation Reminder

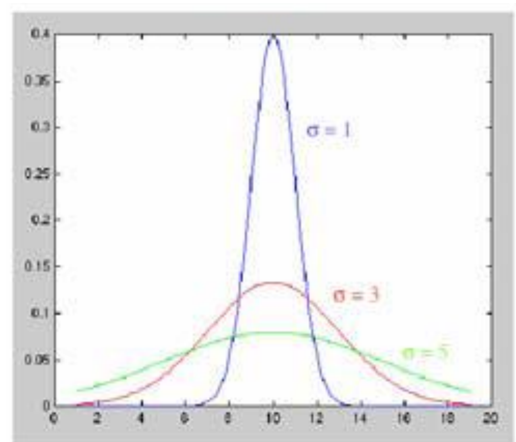
$$\mathbf{x} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

- Random variable with Gaussian probability distribution that has the mean vector μ and covariance matrix Σ.
- $\mathbf{x}$ and μ are d-dimensional, Σ is $d \times d$.

***d*=2**



***d*=1**



If $\mathbf{x}$ is 1D, we just have one Σ parameter: the variance σ^2

Linear Dynamic Models

- **Dynamics model**
  - **State undergoes linear tranformation D_t plus Gaussian noise**

$$\mathbf{x}_t \sim N\left(\mathbf{D}_t \mathbf{x}_{t-1}, \Sigma_{d_t}\right)$$

$\mathbf{x}_t$: $n\times 1$, $\mathbf{D}_t$: $n\times n$, $\mathbf{x}_{t-1}$: $n\times 1$

- **Observation model**
  - **Measurement is linearly transformed state plus Gaussian noise**

$$\mathbf{y}_t \sim N\left(\mathbf{M}_t \mathbf{x}_t, \Sigma_{m_t}\right)$$

$\mathbf{y}_t$: $m\times 1$, $\mathbf{M}_t$: $m\times n$, $\mathbf{x}_t$: $n\times 1$

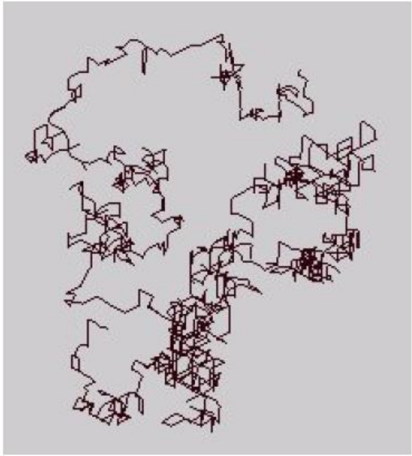
Example: Randomly Drifting Points

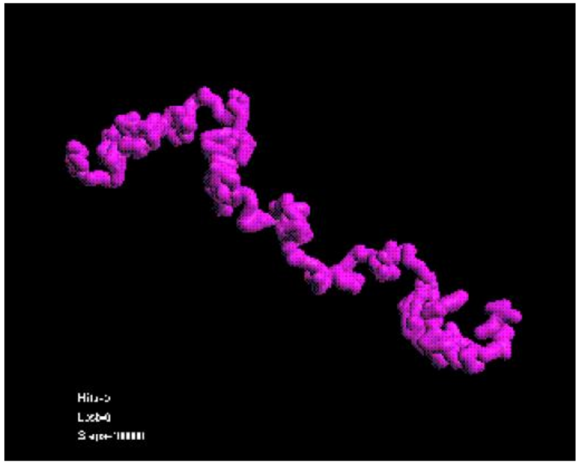
- Consider a stationary object, with state as position.
  - Position is constant, only motion due to random noise term.

$$x_t = p_t \qquad p_t = p_{t-1} + \varepsilon$$

$\Rightarrow$ State evolution is described by identity matrix D=I

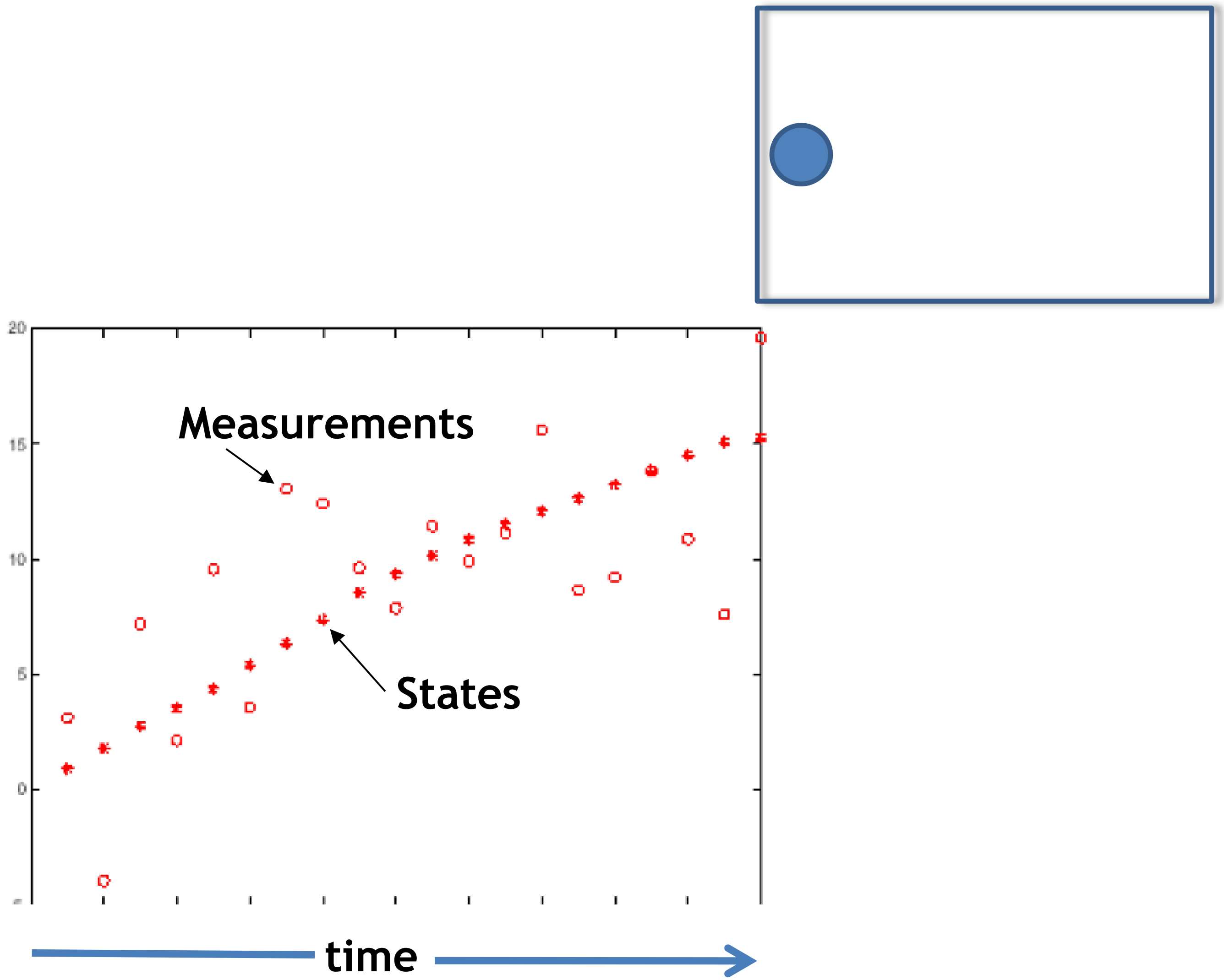
$$x_t = D_t x_{t-1} + noise = I p_{t-1} + noise$$





