Computer Vision II - Lecture 12

Multi-Object Tracking II

01.07.2014

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Course Outline

• Single-Object Tracking
• Bayesian Filtering
  ➢ Kalman filters
  ➢ Particle filters
  ➢ Case studies
• Multi-Object Tracking
  ➢ Introduction
  ➢ MHT, JPDAF
  ➢ Network Flow Optimization
• Articulated Tracking
Topics of This Lecture

• Recap: Track-Splitting Filter
  - Motivation
  - Ambiguities

• Multi-Hypothesis Tracking (MHT)
  - Basic idea
  - Hypothesis Generation
  - Assignment
  - Measurement Likelihood
  - Practical considerations
Recap: Motion Correspondence Ambiguities

1. Predictions may not be supported by measurements
   - Have the objects ceased to exist, or are they simply occluded?

2. There may be unexpected measurements
   - Newly visible objects, or just noise?

3. More than one measurement may match a prediction
   - Which measurement is the correct one (what about the others)?

4. A measurement may match to multiple predictions
   - Which object shall the measurement be assigned to?
Let’s Formalize This

- **Multi-Object Tracking problem**
  - We represent a track by a state vector $x$, e.g.,
    \[ x = [x, y, u_x, u_y]^T \]
  - As the track evolves, we denote its state by the time index $k$:
    \[ x^{(k)} = [x^{(k)}, y^{(k)}, u_x^{(k)}, u_y^{(k)}]^T \]
  - At each time step, we get a set of observations (measurements)
    \[ Y^{(k)} = \{ y_1^{(k)}, \ldots, y_{M_k}^{(k)} \} \]
  - We now need to make the data association between tracks
    \[ \{ x_1^{(k)}, \ldots, x_{N_k}^{(k)} \} \text{ and observations } \{ y_1^{(k)}, \ldots, y_{M_k}^{(k)} \} : \]
    \[ z_i^{(k)} = j \text{ iff } y_j^{(k)} \text{ is associated with } x_i^{(k)} \]

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Recap: Reducing Ambiguities

- **Gating**
  - Only consider measurements within a certain area around the predicted location.
  - Large gain in efficiency, since only a small region needs to be searched

- **Nearest-Neighbor Filter**
  - Among the candidates in the gating region, only take the one closest to the prediction $x_p$
  
  $$
  z_l^{(k)} = \arg \min_j (x_p^{(k)} - y_j^{(k)})^T (x_p^{(k)} - y_j^{(k)})
  $$
  - Better: the one most likely under a Gaussian prediction model
  
  $$
  z_l^{(k)} = \arg \max_j \mathcal{N}(y_j^{(k)}; x_p^{(k)}, \Sigma_p^{(k)})
  $$

  which is equivalent to taking the Mahalanobis distance

  $$
  z_l = \arg \min_j (x_p, l - y_j)^T \Sigma_{p,l}^{-1} (x_p, l - y_j)
  $$

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Recap: Track-Splitting Filter

• Idea
  - Instead of assigning the measurement that is currently closest, as in the NN algorithm, select the *sequence* of measurements that minimizes the *total* Mahalanobis distance over some interval!
  - Form a track tree for the different association decisions
  - Modified log-likelihood provides the merit of a particular node in the track tree.
  - Cost of calculating this is low, since most terms are needed anyway for the Kalman filter.

• Problem
  - The track tree grows exponentially, may generate a very large number of possible tracks that need to be maintained.
Recap: Pruning Strategies

- In order to keep this feasible, need to apply pruning
  - Deleting unlikely tracks
    - May be accomplished by comparing the modified log-likelihood \( \lambda(k) \), which has a \( \chi^2 \) distribution with \( kn_z \) degrees of freedom, with a threshold \( \alpha \) (set according to \( \chi^2 \) distribution tables).
    - Problem for long tracks: modified log-likelihood gets dominated by old terms and responds very slowly to new ones.
      \( \Rightarrow \) Use sliding window or exponential decay term.
  - Merging track nodes
    - If the state estimates of two track nodes are similar, merge them.
    - E.g., if both tracks validate identical subsequent measurements.
  - Only keeping the most likely \( N \) tracks
    - Rank tracks based on their modified log-likelihood.
Summary: Track-Splitting Filter

- **Properties**
  - Very old algorithm
  - Improvement over NN assignment.
  - Assignment decisions are delayed until more information is available.

