Multi-Object Tracking II (02.06.2016)

Topics of This Lecture

- Recap: Track-Splitting Filter
  - Motivation
  - Ambiguities
- Multi-Hypothesis Tracking (MHT)
  - Basic idea
  - Hypothesis Generation
  - Assignment
  - Measurement Likelihood
  - Practical considerations

Let’s Formalize This

- Multi-Object Tracking problem
  - We represent a track by a state vector \( x \), e.g.,
    \[ x = \begin{bmatrix} x_1, y_1, v_x, v_y \end{bmatrix}^T \]
  - As the track evolves, we denote its state by the time index \( k \):
    \[ x^{(k)} = \begin{bmatrix} x_1^{(k)}, y_1^{(k)}, v_x^{(k)}, v_y^{(k)} \end{bmatrix}^T \]
  - At each time step, we get a set of observations (measurements)
    \[ Y^{(k)} = \left\{ y_1^{(k)}, \ldots, y_{Nt}^{(k)} \right\} \]
  - We now need to make the data association between tracks
    \( \{ x_1^{(k)}, \ldots, x_{Nt}^{(k)} \} \) and observations \( \{ y_1^{(k)}, \ldots, y_{Nt}^{(k)} \} \):
    \[ \hat{y}_j^{(k)} = \{ y_j^{(k)} \} \text{ is associated with } x_i^{(k)} \]

Recap: Motion Correspondence Ambiguities

1. Predictions may not be supported by measurements
   - Have the objects ceased to exist, or are they simply occluded?
2. There may be unexpected measurements
   - Newly visible objects, or just noise?
3. More than one measurement may match a prediction
   - Which measurement is the correct one (what about the others)?
4. A measurement may match to multiple predictions
   - Which object shall the measurement be assigned to?

Mahalanobis Distance

- Additional notation
  - Our KF state of track \( x_i \) is given by the prediction \( \tilde{x}_i^{(k)} \) and covariance \( \Sigma_{i}^{(k)} \)
  - We define the innovation that measurement \( y_j \) brings to track \( x_i \) at time \( k \) as
    \[ v_{ij}^{(k)} = (y_j - \tilde{x}_i^{(k)}) \]
  - With this, we can write the observation likelihood shortly as
    \[ p(y_j | x_i^{(k)}) \sim \exp \left\{ -\frac{1}{2} v_{ij}^{(k)} \Sigma_{ii}^{-1} v_{ij}^{(k)} \right\} \]
  - We define the ellipsoidal gating or validation volume as
    \[ V^{(k)}(\gamma) = \left\{ y | (y - x_i^{(k)})^T \Sigma_{ii}^{-1} (y - x_i^{(k)}) \leq \gamma \right\} \]
Recap: Track-Splitting Filter

• Idea
  - Instead of assigning the measurement that is currently closest, as in the NN algorithm, select the sequence of measurements that minimizes the total Mahalanobis distance over some interval!
  - Form a track tree for the different association decisions
  - Modified log-likelihood provides the merit of a particular node in the track tree.
  - Cost of calculating this is low, since most terms are needed anyway for the Kalman filter.

• Problem
  - The track tree grows exponentially, may generate a very large number of possible tracks that need to be maintained.

Recap: Pruning Strategies

• In order to keep this feasible, need to apply pruning
  - Deleting unlikely tracks
    - May be accomplished by comparing the modified log-likelihood $\lambda(k)$, which has a $\chi^2$ distribution with $kn$ degrees of freedom, with a threshold $\alpha$ (set according to $\chi^2$ distribution tables).
    - Problem for long tracks: modified log-likelihood gets dominated by old terms and responds very slowly to new ones.
      - Use sliding window or exponential decay term.
  - Merging track nodes
    - If the state estimates of two track nodes are similar, merge them.
    - E.g., if both tracks validate identical subsequent measurements.
  - Only keeping the most likely $N$ tracks
    - Rank tracks based on their modified log-likelihood.

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• Recap: Track-Splitting Filter
  - Motivation
  - Ambiguities
• Multi-Hypothesis Tracking (MHT)
  - Basic Idea
    - Hypothesis Generation
    - Assignment
    - Measurement Likelihood
    - Practical considerations

Multi-Hypothesis Tracking (MHT)

• Ideas
  - Again associate sequences of measurements.
  - Evaluate the probabilities of all association hypotheses.
  - For each sequence of measurements (a hypothesized track), a standard KF yields the state estimate and covariance

• Differences to Track-Splitting Filter
  - Instead of forming a track tree, keep a set of hypotheses that generate child hypotheses based on the associations.
  - After each hypothesis generation step, merge and prune the current hypothesis set to keep the approach feasible.
  - Integrate track generation into the assignment process.