cic.nist.gov/lipman/sciviz/images/random3.gif
http://www.grunch.net/synergetics/images/ranc
3.jpg

Example: Constant Velocity (1D Points)



Slide credit: Kristen Grauman

Figure from Forsyth & Ponce

Example: Constant Velocity (1D Points)

- State vector: position p and velocity v

$$x_t = \begin{bmatrix} p_t \\ v_t \end{bmatrix} \qquad p_t =$$

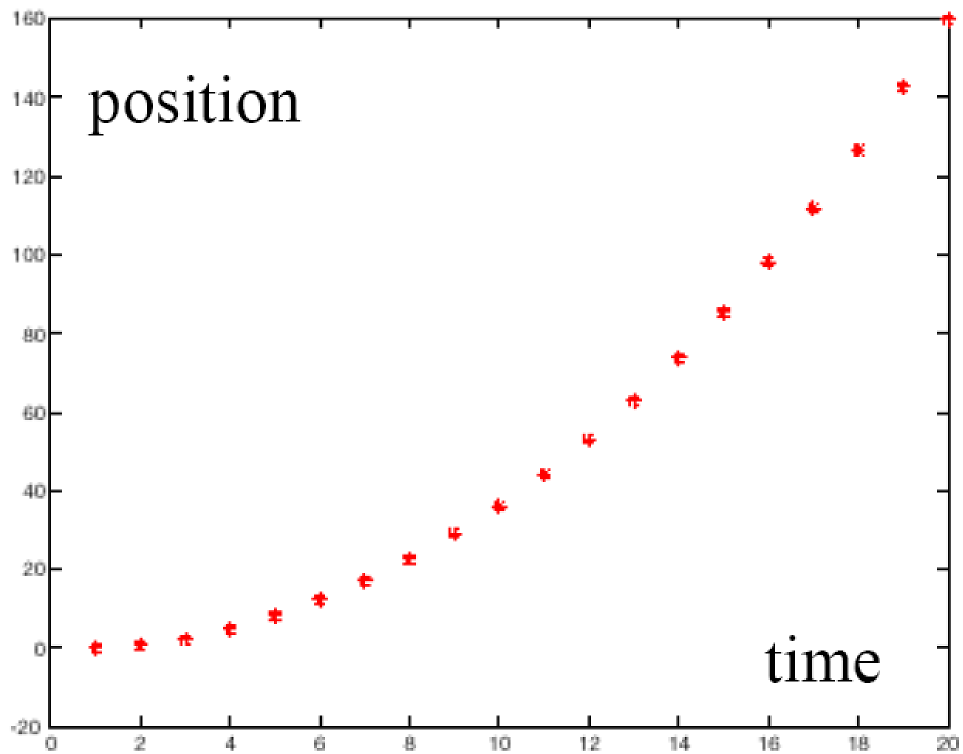
(greek letters denote noise terms)

$$x_t = D_t x_{t-1} + noise =$$

- Measurement is position only

$$y_t = M x_t + noise =$$

Example: Constant Acceleration (1D Points)



Slide credit: Kristen Grauman

Figure from Forsyth & Ponce

Example: Constant Acceleration (1D Points)

- State vector: position p, velocity v, and acceleration a.

$$x_t = \begin{bmatrix} p_t \\ v_t \\ a_t \end{bmatrix} \qquad \begin{aligned} p_t &= p_{t-1} + (\Delta t) v_{t-1} + \varepsilon \\ v_t &= \\ a_t &= \end{aligned}$$

(greek letters denote noise terms)

$$x_t = D_t x_{t-1} + noise =$$

- Measurement is position only

$$y_t = Mx_t + noise =$$

Slide credit: S. Lazebnik, K. Grauman

Example: General Motion Models

- Assuming we have differential equations for the motion
  - E.g. for (undampened) periodic motion of a pendulum

$$\frac{d^2 p}{dt^2} = -p$$

- Substitute variables to transform this into linear system

$$p_1 = p \qquad p_2 = \frac{dp}{dt} \qquad p_3 = \frac{d^2 p}{dt^2}$$

- Then we have

$$x_t = \begin{bmatrix} p_{1,t} \\ p_{2,t} \\ p_{3,t} \end{bmatrix} \qquad \begin{aligned} p_{1,t} &= p_{1,t-1} + (\Delta t) p_{2,t-1} + \varepsilon \\ p_{2,t} &= p_{2,t-1} + (\Delta t) p_{3,t-1} + \xi \\ p_{3,t} &= -p_{1,t-1} + \zeta \end{aligned} \qquad D_t = \begin{bmatrix} 1 & \Delta t & 0 \\ 0 & 1 & \Delta t \\ -1 & 0 & 0 \end{bmatrix}$$

