Computer Vision - Lecture 17

Epipolar Geometry & Stereo Basics

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Announcements

- Exam Dates
  - 1st try: 29.02. 13:30 - 17:30h in AH I/II
  - 2nd try: 31.03. 09:40 - 12:40h in UMIC 025
  - We will send around an email announcing the precise start/end times and your assigned exam rooms.

+ AH VI, UMIC 025
+ AH IV
Announcements (2)

• Seminar in the summer semester
  - “Current Topics in Computer Vision and Machine Learning”
  - Block seminar, presentations at beginning of semester break
  - [https://www.graphics.rwth-aachen.de/apse/check.php](https://www.graphics.rwth-aachen.de/apse/check.php)
Course Outline

- Image Processing Basics
- Segmentation & Grouping
- Object Recognition
- Local Features & Matching
- Object Categorization
- 3D Reconstruction
  - Epipolar Geometry and Stereo Basics
  - Camera calibration & Uncalibrated Reconstruction
  - Multi-view Stereo
- Optical Flow
Recap: Convolutional Neural Networks

- Neural network with specialized connectivity structure
  - Stack multiple stages of feature extractors
  - Higher stages compute more global, more invariant features
  - Classification layer at the end


Slide credit: Svetlana Lazebnik
The Learned Features are Generic

- **Experiment: feature transfer**
  - Train AlexNet-like network on ImageNet
  - Chop off last layer and train classification layer on CalTech256

  ⇒ State of the art accuracy already with only 6 training images!

*Image source: M. Zeiler, R. Fergus*
Transfer Learning with CNNs

1. Train on ImageNet

2. If small dataset: fix all weights (treat CNN as fixed feature extractor), retrain only the classifier

I.e., swap the Softmax layer at the end
Transfer Learning with CNNs

1. Train on ImageNet

3. If you have medium sized dataset, “finetune” instead: use the old weights as initialization, train the full network or only some of the higher layers.

Retrain bigger portion of the network
Other Tasks: Detection

**R-CNN: Regions with CNN features**

1. Input image
2. Extract region proposals (~2k)
3. Compute CNN features
4. Classify regions

- **Results on PASCAL VOC Detection benchmark**
  - Pre-CNN state of the art: 35.1% mAP [Uijlings et al., 2013]
  - R-CNN: 53.7% mAP
  - DPM

Other Tasks: Semantic Segmentation

[Farabet et al. ICML 2012, PAMI 2013]
Other Tasks: Semantic Segmentation

[Farabet et al. ICML 2012, PAMI 2013]
Other Tasks: Face Verification

Y. Taigman, M. Yang, M. Ranzato, L. Wolf, DeepFace: Closing the Gap to Human-Level Performance in Face Verification, CVPR 2014

Slide credit: Svetlana Lazebnik
Commercial Recognition Services

- E.g., clarifai

Try it out with your own media

Upload an image or video file under 100mb or give us a direct link to a file on the web.

*By using the demo you agree to our terms of service
Commercial Recognition Services

- Be careful when testing with images from Google Search
  - Chances are they may have been seen in the training set...

Image source: clarifai.com
Topics of This Lecture

• Geometric vision
  - Visual cues
  - Stereo vision

• Epipolar geometry
  - Depth with stereo
  - Geometry for a simple stereo system
  - Case example with parallel optical axes
  - General case with calibrated cameras

• Stereopsis & 3D Reconstruction
  - Correspondence search
  - Additional correspondence constraints
  - Possible sources of error
  - Applications
Geometric vision

- Goal: Recovery of 3D structure
  - What cues in the image allow us to do this?
Visual Cues

- Shading

Merle Norman Cosmetics, Los Angeles

Slide credit: Steve Seitz
Visual Cues

- Shading
- Texture

*The Visual Cliff*, by William Vandivert, 1960

Slide credit: Steve Seitz
Visual Cues

- Shading
- Texture
- Focus

From *The Art of Photography*, Canon

Slide credit: Steve Seitz
Visual Cues

- Shading
- Texture
- Focus
- Perspective
Visual Cues

- Shading
- Texture
- Focus
- Perspective
- Motion

Figures from L. Zhang

Slide credit: Steve Seitz, Kristen Grauman

http://www.brainconnection.com/teasers/?main=illusion/motion-shape
Our Goal: Recovery of 3D Structure

- We will focus on perspective and motion
- We need *multi-view geometry* because recovery of structure from one image is inherently ambiguous
To Illustrate This Point...