- **Many problems remain**
  - Exponential complexity, heuristic pruning needed.
  - Merging of track nodes is necessary, because tracks may share measurements, which is physically unrealistic.
  - Would need to add exclusion constraints such that each measurement may only belong to a single track.
  - Impossible in this framework...
Topics of This Lecture

• Recap: Track-Splitting Filter
  ➢ Motivation
  ➢ Ambiguities

• Multi-Hypothesis Tracking (MHT)
  ➢ Basic idea
  ➢ Hypothesis Generation
  ➢ Assignment
  ➢ Measurement Likelihood
  ➢ Practical considerations
Multi-Hypothesis Tracking (MHT)

• Ideas
  - Again associate sequences of measurements.
  - Evaluate the probabilities of all association hypotheses.
  - For each sequence of measurements (a hypothesized track), a standard KF yields the state estimate and covariance.

• Differences to Track-Splitting Filter
  - Instead of forming a track tree, keep a set of hypotheses that generate child hypotheses based on the associations.
  - After each hypothesis generation step, merge and prune the current hypothesis set to keep the approach feasible.
  - Integrate track generation into the assignment process.

Target vs. Measurement Orientation

- **Target-oriented approaches**
  - Evaluate the probability that a measurement belongs to an established target.

- **Measurement-oriented approaches**
  - Evaluate the probability that an established target or a new target gave rise to a certain measurement sequence.
  - This makes it possible to include track initiation of new targets within the algorithmic framework.

- **MHT**
  - Measurement-oriented
  - Handles track initialization and termination
Challenge: Exponential Complexity

• Strategy
  - Generate all possible hypotheses and then depend on pruning these hypotheses to avoid the combinatorial explosion.  
  ⇒ Exhaustive search
  - Tree data structures are used to keep this search efficient

• Commonly used pruning techniques
  - Clustering to reduce the combinatorial complexity
  - Pruning of low-probability hypotheses
  - N-scan pruning
  - Merging of similar hypotheses
MHT Outline

Hypotheses at time \( k-1 \)
\( \Omega^{k-1} \)

Delay

Hypotheses at time \( k \)
\( \Omega^k \)

Hypothesis Management (pruning, merging)

Hypothesis Generation

For Each Hypothesis \( \Theta_i^{k-1} \)
Generate Predictions

\( \hat{X}(k) \)
Predicted Features

Matching

Hypothesis Matrix

\( Y(k) \)
Observed Features

Feature Extraction

Raw Sensor Data

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Hypothesis Generation

• Formalization
  - Set of hypotheses at time $k$: \( \Omega^{(k)} = \left\{ \Omega_j^{(k)} \right\} \)
  - This set is obtained from \( \Omega^{(k-1)} \) and the latest set of measurements
    \[
    Y^{(k)} = \left\{ y_1^{(k)}, \ldots, y_{M_k}^{(k)} \right\}
    \]
  - The set \( \Omega^{(k)} \) is generated from \( \Omega^{(k-1)} \) by performing all feasible associations between the old hypotheses and the new measurements \( Y^{(k)} \).

• Feasible associations can be
  - A continuation of a previous track
  - A false alarm
  - A new target
Hypothesis Matrix

• Visualize feasible associations by a hypothesis matrix

\[ \Theta = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \]

\[ \begin{array}{c} y_1 \\ y_2 \\ y_3 \\ y_4 \end{array} \]

• Interpretation
  
  ➢ Columns represent tracked objects
  ➢ Rows represent measurements
  ➢ A non-zero element at matrix position \((i,j)\) denotes that measurement \(y_i\) is contained in the validation region of track \(x_j\).
  ➢ Extra column \(x_{fa}\) for association as *false alarm*.
  ➢ Extra column \(x_{nt}\) for association as *new track*. 

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Assignments

• Turning feasible associations into assignments
  ➢ For each feasible association, we generate a new hypothesis.
  ➢ Let $\Omega_j^{(k)}$ be the $j$-th hypothesis at time $k$ and $\Omega_{p(j)}^{(k-1)}$ be the parent hypothesis from which $\Omega_j^{(k)}$ was derived.
  ➢ Let $Z_j^{(k)}$ denote the set of assignments that gives rise to $\Omega_j^{(k)}$.
  ➢ Assignments are again best visualized in matrix form

<table>
<thead>
<tr>
<th>$Z_j$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_{fa}$</th>
<th>$x_{nt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_1$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$y_2$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$y_3$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$y_4$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
Assignments

<table>
<thead>
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</tr>
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<td>0</td>
</tr>
<tr>
<td>$y_3$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$y_4$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

- **Impose constraints**
  - A measurement can originate from only one object.
    - \( \Rightarrow \) Any row has only a single non-zero value.
  - An object can have at most one associated measurement per time step.
    - \( \Rightarrow \) Any column has only a single non-zero value, except for $x_{fa}$, $x_{nt}$
Calculating Hypothesis Probabilities