Topics of This Lecture

- Tracking with Dynamics
  - Detection vs. Tracking
  - Tracking as probabilistic inference
  - Prediction and Correction
- Linear Dynamic Models
  - Zero velocity model
  - Constant velocity model
  - Constant acceleration model
- **The Kalman Filter**
  - **Kalman filter for 1D state**
  - **General Kalman filter**
  - **Limitations**

The Kalman Filter

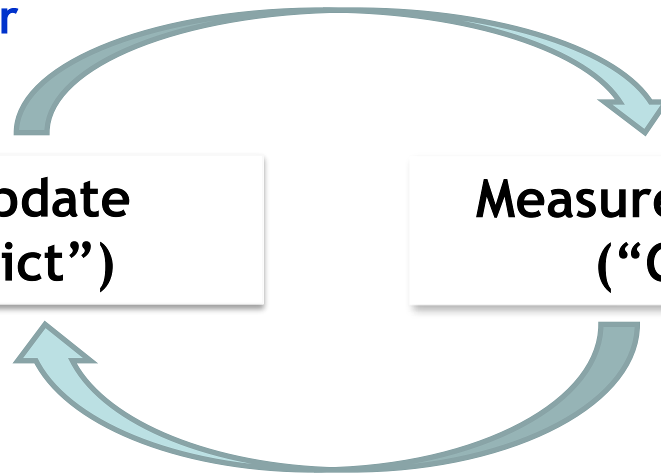
- **Kalman filter**
  - Method for tracking linear dynamical models in Gaussian noise
- **The predicted/corrected state distributions are Gaussian**
  - You only need to maintain the mean and covariance.
  - The calculations are easy (all the integrals can be done in closed form).

The Kalman Filter

Know corrected state from previous time step, and all measurements up to the current one
→ Predict distribution over next state.

Receive measurement

Know prediction of state, and next measurement
→Update distribution over current state.



Time update ("Predict")

Measurement update ("Correct")

$$P\left(X_t \middle| y_0, \ldots, y_{t-1}\right)$$

$$P\left(X_t \middle| y_0, \ldots, y_t\right)$$

Mean and std. dev. of predicted state:

$$\mu_t^-, \sigma_t^-$$

Time advances: t++

Mean and std. dev. of corrected state:

$$\mu_t^+, \sigma_t^+$$

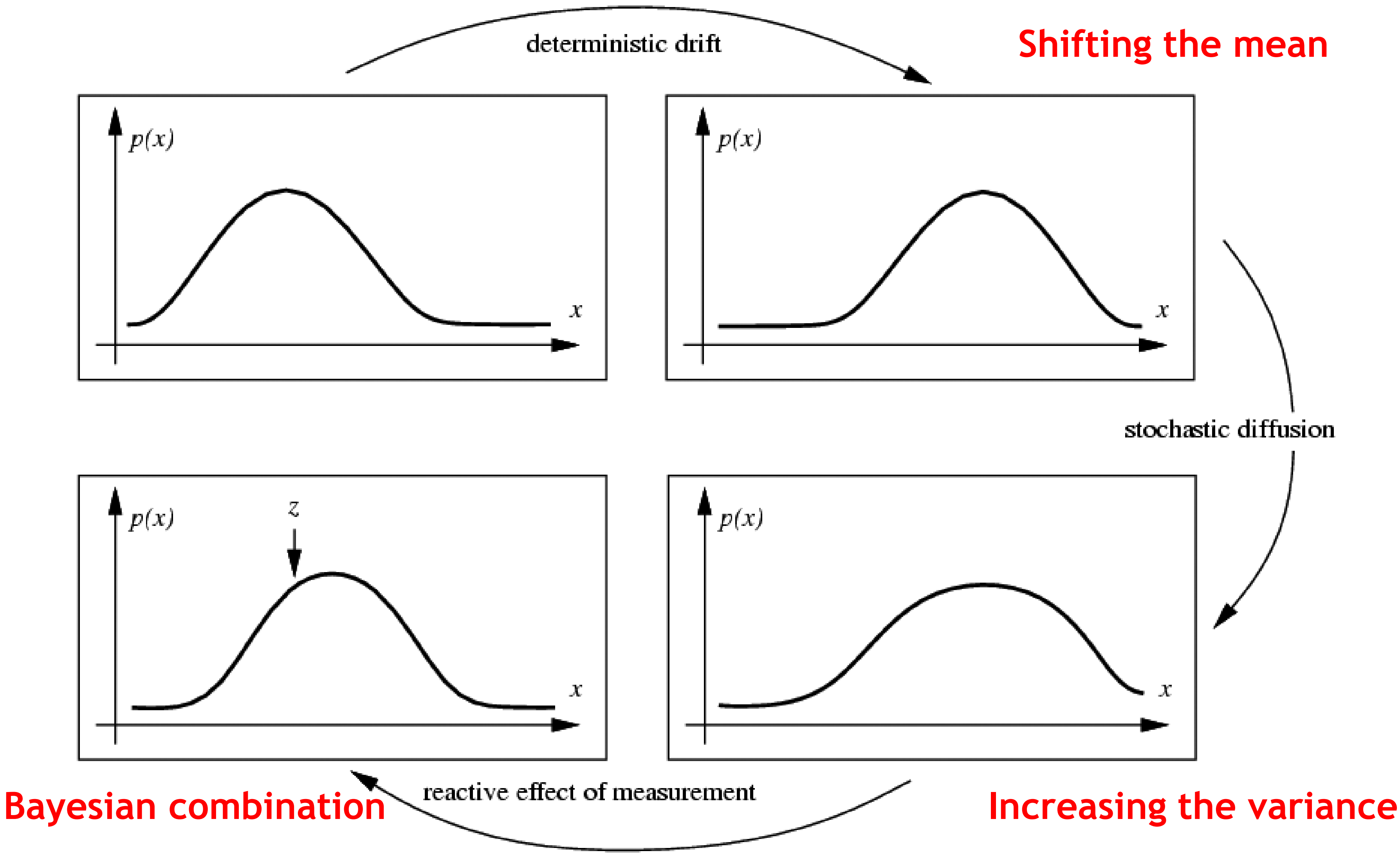
Kalman Filter for 1D State

Want to represent and update

$$P\left(x_t \mid y_0, \ldots, y_{t-1}\right) = N\left(\mu_t^-, (\sigma_t^-)^2\right)$$

$$P\left(x_t \mid y_0, \ldots, y_t\right) = N\left(\mu_t^+, (\sigma_t^+)^2\right)$$

Propagation of Gaussian densities



Slide credit: Svetlana Lazebnik

1D Kalman Filter: Prediction

- Have linear dynamic model defining predicted state evolution, with noise

$$X_t \sim N\left(dx_{t-1}, \sigma_d^2\right)$$

- Want to estimate predicted distribution for next state

$$P\left(X_t \middle| y_0, \ldots, y_{t-1}\right) = N\left(\mu_t^-, (\sigma_t^-)^2\right)$$

- Update the mean:

$$\mu_t^- = d\mu_{t-1}^+$$

for derivations, see F&P Chapter 17.3

- Update the variance:

$$(\sigma_t^-)^2 = \sigma_d^2 + (d\sigma_{t-1}^+)^2$$

1D Kalman Filter: Correction

- Have linear model defining the mapping of state to measurements: $Y_t \sim N\left(mx_t, \sigma_m^2\right)$

- Want to estimate corrected distribution given latest measurement: $P\left(X_t \middle| y_0, \ldots, y_t\right) = N\left(\mu_t^+, (\sigma_t^+)^2\right)$

- Update the mean:

$$\mu_t^+ = \frac{\mu_t^- \sigma_m^2 + m y_t (\sigma_t^-)^2}{\sigma_m^2 + m^2 (\sigma_t^-)^2}$$

- Update the variance:

$$(\sigma_t^+)^2 = \frac{\sigma_m^2 (\sigma_t^-)^2}{\sigma_m^2 + m^2 (\sigma_t^-)^2}$$

Slide credit: Kristen Grauman

Derivations: F&P Chapter 17.3

Prediction vs. Correction

$$\mu_t^+ = \frac{\mu_t^- \sigma_m^2 + m y_t (\sigma_t^-)^2}{\sigma_m^2 + m^2 (\sigma_t^-)^2} \qquad (\sigma_t^+)^2 = \frac{\sigma_m^2 (\sigma_t^-)^2}{\sigma_m^2 + m^2 (\sigma_t^-)^2}$$

- **What if there is no prediction uncertainty** $(\sigma_t^- = 0)$?

$$\mu_t^+ = \mu_t^- \qquad (\sigma_t^+)^2 = 0$$

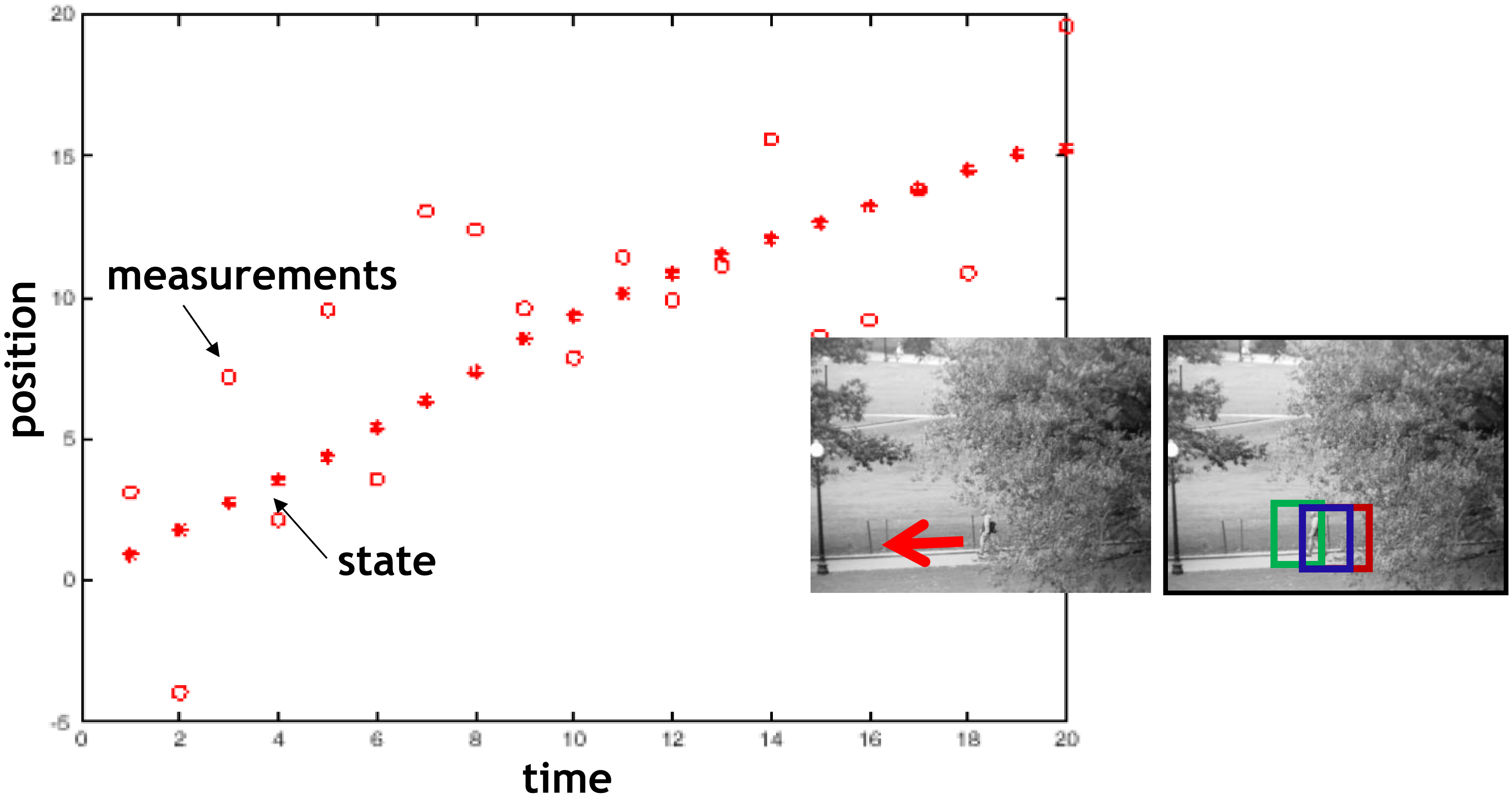
The measurement is ignored!