- Structure and depth are inherently ambiguous from single views.

Slide credit: Svetlana Lazebnik, Kristen Grauman  B. Leibe
Stereo Vision

http://www.well.com/~jimg/stereo/stereo_list.html

Slide credit: Kristen Grauman
What Is Stereo Vision?

• Generic problem formulation: given several images of the same object or scene, compute a representation of its 3D shape
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- Narrower formulation: given a calibrated binocular stereo pair, fuse it to produce a depth image
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  - Humans can do it

Stereograms: Invented by Sir Charles Wheatstone, 1838
What Is Stereo Vision?

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  - Humans can do it

Autostereograms: [http://www.magiceye.com](http://www.magiceye.com)
What Is Stereo Vision?

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Slide credit: Svetlana Lazebnik, Steve Seitz
Application of Stereo: Robotic Exploration

Nomad robot searches for meteorites in Antarctica

Real-time stereo on Mars

Slide credit: Steve Seitz
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Depth with Stereo: Basic Idea

- **Basic Principle: Triangulation**
  - Gives reconstruction as intersection of two rays
  - Requires
    - Camera pose (calibration)
    - Point correspondence
Camera Calibration

- Parameters
  - **Extrinsic**: rotation matrix and translation vector
  - **Intrinsic**: focal length, pixel sizes (mm), image center point, radial distortion parameters

  We’ll assume for now that these parameters are given and fixed.
Geometry for a Simple Stereo System

- First, assuming parallel optical axes, known camera parameters (i.e., calibrated cameras):
Focal length

Baseline

Depth of p

World point

Image point (left)

Optical center (left)

Image point (right)

Optical center (right)

Slade credit: Kristen Grauman
Geometry for a Simple Stereo System

- Assume parallel optical axes, known camera parameters (i.e., calibrated cameras). We can triangulate via:

  Similar triangles \((p_l, P, p_r)\) and \((O_l, P, O_r)\):

  \[
  \frac{T - (x_r - x_l)}{Z - f} = \frac{T}{Z}
  \]

  \[
  Z = f \frac{T}{x_r - x_l}
  \]

  “disparity”
Depth From Disparity

Image $I(x,y)$ \hspace{1cm} Disparity map $D(x,y)$ \hspace{1cm} Image $I'(x',y')$

$$(x', y') = (x + D(x, y), y)$$
General Case With Calibrated Cameras

- The two cameras need not have parallel optical axes.
Stereo Correspondence Constraints

• Given \( p \) in the left image, where can the corresponding point \( p' \) in the right image be?
Stereo Correspondence Constraints

- Given $p$ in the left image, where can the corresponding point $p'$ in the right image be?
Stereo Correspondence Constraints
Stereo Correspondence Constraints

- Geometry of two views allows us to constrain where the corresponding pixel for some image point in the first view must occur in the second view.

- Epipolar constraint: Why is this useful?
  - Reduces correspondence problem to 1D search along conjugate epipolar lines.
Epipolar Geometry

- Epipolar Plane
- Epipoles
- Baseline
- Epipolar Lines

Slide adapted from Marc Pollefeys
Epipolar Geometry: Terms

- **Baseline**
  - Line joining the camera centers
- **Epipole**
  - Point of intersection of baseline with the image plane
- **Epipolar plane**
  - Plane containing baseline and world point
- **Epipolar line**
  - Intersection of epipolar plane with the image plane

- **Properties**
  - All epipolar lines intersect at the epipole.
  - An epipolar plane intersects the left and right image planes in epipolar lines.
Epipolar Constraint

- Potential matches for \( p \) have to lie on the corresponding epipolar line \( l' \).
- Potential matches for \( p' \) have to lie on the corresponding epipolar line \( l \).

http://www.ai.sri.com/~luong/research/Meta3DViewer/EpipolarGeo.html

Slide credit: Marc Pollefeys
Example

[Slide with images showing examples of computer vision techniques, annotated with green lines and points.]

Slide credit: Kristen Grauman
Example: Converging Cameras

As position of 3D point varies, epipolar lines “rotate” about the baseline.
Example: Motion Parallel With Image Plane

Figure from Hartley & Zisserman

Slide credit: Kristen Grauman
Example: Forward Motion

- Epipole has same coordinates in both images.
- Points move along lines radiating from $e$: “Focus of expansion”
Let’s Formalize This!