Target vs. Measurement Orientation

• Target-oriented approaches
  - Evaluate the probability that a measurement belongs to an established target.
  - Measurement-oriented approaches
  - Evaluate the probability that an established target or a new target gave rise to a certain measurement sequence.
  - This makes it possible to include track initiation of new targets within the algorithmic framework.

• MHT
  - Measurement-oriented
  - Handles track initialization and termination

Challenge: Exponential Complexity

• Strategy
  - Generate all possible hypotheses and then depend on pruning these hypotheses to avoid the combinatorial explosion.
    - Exhaustive search
      - Tree data structures are used to keep this search efficient
  - Commonly used pruning techniques
    - Clustering to reduce the combinatorial complexity
    - Pruning of low-probability hypotheses
    - N-scan pruning
      - Select a single best hypothesis at frame $k$ and prune all tracks that do not share the predecessor track at the $(k-\lambda)^{th}$ frame.
      - Merging of similar hypotheses
Multi-Hypothesis Tracking (MHT)

- Ideas
  - Instead of forming a track tree, keep a set of hypotheses that generate child hypotheses based on the associations.
  - Enforce exclusion constraints between tracks and measurements in the assignment.
  - Integrate track generation into the assignment process.
  - After hypothesis generation, merge and prune the current hypothesis set.


Hypothesis Generation

- Formalization
  - Set of hypotheses at time $k$: $\Omega^{(k)} = \Omega^{(k-1)}$
  - This set is obtained from $\Omega^{(k-1)}$ and the latest set of measurements $Y^{(k)}$.
  - The set $\Omega^{(k)}$ is generated from $\Omega^{(k-1)}$ by performing all feasible associations between the old hypotheses and the new measurements $Y^{(k)}$.

- Feasible associations can be
  - A continuation of a previous track
  - A false alarm
  - A new target

Hypothesis Matrix

- Visualize feasible associations by a hypothesis matrix

  $\Theta = \begin{bmatrix}
  1 & 0 & 1 & 1 & Y_1 \\
  1 & 1 & 1 & 1 & Y_2 \\
  0 & 1 & 1 & 1 & Y_3 \\
  0 & 0 & 1 & 1 & Y_4 \\
  \end{bmatrix}$

- Interpretation
  - Columns represent tracked objects
  - Rows represent measurements
  - A non-zero element at matrix position $(i,j)$ denotes that measurement $y_i$ is contained in the validation region of track $x_j$.
  - Extra column $x_{fa}$ for association as false alarm.
  - Extra column $x_{nt}$ for association as new track.

Assignments

- Turning feasible associations into assignments
  - For each feasible association, we generate a new hypothesis.
  - Let $Z_j^{(k)}$ be the $j$-th hypothesis at time $k$ and $\Omega^{(k-1)}$ be the parent hypothesis from which $Z_j^{(k)}$ was derived.
  - Let $Z_j^{(k)}$ denote the set of assignments that gives rise to $Z_j^{(k)}$.
  - Assignments are again best visualized in matrix form

  $Z_j^{(k)} = \begin{bmatrix}
  y_1 & 0 & 0 & 1 & 0 \\
  y_2 & 1 & 0 & 0 & 0 \\
  y_3 & 0 & 1 & 0 & 0 \\
  y_4 & 0 & 0 & 1 & 0 \\
  \end{bmatrix}$

- Impose constraints
  - A measurement can originate from only one object.
    $\Rightarrow$ Any row has only a single non-zero value.
  - An object can have at most one associated measurement per time step.
    $\Rightarrow$ Any column has only a single non-zero value, except for $x_{fa}$, $x_{nt}$

Calculating Hypothesis Probabilities

- Probabilistic formulation
  - It is straightforward to enumerate all possible assignments.
  - However, we also need to calculate the probability of each child hypothesis.
  - This is done recursively:

  $p(T_j^{(k)}|Y^{(k)}) = p(Z_j^{(k)}|\Omega_0^{(k)}, Y^{(k)})$

  $= \frac{\prod_{i=1}^{N} p(Y_i|Z_i^{(k)}, \Omega_0^{(k)}, Y^{(k-1)}) p(Z_i^{(k)}|\Omega_0^{(k)}, Y^{(k-1)})}{\prod_{i=1}^{N} p(Y_i|\Omega_0^{(k)}, Y^{(k-1)}) p(Z_i^{(k)}|\Omega_0^{(k)}, Y^{(k-1)})}$