- **What if there is no measurement uncertainty** $(\sigma_m = 0)$?

$$\mu_t^+ = \frac{y_t}{m} \qquad (\sigma_t^+)^2 = 0$$

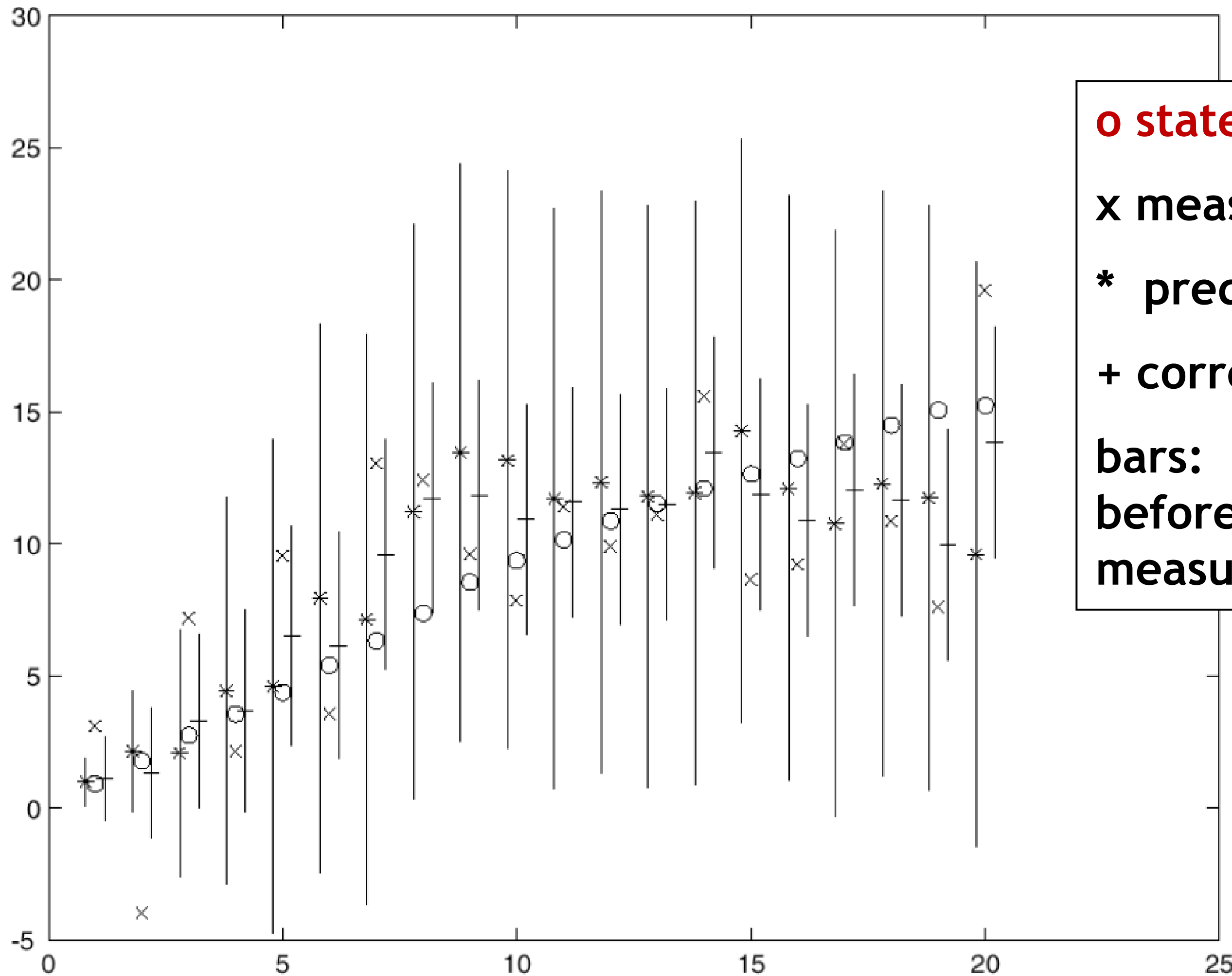
The prediction is ignored!