• For a given stereo rig, how do we express the epipolar constraints algebraically?

• For this, we will need some linear algebra.

• But don’t worry! We’ll go through it step by step...
Stereo Geometry With Calibrated Cameras

- If the rig is calibrated, we know:
  - How to rotate and translate camera reference frame 1 to get to camera reference frame 2.
    - Rotation: 3 x 3 matrix; translation: 3 vector.
Rotation Matrix

Express 3D rotation as series of rotations around coordinate axes by angles $\alpha$, $\beta$, $\gamma$

Overall rotation is product of these elementary rotations:

$$R = R_x(\alpha) R_y(\beta) R_z(\gamma)$$
3D Rigid Transformation

\[
\begin{bmatrix}
X' \\
Y' \\
Z'
\end{bmatrix} = \begin{bmatrix}
r_{11} & r_{12} & r_{13} \\
r_{21} & r_{22} & r_{23} \\
r_{31} & r_{32} & r_{33}
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} + \begin{bmatrix}
T_x \\
T_y \\
T_z
\end{bmatrix}
\]

\[X' = RX + T\]
Stereo Geometry With Calibrated Cameras

Camera-centered coordinate systems are related by known rotation $\mathbf{R}$ and translation $\mathbf{T}$:

$$\mathbf{X}' = \mathbf{R} \mathbf{X} + \mathbf{T}$$
Excursion: Cross Product

\[ \vec{a} \times \vec{b} = \vec{c} \]
\[ \vec{a} \cdot \vec{c} = 0 \]
\[ \vec{b} \cdot \vec{c} = 0 \]

- Vector cross product takes two vectors and returns a third vector that’s perpendicular to both inputs.

- So here, \( \vec{c} \) is perpendicular to both \( \vec{a} \) and \( \vec{b} \), which means the dot product is 0.
From Geometry to Algebra

\[ X' = RX + T \]

\[ T \times X' = T \times RX + T \times T \]

Normal to the plane

\[ = T \times RX \]

\[ X' \cdot (T \times X') = X' \cdot (T \times RX) \]

\[ 0 = X' \cdot (T \times RX) \]

Slide credit: Kristen Grauman
Matrix Form of Cross Product

\[ \vec{a} \times \vec{b} = \vec{c} \]

\[ \vec{a} \cdot \vec{c} = 0 \]
\[ \vec{b} \cdot \vec{c} = 0 \]

“skew symmetric” matrix

\[ [a_x] = \begin{bmatrix}
0 & -a_z & a_y \\
a_z & 0 & -a_x \\
-a_y & a_x & 0
\end{bmatrix} \]

\[ \vec{a} \times \vec{b} = [a_x] \vec{b} \]
From Geometry to Algebra

\[ X' = RX + T \]

\[ T \times X' = T \times RX + T \times T \]

Normal to the plane

\[ 0 = X' \cdot (T \times RX) \]
**Essential Matrix**

\[
X' \cdot (T \times RX) = 0
\]

\[
X' \cdot (T_x \ RX) = 0
\]

Let \( E = T_x R \)

\[
X'^T E X = 0
\]

- This holds for the rays \( p \) and \( p' \) that are parallel to the camera-centered position vectors \( X \) and \( X' \), so we have: \[ p'^T E p = 0 \]

- \( E \) is called the **essential matrix**, which relates corresponding image points [Longuet-Higgins 1981]
Essential Matrix and Epipolar Lines

\[ p'^T E p = 0 \]

Epipolar constraint: if we observe point \( p \) in one image, then its position \( p' \) in second image must satisfy this equation.

\[ l' = E p \]

is the coordinate vector representing the epipolar line for point \( p \)

(i.e., the line is given by: \( l'^T x = 0 \))

\[ l = E^T p' \]

is the coordinate vector representing the epipolar line for point \( p' \)

Slide credit: Kristen Grauman
Essential Matrix: Properties

- Relates image of corresponding points in both cameras, given rotation and translation.
- Assuming intrinsic parameters are known

\[ E = T_x R \]
Essential Matrix Example: Parallel Cameras

For the parallel cameras, image of any point must lie on same horizontal line in each image plane.

\[ \mathbf{p}'^T \mathbf{E} \mathbf{p} = 0 \]
Essential Matrix Example: Parallel Cameras

For the parallel cameras, image of any point must lie on same horizontal line in each image plane.