  $= \frac{\prod_{i=1}^{N} \text{Normalization factor} \cdot \text{Measurement likelihood} \cdot \text{Prob. of assignment set} \cdot \text{Prob. of parent}}{\prod_{i=1}^{N} \text{Normalization factor} \cdot \text{Measurement likelihood} \cdot \text{Prob. of assignment set} \cdot \text{Prob. of parent}}$

- Interpretation
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Measurement Likelihood

- Use KF prediction
  - Assume that a measurement $z^{(k)}_j$ associated to a track $x_j$ has a Gaussian pdf centered around the measurement prediction $\hat{z}^{(k)}_j$ with innovation covariance $\sum_j^{(k)}$.
  - Further assume that the pdf of a measurement belonging to a new track or false alarm is uniform in the observation volume $W$ (the sensor’s field-of-view) with probability $W^{-1}$.
  - Thus, the measurement likelihood can be expressed as
  \[
  p(z^{(k)}_j | \omega^{(k-1)}_j) = \prod_{j=1}^{M} \mathcal{N}(z^{(k)}_j | \hat{z}^{(k)}_j, \sum_j^{(k)} W^{-1} - 1).
  \]

Probability of an Assignment Set

- Composed of three terms
  1. Probability of the number of tracks $N_{\text{det}}, N_{\text{fal}}, N_{\text{new}}$
     - Assumption 1: $N_{\text{det}}$ follows a binomial distribution
     - Assumption 2: $N_{\text{fal}}$ and $N_{\text{new}}$ both follow a Poisson distribution with expected number of events $\lambda_{\text{fal}}W$ and $\lambda_{\text{new}}W$.
  \[
  p(N_{\text{det}}, N_{\text{fal}}, N_{\text{new}}|\omega^{(k-1)}) = \binom{N}{N_{\text{det}}} p_{\text{det}}(1-p_{\text{det}})^{(N-N_{\text{det}})}
  \]
  where $N$ is the number of tracks in the parent hypothesis.
  - Assumption 2: $N_{\text{fal}}$ and $N_{\text{new}}$ both follow a Poisson distribution with expected number of events $\lambda_{\text{fal}}W$ and $\lambda_{\text{new}}W$.
  \[
  p(N_{\text{det}}, N_{\text{fal}}, N_{\text{new}}|\omega^{(k-1)}) = \binom{N}{N_{\text{det}}} p_{\text{det}}(1-p_{\text{det}})^{(N-N_{\text{det}})}
  \]

- Probability of a specific assignment of measurements
  - Such that $M_k = N_{\text{det}} + N_{\text{fal}} + N_{\text{new}}$ holds.
  - This is determined as $1$ over the number of combinations $\binom{M_k}{N_{\text{det}}}, \binom{M_k-N_{\text{det}}}{N_{\text{fal}}}, \binom{N_{\text{new}}}{N_{\text{new}}}$.

- Probability of a specific assignment of tracks
  - Given that a track can be either detected or not detected.
  - This is determined as $1$ over the number of assignments $\binom{N}{N_{\text{det}}} (N-N_{\text{det}})!

\Rightarrow$ When combining the different parts, many terms cancel out!

Measurement Likelihood

- Combining all the different parts
  - Nice property: many terms cancel out!
  - (Derivation left as exercise)
  \[
  \Rightarrow \text{The final probability } p(z^{(k)}_j | \omega^{(k-1)}) \text{ can be computed in a very simple form.}
  \]
  - This was the main contribution by Reid and it is one of the reasons why the approach is still popular.

- Practical issues
  - Exponential complexity remains
  - Heuristic pruning strategies must be applied to contain the growth of the hypothesis set.
  - E.g., dividing hypotheses into spatially disjoint clusters.

Laser-based Leg Tracking using MHT


Laser-based People Tracking using MHT

Multi Hypothesis Tracking of People
Matthias Luber, Gian Diego Tipaldi and Kai O. Arras

Laser Based People Tracking using MHT
Data from all finding, laboratory Results projected into MuDoDA.

Social Robotics Laboratory
References and Further Reading

- A good tutorial on Data Association

- Reid's original MHT paper