Recall: Constant Velocity Example



State is 2D: position + velocity
Measurement is 1D: position

Constant Velocity Model



o state

x measurement

*** predicted mean estimate**

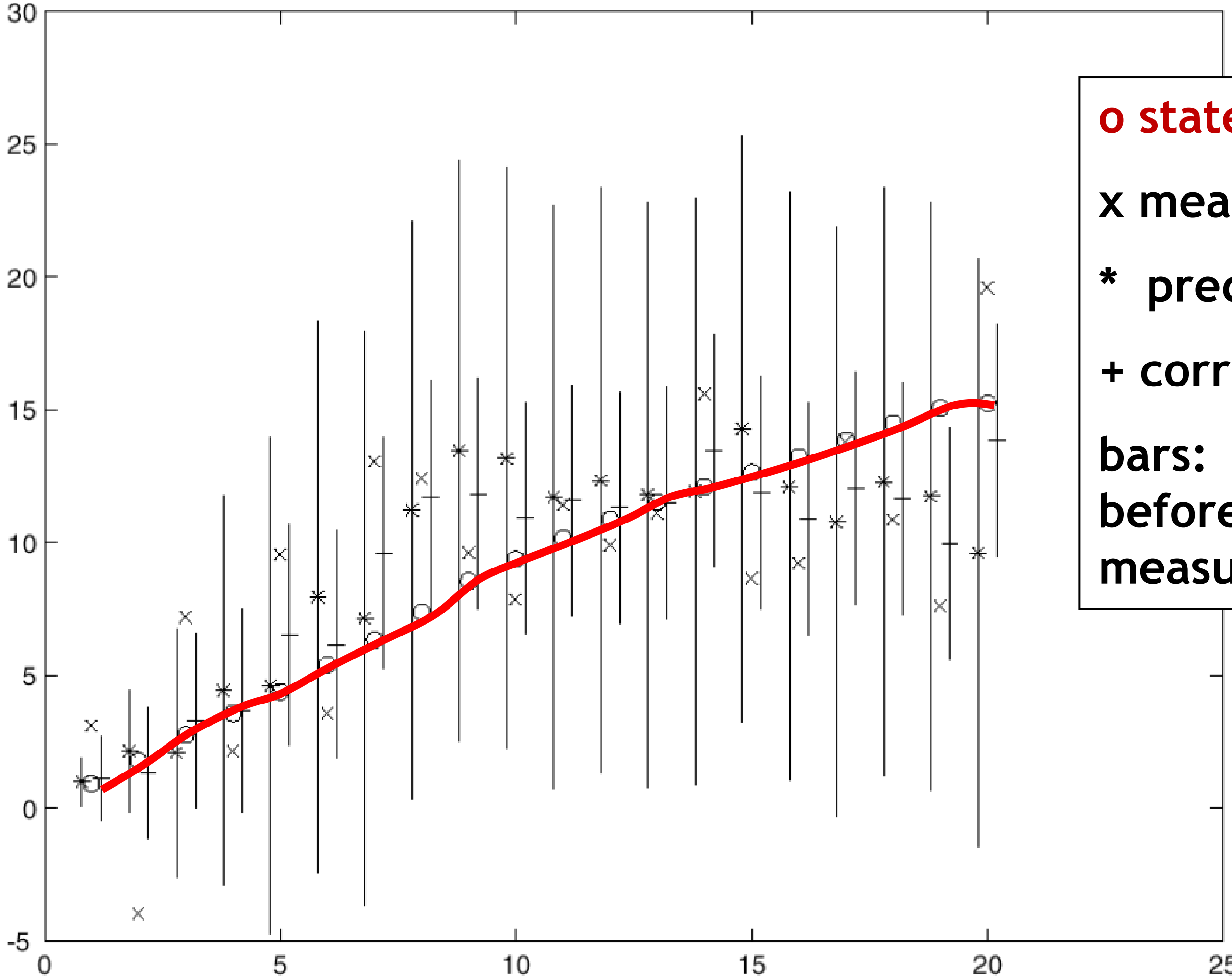
+ corrected mean estimate

bars: variance estimates before and after measurements

Slide credit: Kristen Grauman

Figure from Forsyth & Ponce

Constant Velocity Model



o state

x measurement

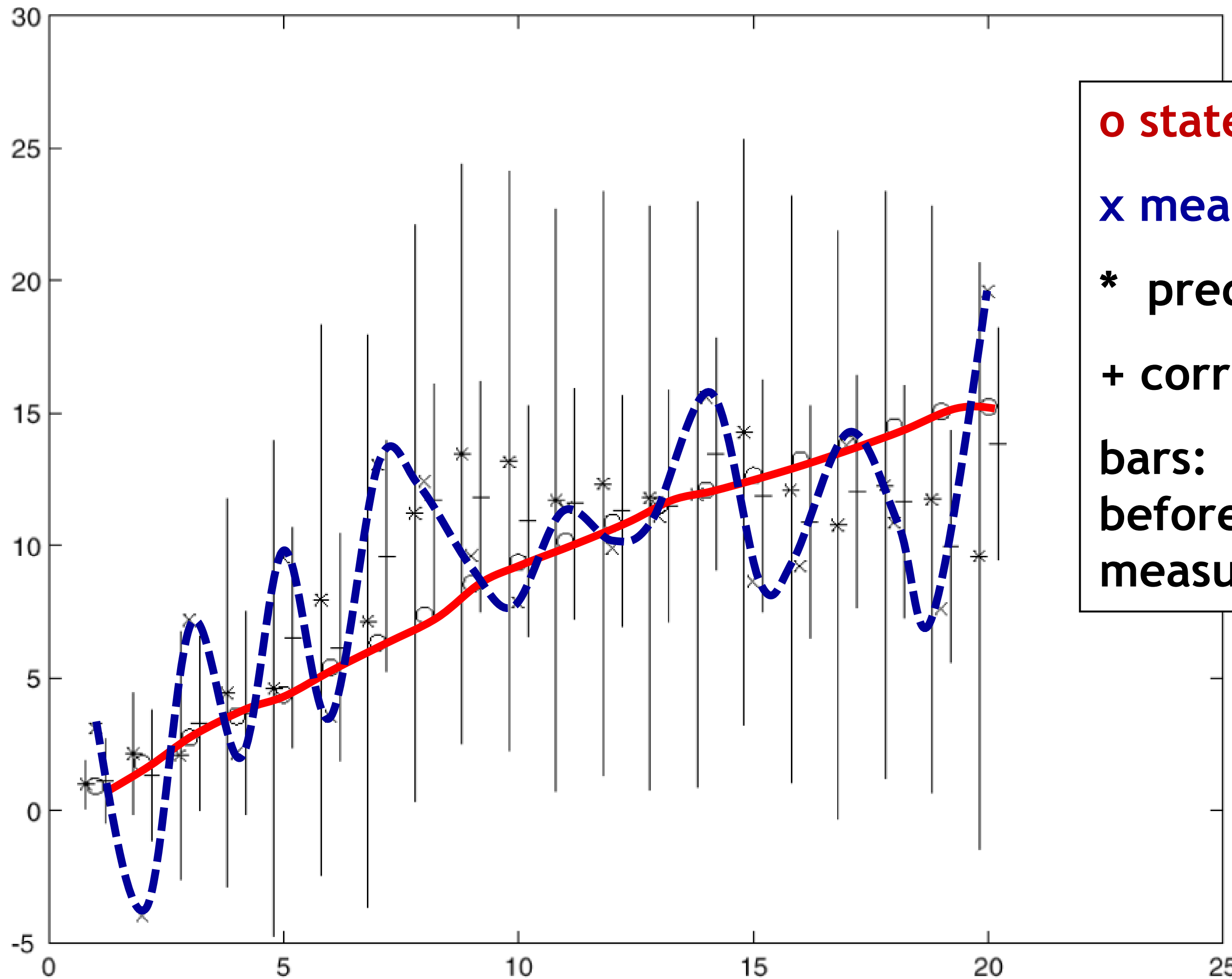
* predicted mean estimate