\[
\begin{align*}
R &= I \\
T &= [-d, 0, 0]^T \\
E &= [Tx]R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & d \\ 0 & -d & 0 \end{pmatrix}
\end{align*}
\]

\[
p'^T E p = 0
\]

\[
\begin{bmatrix} x' & y' & f \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & d \\ 0 & -d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ f \end{bmatrix} = 0
\]

\[
\Leftrightarrow \begin{bmatrix} x' & y' & f \end{bmatrix} \begin{bmatrix} 0 \\ df \\ -dy \end{bmatrix} = 0
\]

\[
\Leftrightarrow y = y'
\]
More General Case

Image $I(x,y)$  Disparity map $D(x,y)$  Image $I'(x',y')$

$$(x', y') = (x + D(x, y), y)$$

What about when cameras’ optical axes are not parallel?

Slide credit: Kristen Grauman
Stereo Image Rectification

- In practice, it is convenient if image scanlines are the epipolar lines.

- Algorithm
  - Reproject image planes onto a common plane parallel to the line between optical centers
  - Pixel motion is horizontal after this transformation
  - Two homographies ($3 \times 3$ transforms), one for each input image reprojection

Stereo Image Rectification: Example

Source: Alyosha Efros
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   Applications
Stereo Reconstruction

- **Main Steps**
  - Calibrate cameras
  - Rectify images
  - **Compute disparity**
  - Estimate depth
Correspondence Problem

Multiple match hypotheses satisfy epipolar constraint, but which is correct?

Figure from Gee & Cipolla 1999
Dense Correspondence Search

- For each pixel in the first image
  - Find corresponding epipolar line in the right image
  - Examine all pixels on the epipolar line and pick the best match (e.g. SSD, correlation)
  - Triangulate the matches to get depth information

- This is easiest when epipolar lines are scanlines
  ⇒ Rectify images first

adapted from Svetlana Lazebnik, Li Zhang
Example: Window Search

- Data from University of Tsukuba

![Scene](image1)

![Ground truth](image2)

Slide credit: Kristen Grauman

B. Leibe
Example: Window Search

- Data from University of Tsukuba

Window-based matching
(best window size)

Ground truth

Slide credit: Kristen Grauman
Effect of Window Size

\[ W = 3 \]

\[ W = 20 \]

Want window large enough to have sufficient intensity variation, yet small enough to contain only pixels with about the same disparity.

Slide credit: Kristen Grauman

Figures from Li Zhang
Alternative: Sparse Correspondence Search

- Idea: Restrict search to sparse set of detected features
- Rather than pixel values (or lists of pixel values) use feature descriptor and an associated feature distance
- Still narrow search further by epipolar geometry

*What would make good features?*

Slide credit: Kristen Grauman
Dense vs. Sparse

• Sparse
  - Efficiency
  - Can have more reliable feature matches, less sensitive to illumination than raw pixels
  - But...
    - Have to know enough to pick good features
    - Sparse information

• Dense
  - Simple process
  - More depth estimates, can be useful for surface reconstruction
  - But...
    - Breaks down in textureless regions anyway
    - Raw pixel distances can be brittle
    - Not good with very different viewpoints
Difficulties in Similarity Constraint

Untextured surfaces

Occlusions

Slide credit: Kristen Grauman
Possible Sources of Error?

• Low-contrast / textureless image regions
• Occlusions
• Camera calibration errors
• Violations of *brightness constancy* (e.g., specular reflections)
• Large motions
Application: View Interpolation

Right Image

Slide credit: Svetlana Lazebnik
Application: View Interpolation

Left Image

Slide credit: Svetlana Lazebnik
Application: View Interpolation

Disparity

Slide credit: Svetlana Lazebnik
Application: View Interpolation

Slide credit: Svetlana Lazebnik
Application: Free-Viewpoint Video

http://www.liberovision.com

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Summary: Stereo Reconstruction

• Main Steps
  - Calibrate cameras
  - Rectify images
  - Compute disparity
  - Estimate depth

• So far, we have only considered calibrated cameras...

• Next lecture
  - Uncalibrated cameras
  - Camera parameters
  - Revisiting epipolar geometry
  - Robust fitting

Slide credit: Kristen Grauman
References and Further Reading

- Background information on epipolar geometry and stereopsis can be found in Chapters 10.1-10.2 and 11.1-11.3 of
  

- More detailed information (if you really want to implement 3D reconstruction algorithms) can be found in Chapters 9 and 10 of
  