- Probabilistic formulation
  - It is straightforward to enumerate all possible assignments.
  - However, we also need to calculate the probability of each child hypothesis.
  - This is done recursively:

\[
p(\Omega_{j}^{(k)}|\mathbf{Y}^{(k)}) = p(Z_{j}^{(k)}, \Omega_{p(j)}^{(k-1)}|\mathbf{Y}^{(k)})
\]

Bayes
\[
= \eta p(\mathbf{Y}^{(k)}|Z_{j}^{(k)}, \Omega_{p(j)}^{(k-1)}) p(Z_{j}^{(k)}, \Omega_{p(j)}^{(k-1)})
\]

\[
= \eta p(\mathbf{Y}^{(k)}|Z_{j}^{(k)}, \Omega_{p(j)}^{(k-1)}) p(Z_{j}^{(k)}|\Omega_{p(j)}^{(k-1)}) p(\Omega_{p(j)}^{(k-1)})
\]
Measurement Likelihood

- Use KF prediction
  - Assume that a measurement $y_i^{(k)}$ associated to a track $x_j$ has a Gaussian pdf centered around the measurement prediction $\hat{x}_j^{(k)}$ with innovation covariance $\hat{\Sigma}_j^{(k)}$.
  - Further assume that the pdf of a measurement belonging to a new track or false alarm is uniform in the observation volume $W$ (the sensor’s field-of-view) with probability $W^{-1}$.
  - Thus, the measurement likelihood can be expressed as

\[
p \left( Y^{(k)} \mid Z_j^{(k)}, \Omega_{p(j)}^{(k-1)} \right) = \prod_{i=1}^{M_k} \mathcal{N} \left( y_i^{(k)} ; \hat{x}_j, \hat{\Sigma}_j^{(k)} \right)^{\delta_i} W^{-(1-\delta_i)}
\]

\[
= W^{-(N_{fal}+N_{new})} \prod_{i=1}^{M_k} \mathcal{N} \left( y_i^{(k)} ; \hat{x}_j, \hat{\Sigma}_j^{(k)} \right)^{\delta_i}
\]

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Probability of an Assignment Set

\[ p(Z_j^{(k)} | \Omega_{p(j)}^{(k-1)}) \]

- Composed of three terms
  1. Probability of the number of tracks \( N_{det}, N_{fal}, N_{new} \)
     - Assumption 1: \( N_{det} \) follows a binomial distribution

\[
p(N_{det} | \Omega_{p(j)}^{(k-1)}) = \binom{N}{N_{det}} p_{det}^{N_{det}} (1 - p_{det})^{(N-N_{det})}
\]

where \( N \) is the number of tracks in the parent hypothesis

- Assumption 2: \( N_{fal} \) and \( N_{new} \) both follow a Poisson distribution with expected number of events \( \lambda_{fal} W \) and \( \lambda_{new} W \)

\[
p(N_{det}, N_{fal}, N_{new} | \Omega_{p(j)}^{(k-1)}) = \binom{N}{N_{det}} p_{det}^{N_{det}} (1 - p_{det})^{(N-N_{det})} \cdot \mu(N_{fal}; \lambda_{fal} W) \cdot \mu(N_{new}; \lambda_{new} W)
\]
Probability of an Assignment Set

2. Probability of a specific assignment of measurements
   - Such that $M_k = N_{det} + N_{fal} + N_{new}$ holds.
   - This is determined as $1$ over the number of combinations
     \[
     \binom{M_k}{N_{det}} \binom{M_k - N_{det}}{N_{fal}} \binom{M_k - N_{det} - N_{fal}}{N_{new}}
     \]

3. Probability of a specific assignment of tracks
   - Given that a track can be either detected or not detected.
   - This is determined as $1$ over the number of assignments
     \[
     \frac{N!}{(N - N_{det})!} \binom{N - N_{det}}{N_{det}}
     \]
Measurement Likelihood

- Combining all the different parts
  - Nice property: many terms cancel out!
  - (Derivation left as exercise)

\[ p \left( \Omega_j^{(k)} | Y^{(k)} \right) \] can be computed in a very simple form.

- This was the main contribution by Reid and it is one of the reasons why the approach is still popular.

- Practical issues
  - Exponential complexity remains
  - Heuristic pruning strategies must be applied to contain the growth of the hypothesis set.
  - E.g., dividing hypotheses into spatially disjoint clusters.
References and Further Reading

• A good tutorial on Data Association

• Reid’s original MHT paper