+ corrected mean estimate

bars: variance estimates before and after measurements

Figure from Forsyth & Ponce

Constant Velocity Model



o state

x measurement

* predicted mean estimate

+ corrected mean estimate

bars: variance estimates before and after measurements

Constant Velocity Model

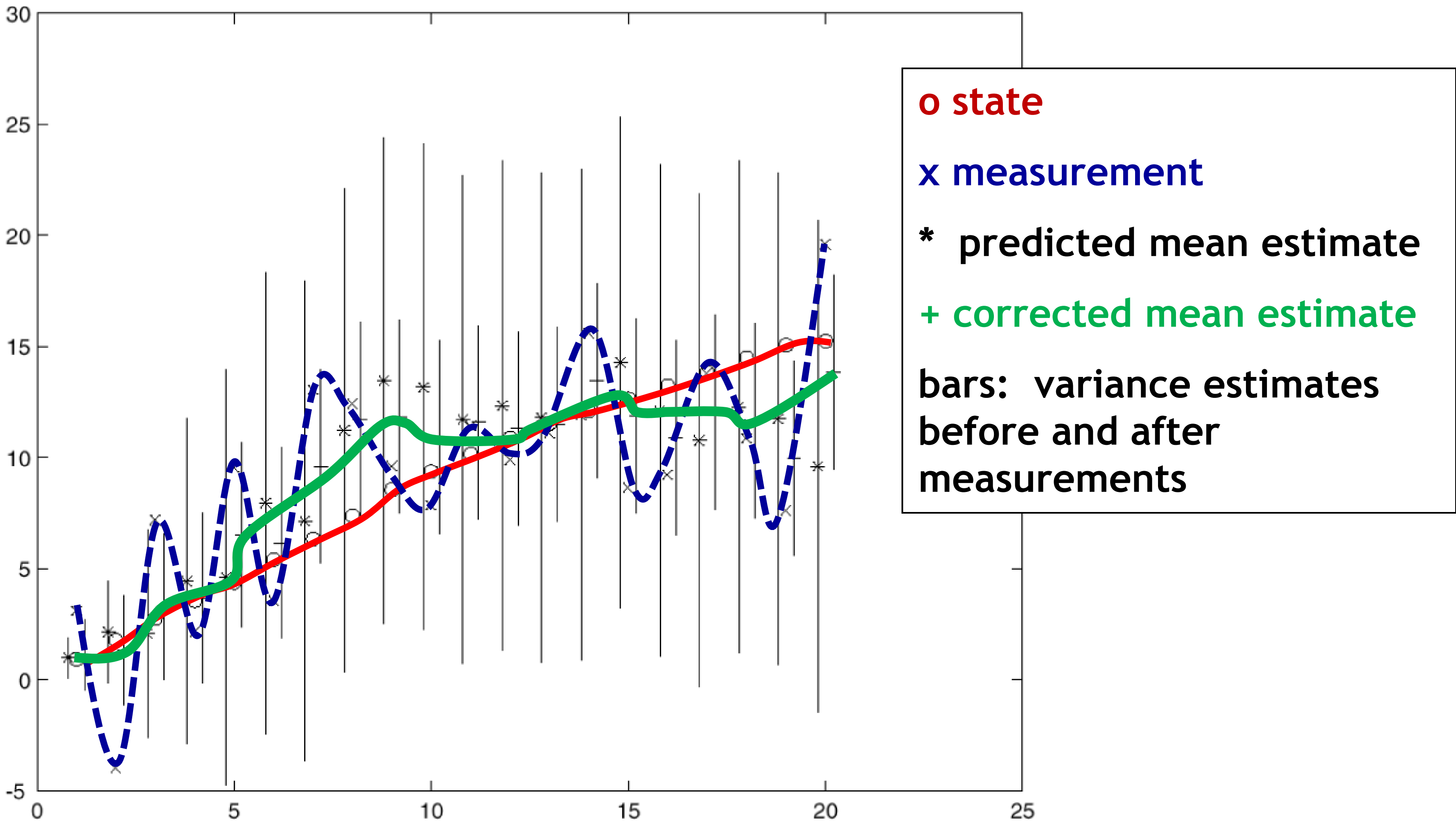


Figure from Forsyth & Ponce

Kalman Filter: General Case (>1dim)

- What if state vectors have more than one dimension?

PREDICT

$$x_t^- = D_t x_{t-1}^+$$

$$\Sigma_t^- = D_t \Sigma_{t-1}^+ D_t^T + \Sigma_{d_t}$$

CORRECT

$$K_t = \Sigma_t^- M_t^T \left(M_t \Sigma_t^- M_t^T + \Sigma_{m_t} \right)^{-1}$$

$$x_t^+ = x_t^- + K_t \left(y_t - M_t x_t^- \right)$$

"residual"

$$\Sigma_t^+ = \left(I - K_t M_t \right) \Sigma_t^-$$

More weight on residual when measurement error covariance approaches 0.

Less weight on residual as a priori estimate error covariance approaches 0.

for derivations, see F&P Chapter 17.3

Summary: Kalman Filter

- **Pros:**
  - Gaussian densities everywhere
  - Simple updates, compact and efficient
  - Very established method, very well understood

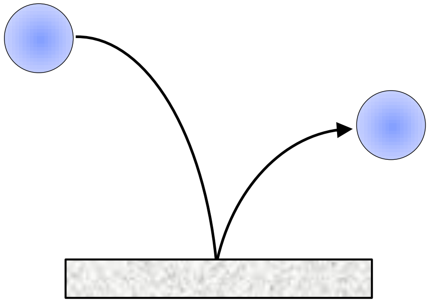
- **Cons:**
  - Unimodal distribution, only single hypothesis
  - Restricted class of motions defined by linear model

Slide adapted from Svetlana Lazebnik

Why Is This A Restriction?

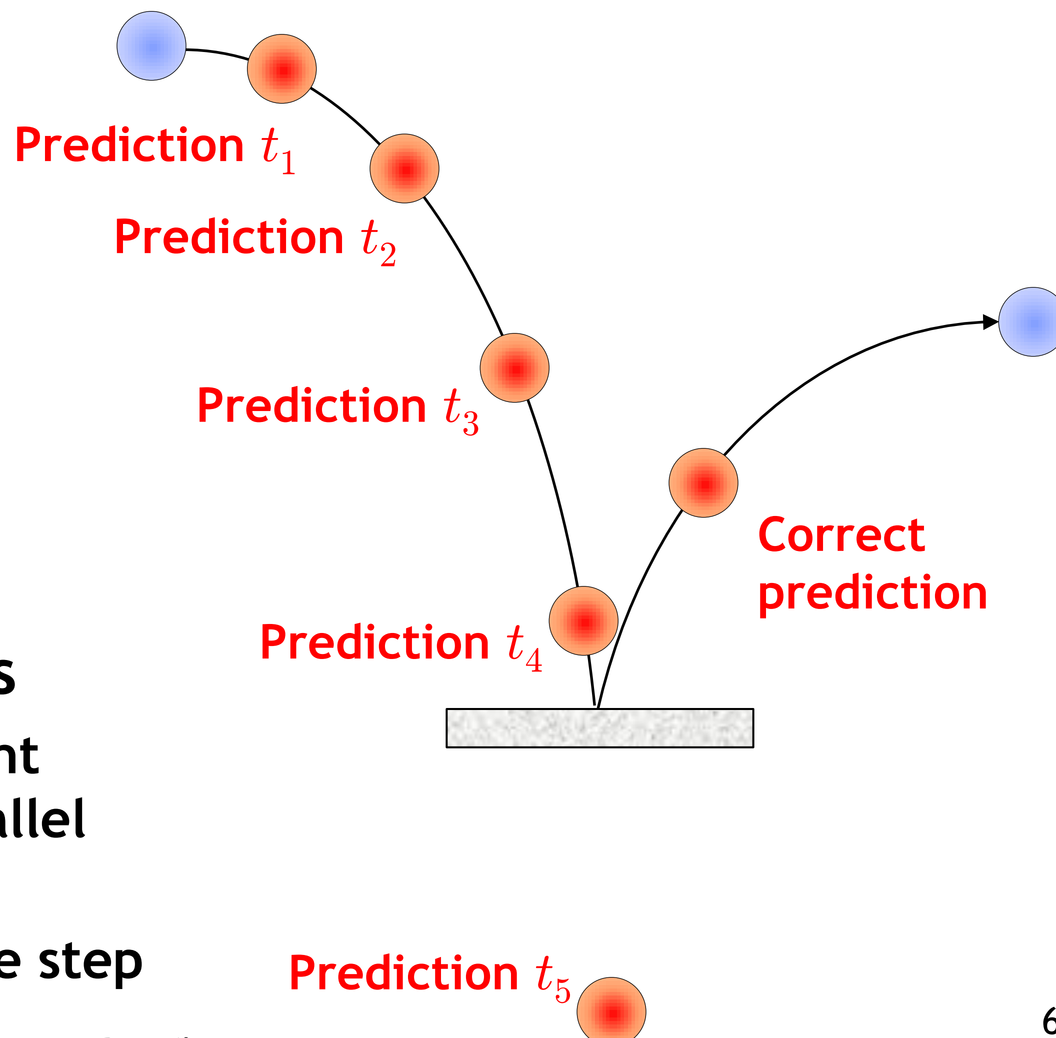
- Many interesting cases don't have linear dynamics
  - E.g. pedestrians walking

  - E.g. a ball bouncing



Ball Example: What Goes Wrong Here?

- Assuming constant acceleration model



- Prediction is too far from true position to compensate…
- Possible solution: Keep multiple models
  - Keep multiple different motion models in parallel
  - I.e. would check for bouncing at each time step

References and Further Reading

- A very good introduction to tracking with linear dynamic models and Kalman filters can be found in Chapter 17 of
  - D. Forsyth, J. Ponce,
    Computer Vision – A Modern Approach.
    Prentice Hall, 2003